

University of Toronto

Term Test 1

Date - Oct 14, 2009

Duration: 1.5 hrs

ECE512 — Analog Signal Processing

Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS USING BACKS IF NECESSARY

1. Both programmable and non-programmable calculators allowed.
 2. Equation sheet on last page of this test.
 3. **Only tests written in pen will be considered for a re-mark.**
 4. Grading indicated by []. Attempt all questions since a blank answer will certainly get 0.
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Last Name: ANSWERS

First Name: _____

Student #: _____

Question	Mark
1	
2	
3	
4	
5	
Total	

(max grade = 25)

[5] **Question 1:** Answer the True [T] or False [F] questions below by **circling** the correct answer. Each correct answer is worth 0.5 marks.

- T F A filter with a linear phase response in its passband has a constant group delay in its passband.
- T F The units of group delay is “radians”.
- T F A complex number of “1+j” has a magnitude of $\sqrt{2}$ and a phase of $\pi/2$ radians
- T F $|H(j\omega)|^2$ is always an odd function
- T F $|H(j\omega)|^2$ is always an even function
- T F A passive RC circuit has all its poles on the real axis
- T F The numerator of an all-pole lowpass filter is a constant.
- T F If zeros at ∞ are included, the number of zeros equals the numbers of poles of $H(s)$
- T F An integrator is equivalent to a first-order all-pole filter with a pole at $s = 0$
- T F Integrators rather than differentiators are generally used to build filters because integrators are smaller.

[5] Question 2:

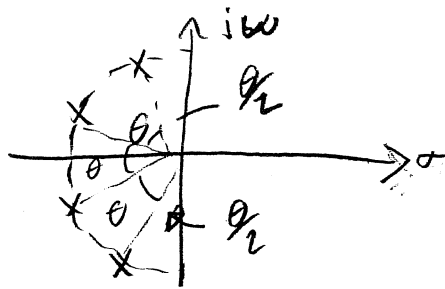
Find $H(s)$ for a 4'th order Butterworth filter having a passband ripple of 2 dB from dc to 1 rad/s and a dc gain of 1. Write $H(s)$ in root form using ω_0 and Q notation in describing each pair of poles.

$$A_{MAX} = 20 \log \sqrt{1+\epsilon^2} = 10 \log(1+\epsilon^2)$$

$$1+\epsilon^2 = 10^{\frac{2}{10}} = 1.5849 \Rightarrow \epsilon^2 = 0.5849$$

$$\epsilon = 0.7648$$

$$\omega_{01} = \omega_{02} = \left(\frac{1}{\epsilon}\right)^{\frac{1}{N}} = \left(\frac{1}{0.7648}\right)^{\frac{1}{4}} = 1.0693$$



$$\theta = \frac{\pi}{4} \text{ RAD} = 45^\circ$$

$$\frac{\omega_{01}}{2Q_1} = \omega_{01} \cos(90^\circ - 22.5^\circ) \Rightarrow Q_1 = \frac{1}{2 \cos(67.5^\circ)}$$

$$Q_1 = 1.307$$

$$\frac{\omega_{02}}{2Q_2} = \omega_{02} \cos(90^\circ - 67.5^\circ) \Rightarrow Q_2 = \frac{1}{2 \cos(22.5^\circ)}$$

$$Q_2 = 0.5412$$

$$H(s) = \frac{k}{\left(s^2 + \frac{\omega_{01}}{Q_1}s + \omega_{01}^2\right) \left(s^2 + \frac{\omega_{02}}{Q_2}s + \omega_{02}^2\right)}$$

$$H(0) = 1 \Rightarrow \frac{k}{\omega_{01}^2 \omega_{02}^2} = 1 \Rightarrow k = 1.0693^4 = 1.3074$$

$$H(s) = \frac{1.3074}{\left(s^2 + \left(\frac{1.0693}{1.307}\right)s + 1.0693^2\right) \left(s^2 + \left(\frac{1.0693}{0.5412}\right)s + 1.0693^2\right)}$$

[5] Question 3:

Find $H(s)$ in polynomial form for a high-pass notch (HPN) biquad filter having a gain of 1 at high frequency, zeros at 50 Hz, pole-frequencies of 110 Hz and pole-Q of $1/\sqrt{2}$. Also find the dc gain of this filter.

$$\omega_0 = 2\pi \times 110 = 691.15 \text{ RAD/S}$$

$$Q = \frac{1}{\sqrt{2}} = 0.7071$$

$$z_1 = 2\pi \times 50 = 314.16 \text{ RAD/S}$$

$$H(s) = \frac{k(s^2 + z_1^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(s) = \frac{k(s^2 + 9.870 \times 10^4)}{s^2 + 977.4s + 4.777 \times 10^5}$$

$$H(\infty) = 1 \Rightarrow k = 1$$

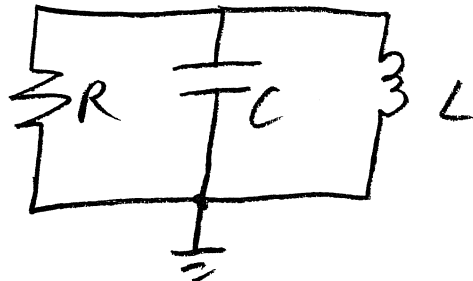
$$H(s) = \frac{s^2 + 9.870 \times 10^4}{s^2 + 977.4s + 4.777 \times 10^5}$$

$$H(0) = \frac{9.870 \times 10^4}{4.777 \times 10^5} = \underline{\underline{0.2066}}$$

$$H_{dB}(0) = 20 \log(0.2066) = \underline{\underline{-13.7 \text{ dB}}}$$

[5] Question 4:

Using only resistors, a capacitor and an inductor, find a circuit that realizes a bandpass filter with a center frequency of 100 MHz, a Q of 10 and a centre frequency gain of 1/3. Let the capacitor have a value of 10 pF. Assume the input is an ideal voltage source and the output is a voltage node in the circuit.



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

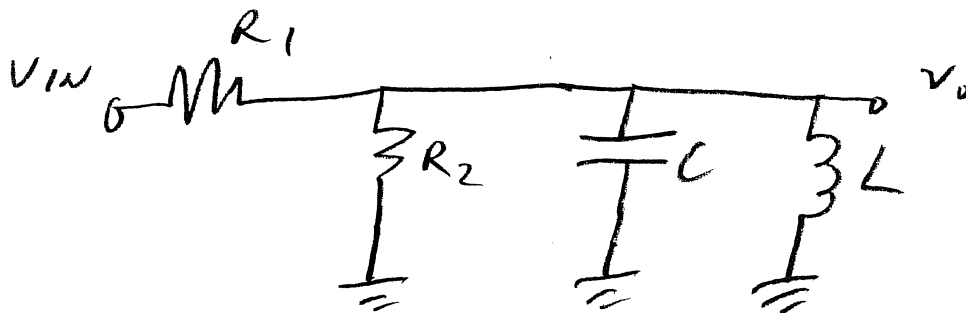
$$Q = \omega_0 CR$$

$$C = 10 \text{ pF} \quad \omega_0 = 2\pi \times 100 \times 10^6 = 6.28 \times 10^8$$

$$Q = 10$$

$$L = \frac{1}{\omega_0^2 C} = 2.533 \times 10^{-7} \text{ H}$$

$$R = \frac{Q}{\omega_0 C} = 1.592 \text{ k}\Omega$$



$$C = 10 \text{ pF}$$

$$L = 2.533 \times 10^{-7} \text{ H}$$

$$= \underline{\underline{0.2533 \mu\text{H}}}$$

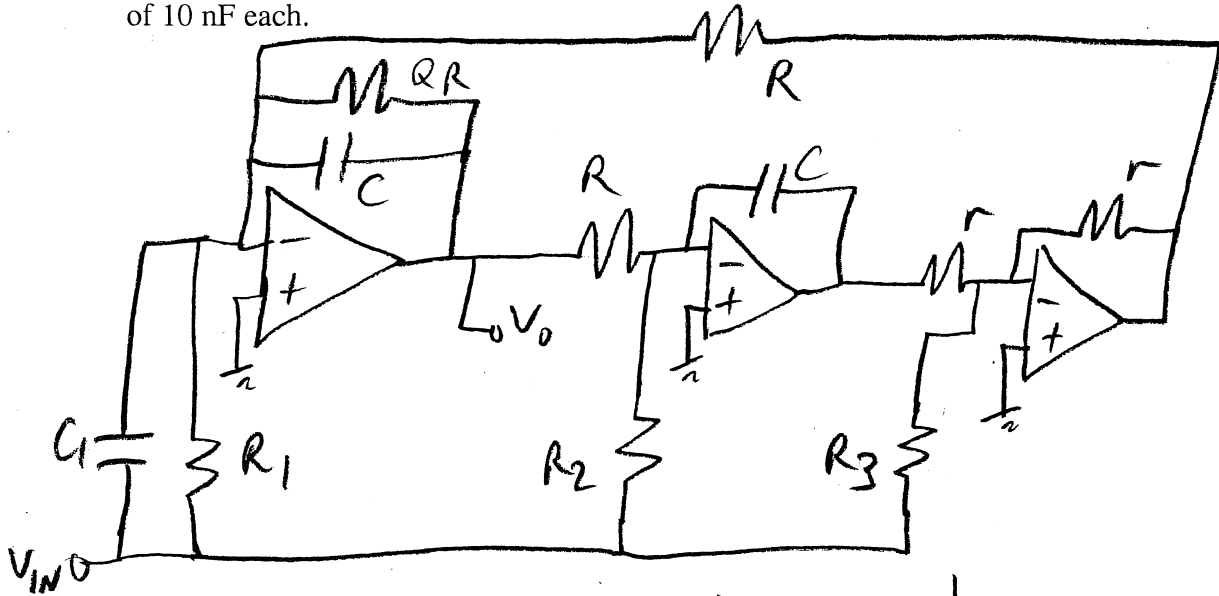
$$\frac{R_2}{R_2 + R_1} = \frac{1}{3} \quad \textcircled{1} \quad R_1 \parallel R_2 = 1.592 \text{ k} = \frac{R_1 R_2}{R_1 + R_2} \quad \textcircled{2}$$

$$\textcircled{2} \div \textcircled{1} \Rightarrow R_1 = \frac{1.592 \text{ k}}{(\frac{1}{3})} = \underline{\underline{4.775 \text{ k}\Omega}}$$

$$R_2 = \left(\frac{1}{1.592} - \frac{1}{4.775} \right)^{-1} \times 10^3 = \underline{\underline{2.391 \text{ k}\Omega}}$$

[5] Question 5:

Design a Tow-Thomas active-RC lowpass notch filter with a dc gain of -1, a notch frequency of 50 kHz, a pole frequency of 5 kHz and a pole-Q of 5. Use integrating capacitors of 10 nF each.



$$C = 10 \text{ nF} \quad RC = \frac{1}{\omega_0} = \frac{1}{2\pi \times 5 \text{ k}}$$

$$R = \frac{1}{(2\pi \times 5 \text{ k})(10 \times 10^{-9})} = \underline{\underline{3.183 \text{ k}\Omega}}$$

$$QR = 5 \times R = \underline{\underline{15.92 \text{ k}\Omega}}$$

$$\Gamma = 10 \text{ k} \quad (\text{ARBITRARY CHOICE})$$

$$R_1 = R_3 = \infty \quad (\text{TO MAKE NOTCH})$$

$$H(s) = \frac{-\left(s^2 \left(\frac{C_1}{C}\right) + \frac{1}{RR_2C^2}\right)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\frac{1}{RR_2C^2} = \omega_0^2$$

$$R_2 = \frac{1}{\omega_0^2 RC^2} = \underline{\underline{3.183 \text{ k}\Omega}}$$

[NOTE $k_1 s^2 + k_2$ HAS ZERO]
 AT $s^2 = -\frac{k_2}{k_1} \Rightarrow \left(\frac{1}{RR_2C^2}\right) = \left(\frac{C_1}{C}\right) = (2\pi \times 50 \text{ e}3)^2$

$$C_1 = \frac{1}{RR_2C(2\pi \times 50 \text{ e}3)^2} = \underline{\underline{0.1 \text{ nF}}}$$

(blank sheet for scratch calculations)

$h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$.

$$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}; T_d(\omega) = -\frac{d\phi(\omega)}{d\omega}; |H(j\omega)|_{\text{dB}} = 20\log|H(j\omega)|; H(s) = \frac{a_m s^m + \dots + a_0}{s^N + b_{n-1}s + \dots + b_0}$$

$$|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega); \text{General Lowpass } |H(j\omega)|^2 = \frac{A_0^2}{1 + F(\omega^2)}$$

$$\text{Butterworth: } F(\omega^2) = \varepsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}; A_{\text{max}} = 20\log\sqrt{1 + \varepsilon^2}; A_{\text{min}} \leq 10\log\left[1 + \varepsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}\right];$$

Poles lie on circle of radius $\omega_p \left(\frac{1}{\varepsilon}\right)^{\frac{1}{N}}$ spaced apart by $\frac{\pi}{N}$ with first half angle from $j\omega$ axis

$$\text{Chebyshev: (for } \omega_p = 1) F(\omega^2) = \varepsilon^2 C_N^2(\omega); C_N(\omega) = \cos(N\cos^{-1}(\omega)) \quad |\omega| \leq 1$$

$$C_N(\omega) = \cosh(N\cosh^{-1}(\omega)) \quad |\omega| \geq 1; C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega);$$

$$A_{\text{max}} = 20\log\sqrt{1 + \varepsilon^2}; A_{\text{min}} \leq 10\log[1 + \varepsilon^2 \cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$$

$$\text{Second-order polynomial: } s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2; \text{ for } Q > 0.5, \text{ poles complex at radius } \omega_0 \text{ and real part is } -\frac{\omega_0}{2Q}$$

$$\text{Lowpass and highpass: peaking occurs if } Q > 1/\sqrt{2} \text{ and } \omega_{\text{max}} = \omega_0 \sqrt{1 - 1/(2Q^2)}$$

Bandpass: for complex poles, peak occurs at ω_0 and has 3dB bandwidth of ω_0/Q

$$\text{LCR: } \omega_0 = 1/\sqrt{LC}; Q = \omega_0 CR$$

$$\text{KHN Biquad: } RC = \frac{1}{\omega_0}; \left(\frac{R_1 || R_2}{(R_1 || R_2) + R_3}\right) = \frac{1}{\xi}; \left(\frac{R_2 || R_3}{(R_2 || R_3) + R_1}\right) = \xi$$

$$\text{Tow-Thomas Biquad: } RC = \frac{1}{\omega_0}; \text{ damping resistor is } QR; \text{ numerator is } -s^2\left(\frac{C_1}{C}\right) - s\left(\frac{1}{C}\right)\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) - \frac{1}{RR_2C^2}$$