University of Toronto

Term Test 1

Date - Oct 14, 2009

Duration: 1.5 hrs

ECE512 — Analog Signal Processing
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Both programmable and non-programmable calculators allowed.
2. Equation sheet on last page of this test.
3. **Only tests written in pen will be considered for a re-mark.**
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: ANSWERS

First Name: ____________

Student #: ____________

Question	Mark
1	
2	
3	
4	
5	
Total	

(max grade = 25)

[5] Question 1: Answer the True [T] or False [F] questions below by **circling** the correct answer. Each correct answer is worth 0.5 marks.

T F A filter with a linear phase response in it's passband has a constant group delay in it's passband.

T F The units of group delay is "radians".

T F A complex number of "1+j" has a magnitude of $\sqrt{2}$ and a phase of $\pi/2$ radians

T F $|H(j\omega)|^2$ is always an odd function

T F $|H(j\omega)|^2$ is always an even function

T F A passive RC circuit has all it's poles on the real axis

T F The numerator of an all-pole lowpass filter is a constant.

T F If zeros at ∞ are included, the number of zeros equals the numbers of poles of $H(s)$

T F An integrator is equivalent to a first-order all-pole filter with a pole at $s = 0$

T F Integrators rather than differentiators are generally used to build filters because integrators are smaller.

[5] Question 2:

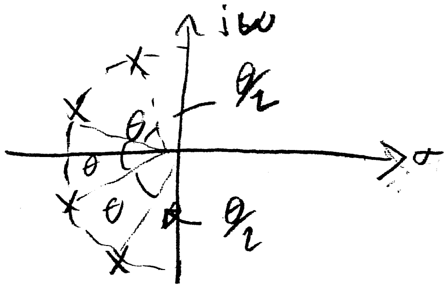
Find $H(s)$ for a 4'th order Butterworth filter having a passband ripple of 2 dB from dc to 1 rad/s and a dc gain of 1. Write $H(s)$ in root form using ω_0 and Q notation in describing each pair of poles.

$$A_{MAX} = 20\log\sqrt{1+\epsilon^2} = 10\log(1+\epsilon^2)$$

$$1+\epsilon^2 = 10^{\frac{2}{10}} = 1.5849 \Rightarrow \epsilon^2 = 0.5849$$

$$\epsilon = 0.7648$$

$$\omega_{01} = \omega_{02} = \left(\frac{1}{\epsilon}\right)^{\frac{1}{N}} = \left(\frac{1}{0.7648}\right)^{\frac{1}{4}} = 1.0693$$



$$\theta = \frac{\pi}{4}\text{ RAD} = 45^\circ$$

$$\frac{\omega_{01}}{2Q_1} = \omega_{01}\cos(90^\circ - 22.5^\circ) \Rightarrow Q_1 = \frac{1}{2\cos(67.5^\circ)}$$

$$Q_1 = 1.307$$

$$\frac{\omega_{02}}{2Q_2} = \omega_{02}\cos(90^\circ - 67.5^\circ) \Rightarrow Q_2 = \frac{1}{2\cos(22.5^\circ)}$$

$$Q_2 = 0.5412$$

$$H(s) = \frac{k}{\left(s^2 + \frac{\omega_{01}}{Q_1}s + \omega_{01}^2\right)\left(s^2 + \frac{\omega_{02}}{Q_2}s + \omega_{02}^2\right)}$$

$$H(0) = 1 \Rightarrow \frac{k}{\omega_{01}^2\,\omega_{02}^2} = 1 \Rightarrow k = 1.0693^4 = 1.3074$$

$$H(s) = \frac{1.3074}{\left(s^2 + \left(\frac{1.0693}{1.307}\right)s + 1.0693^2\right)\left(s^2 + \left(\frac{1.0693}{0.5412}\right)s + 1.0693^2\right)}$$

[5] Question 3:

Find $H(s)$ in polynomial form for a high-pass notch (HPN) biquad filter having a gain of 1 at high frequency, zeros at 50 Hz, pole-frequencies of 110 Hz and pole-Q of $1/\sqrt{2}$. Also find the dc gain of this filter.

$$\omega_0 = 2\pi \times 110 = 691.15 \text{ RAD/S}$$

$$Q = \frac{1}{\sqrt{2}} = 0.7071$$

$$z_1 = 2\pi \times 50 = 314.16 \text{ RAD/S}$$

$$H(s) = \frac{k(s^2 + z_1^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(s) = \frac{k(s^2 + 9.870 \times 10^4)}{s^2 + 977.4s + 4.777 \times 10^5}$$

$$H(\infty) = 1 \Rightarrow k = 1$$

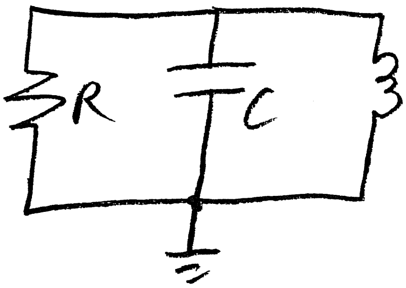
$$\boxed{H(s) = \frac{s^2 + 9.870 \times 10^4}{s^2 + 977.4s + 4.777 \times 10^5}}$$

$$H(0) = \frac{9.870 \times 10^4}{4.777 \times 10^5} = \underline{\underline{0.2066}}$$

$$H\_dB(0) = 20 \log(0.2066) = \underline{\underline{-13.7 \text{ dB}}}$$

[5] Question 4:

Using only resistors, a capacitor and an inductor, find a circuit that realizes a bandpass filter with a center frequency of 100 MHz, a Q of 10 and a centre frequency gain of 1/3. Let the capacitor have a value of 10 pF. Assume the input is an ideal voltage source and the output is a voltage node in the circuit.



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

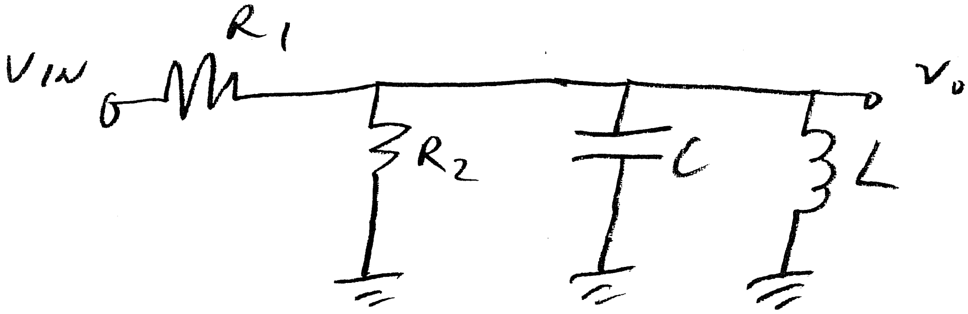
$$Q = \omega_0 C R$$

$C = 10\,pF \qquad \omega_0 = 2\pi \times 100 \times 10^6 = 6.28 \times 10^8$

$$Q = 10$$

$$L = \frac{1}{\omega_0^2 C} = 2.533 \times 10^{-7}\ H$$

$$R = \frac{Q}{\omega_0 C} = 1.592\,k\Omega$$



$C = \underline{10\,pF}$

$L = 2.533 \times 10^{-7}\ H$

$\underline{\underline{= 0.2533\,\mu H}}$

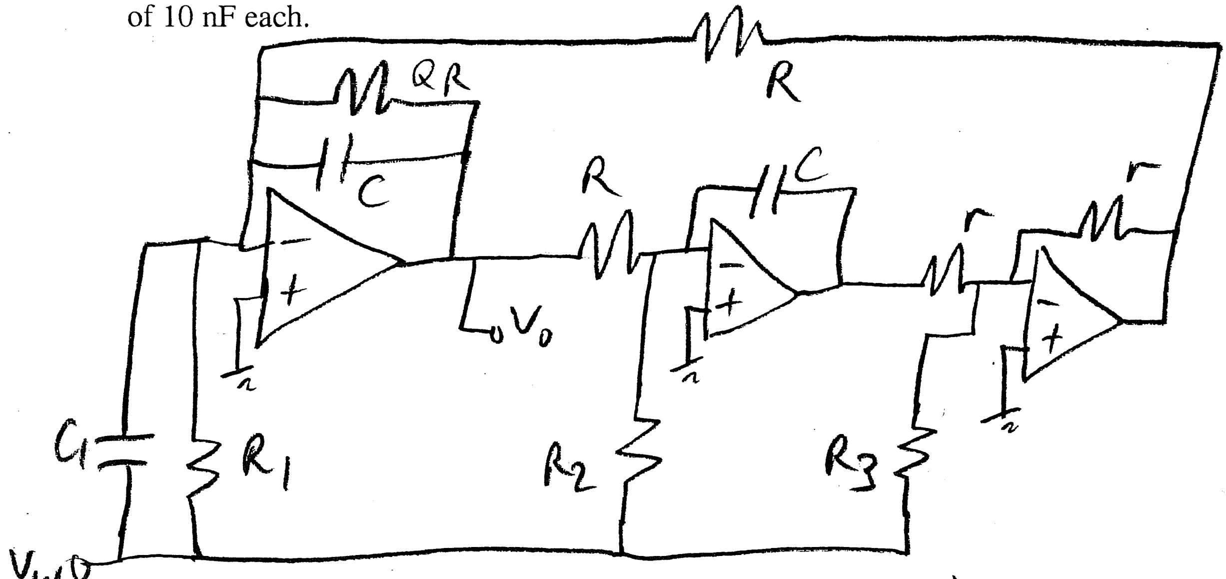
$$\frac{R_2}{R_2 + R_1} = \frac{1}{3} \quad (1) \qquad R_1 \parallel R_2 = 1.592k = \frac{R_1 R_2}{R_1 + R_2} \quad (2)$$

$$(2) \div (1) \Rightarrow R_1 = \frac{1.592k}{(1/3)} = \underline{\underline{4.775\ k\Omega}}$$

$$R_2 = \left(\frac{1}{1.592} - \frac{1}{4.775}\right)^{-1} \times 10^3 = \underline{\underline{2.391\,k\Omega}}$$

[5] Question 5:

Design a Tow-Thomas active-RC lowpass notch filter with a dc gain of -1, a notch frequency of 50 kHz, a pole frequency of 5 kHz and a pole-Q of 5. Use integrating capacitors of 10 nF each.



$\underline{\underline{C = 10\ nF}}$ $\qquad RC = \dfrac{1}{\omega_0} = \dfrac{1}{2\pi \times 5k}$

$$R = \frac{1}{(2\pi \times 5k)(10e-9)} = \underline{\underline{3.183\,k\Omega}}$$

$$QR = 5 \times R = \underline{\underline{15.92\,k\Omega}}$$

$\underline{\underline{r = 10\,k}}$ (ARBITRARY CHOICE)

$\underline{\underline{R_1 = R_3 = \infty}}$ (TO MAKE NOTCH)

$$H(s) = \frac{-\left(s^2\left(\frac{C_1}{C}\right) + \frac{1}{RR_2C^2}\right)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{1}{RR_2C^2} = \omega_0^2$$

$$R_2 = \frac{1}{\omega_0^2 RC^2} = \underline{\underline{3.183\,k\Omega}}$$

$\left[\begin{array}{l}\text{NOTE } k_1 s^2 + k_2 \text{ HAS ZERO} \\ \text{AT } s^2 = -\dfrac{k_2}{k_1} \Rightarrow\end{array}\right] \dfrac{\left(\frac{1}{RR_2C^2}\right)}{\left(C_1/C\right)} = (2\pi \times 50e3)^2$

$$C_1 = \frac{1}{RR_2C(2\pi \times 50e3)^2} = \underline{\underline{0.1\,nF}}$$

(blank sheet for scratch calculations)

ECE512 Analog Signal Processing Equation Sheet

$h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$.

$$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)} \; ; \; T_d(\omega) = -\frac{d\phi(\omega)}{d\omega} \; ; \; |H(j\omega)|_{dB} = 20\log|H(j\omega)| \; ; \; H(s) = \frac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$$

$$|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$$; General Lowpass $|H(j\omega)|^2 = \frac{A_0^2}{1 + F(\omega^2)}$

Butterworth: $F(\omega^2) = \varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}$; $A_{max} = 20\log\sqrt{1 + \varepsilon^2}$; $A_{min} \le 10\log\left[1 + \varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2N}\right]$;

Poles lie on circle of radius $\omega_p\left(\frac{1}{\varepsilon}\right)^{\frac{1}{N}}$ spaced apart by $\frac{\pi}{N}$ with first half angle from $j\omega$ axis

Chebyshev: (for $\omega_p = 1$) $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$

$C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$; $C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$;

$A_{max} = 20\log\sqrt{1 + \varepsilon^2}$; $A_{min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s / \omega_p))]$

Second-order polynomial: $s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2$; for $Q > 0.5$, poles complex at radius ω_0 and real part is $-\frac{\omega_0}{2Q}$

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{max} = \omega_0\sqrt{1 - 1/(2Q^2)}$

Bandpass: for complex poles, peak occurs at ω_0 and has 3dB bandwidth of ω_0 / Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$

KHN Biquad: $RC = \frac{1}{\omega_0}$; $\left(\frac{R_1 || R_2}{(R_1 || R_2) + R_3}\right) = \frac{1}{Q}\left(\frac{R_2 || R_3}{(R_2 || R_3) + R_1}\right) =$,

Tow-Thomas Biquad: $RC = \frac{1}{\omega_0}$; damping resistor is QR ; numerator is $-s^2\left(\frac{C_1}{C}\right) - s\left(\frac{1}{C}\right)\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) - \frac{1}{RR_2C^2}$