University of Toronto

Term Test 1

Date - Oct 13, 2010

Duration: 1.5 hrs

ECE512 — Analog Signal Processing
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Both programmable and non-programmable calculators allowed.
2. Equation sheet on last page of this test.
3. **Only tests written in pen will be considered for a re-mark.**
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: SOLUTIONS

First Name: ____________________

Student #: ____________________

Question	Mark
1	
2	
3	
4	
5	
Total	

(max grade = 25)

[5] Question 1: Answer the True [T] or False [F] questions below by **circling** the correct answer. Each correct answer is worth 0.5 marks.

T F The Laplace transform of the impulse response of a linear time-invariant filter equals the filter's transfer-function.

T F A filter realized using only resistors and capacitors will have all its poles and zeros on the real axis.

T F The transfer-function order of a filter realized using only inductors, capacitors and resistors equals the total number of inductors and capacitors.

T F For a given passband ripple, a Butterworth filter has more stopband attenuation than a Chebyshev filter.

T F The units of group delay is seconds.

T F A direct-form realization of a filter has poor performance even for a second-order filter.

T F When designing a cascade-of-biquads filter, the high-Q poles should be matched with the furthest zeros.

T F The Chebyshev polynomial, $C_3(\omega)$, can be written as $4\omega^3 - 3$

T F In terms of the phase response, having a zero in the s-plane at $z_1 = \sigma_1 + j\omega_1$ has the same phase response effect as having a zero at $-z_1{}^*$

T F For low to moderate frequency filters (0 to 10Mhz), active RC filters are preferred over RLC filters because active RC filters have better linearity performance.

[5] Question 2:

Consider an 8'th order Butterworth filter with a passband frequency at 10kHz and a passband ripple of 2dB. Find the values of ω_o and Q for the pole pair with the highest Q only.

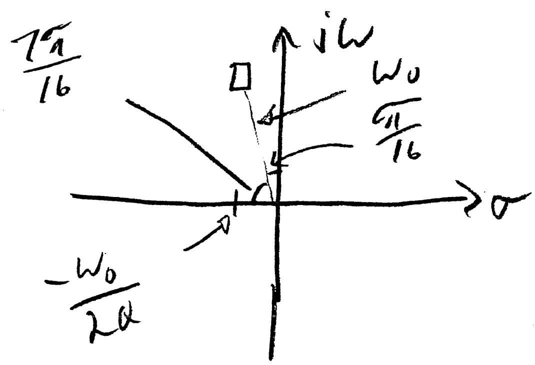
$$A_{MAX} = 2\,dB = 20 \log \sqrt{1+\epsilon^2}$$

$$10^{0.2} = 1+\epsilon^2 \Rightarrow \epsilon^2 = 10^{0.2} - 1 = 0.5849$$

$$\epsilon = 0.7648$$

$$\omega_0 = \omega_P \left(\frac{1}{\epsilon}\right)^{1/N} \qquad \omega_p = 2\pi \times 10K \quad N = 8$$

$$\omega_0 = (2\pi \times 10k)\left(\frac{1}{0.7648}\right)^{1/8} = \underline{\underline{6.50 \times 10^4 \text{ RAD/S}}}$$



$$\cancel{\omega_0} \cos \frac{7\pi}{16} = \frac{\cancel{\omega_0}}{2Q} \Rightarrow Q = \frac{1}{2\cos\left(\frac{7\pi}{16}\right)}$$

$$\underline{\underline{Q = 2.56}}$$

[5] Question 3:

Given $H(s) = \dfrac{1}{(s+1)(s^2+s+1)}$ show that $|H(j\omega)|^2 = \dfrac{1}{1+\omega^6}$

$$|H(j\omega)|^2 = H(s)\,H(-s)\Big|_{s=j\omega}$$

$$H(s)\,H(-s) = \frac{1}{(s+1)(-s+1)(s^2+s+1)(s^2-s+1)}$$

$$= \frac{1}{(1-s^2)(s^4+s^2+1)}$$

$$= \frac{1}{1-s^6}$$

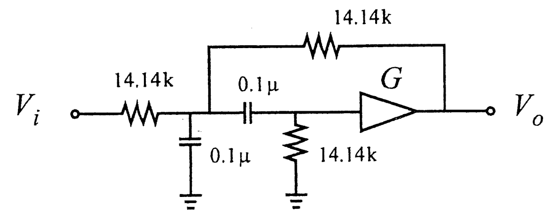
$$|H(j\omega)|^2 = \frac{1}{1+\omega^6}$$

[5] Question 4:

It is known that the circuit below realizes the following voltage transfer-function

$$H(s) = \frac{V_o}{V_i} = \frac{2728.4s}{s^2 + 100s + 10^6}$$

which is a bandpass filter.



G is an ideal gain of 3.859

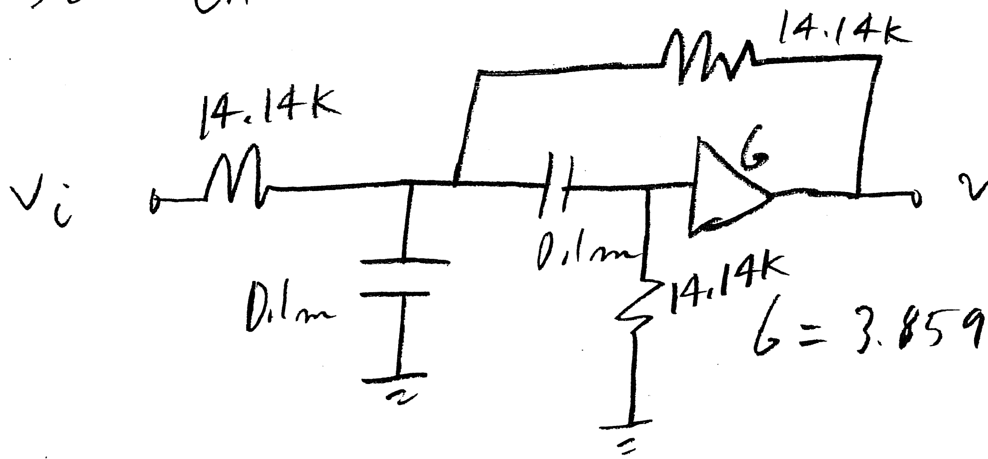
a) Apply a frequency scaling to normalize the band center frequency to 1 rad/s. (you will have to determine the center frequency from the transfer-function above).

H(S) IS BANDPASS SO BAND CENTER FREQ $= \omega_0$

$\omega_0^2 = 10^6$ SO $\omega_0 = 10^3$ RAD/S

NEED TO FREQ SCALE BY 10^{-3}

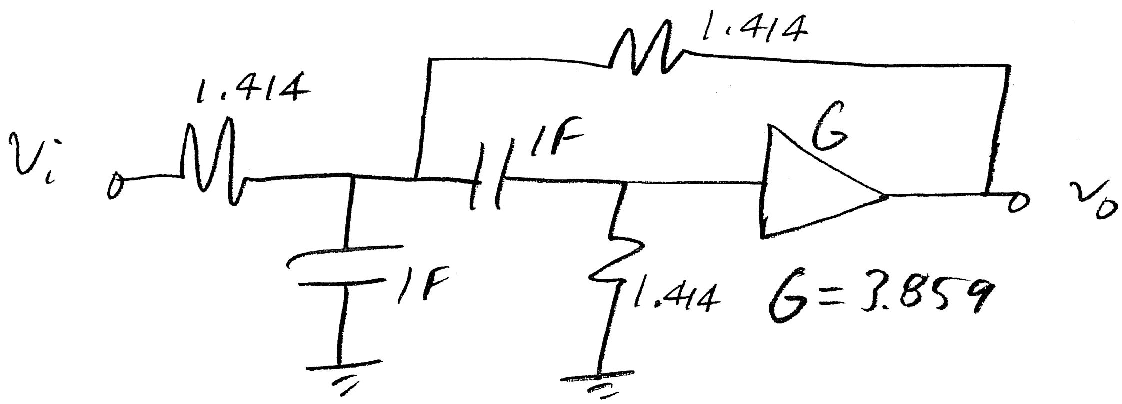
SO CAPACITORS INCREASED 10^3



b) Apply an impedance scaling to the results of a) to normalize the circuit so that both capacitors become 1 F.

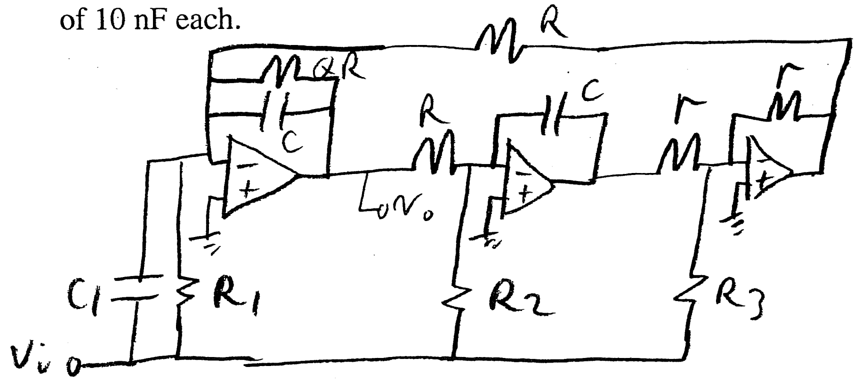
WANT $C = 1F$ SO IMPEDANCE SCALE BY 10^{-4}

$\Rightarrow R = 1.414\,\Omega$



[5] Question 5:

Design a Tow-Thomas highpass notch filter with a high frequency gain of -1, a pole frequency of 5kHz, a pole-Q of 5 and a notch frequency at 500Hz. Use integrating capacitors of 10 nF each.



LET $r = 10k$

(ARBITRARY CHOICE)

$C = 10nF$ $\quad RC = \frac{1}{\omega_0} = \frac{1}{2\pi \times 5k} \Rightarrow R = 3.183k$

$QR = 5 \times R = 15.92k$

$R_1 = R_3 = \infty$ (TO MAKE NOTCH)

$$H(s) = \frac{-\left(s^2\left(\frac{C_1}{C}\right) + \frac{1}{RR_2C^2}\right)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

HIGH FREQ GAIN $= -1 \Rightarrow C_1 = C = 10nF$

$K_1 s^2 + k_2$ HAS ZERO AT $s^2 = -\frac{k_2}{K_1}$

$$\frac{\frac{1}{RR_2C^2}}{1} = (2\pi \times 500)^2$$

$$R_2 = \frac{1}{RC^2(2\pi \times 500)^2} = 318.3\,k$$

(blank sheet for scratch calculations)

ECE512 Analog Signal Processing Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;

General: $h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$; $|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$;

General Lowpass: $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$;

Butterworth: $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega_s}{\omega_p}\right)^{2N}\right]$; Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by π/N with first angle being $\pi/(2N)$

Chebyshev: (for $\omega_p = 1$); $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$; $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$;

Second-order polynomial: $s^2 + (\omega_0/Q)s + \omega_0^2$; for $Q > 0.5$; poles complex at radius ω_0 and real part is $-\omega_0/(2Q)$;

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/2Q^2}$

Bandpass: for complex poles; peak occurs at ω_0 and has 3 dB bandwidth of ω_0/Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$;

KHN Biquad: $RC = 1/\omega_0$; $2((R_1||R_2)/((R_1||R_2) + R_3)) = 1/Q$; $2((R_2||R_3)/((R_2||R_3) + R_1)) = k$;

Tow-Thomas Biquad: $RC = 1/\omega_0$; damping resistor is QR ; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$;

Noise: noise equivalent bandwidth $= (\pi/2)f_{3\mathrm{dB}}$; $V_R^2(f) = 4kTR$; $I_d^2(f) = 2qI_D$; $r_d = (kT)/(qI_D)$; $V_C^2 = (kT)/C$;

Discrete-Time: $X_s(s) = \sum x_c(nT)e^{-snT}$; $X(z) = \sum x_c(nT)z^{-n}$; $p = (z-1)/(z+1)$; $z = (1+p)/(1-p)$; $\Omega = \tan(\omega/2)$;

Switched-Cap: $R_{eq} = T/C$; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$;

Data Converters: $B_{\mathrm{in}} = b_1 2^{-1} + \ldots + b_N 2^{-N}$; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$; $|V_Q| \le 0.5V_{\mathrm{LSB}}$;

$V_{\mathrm{Q(rms)}} = V_{\mathrm{LSB}}/\sqrt{12}$; $\mathrm{SNR} = 6.02N + 1.76$; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}$; $E_{\mathrm{off(A/D)}} = V_{0\ldots01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$; $\Delta t < 1/(2^N\pi f_{\mathrm{in}})$;

$E_{\mathrm{gain(D/A)}} = \left(V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1\ldots1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}\right) - (2^N - 1)$; $E_{\mathrm{gain(A/D)}} = (V_{1\ldots1}/V_{\mathrm{LSB}} - V_{0\ldots01}/V_{\mathrm{LSB}}) - (2^N - 2)$;

Oversampling: $\mathrm{OSR} = f_s/(2f_0)$; $\mathrm{SNR}_0 = 6.02N + 1.76 + 10\log(\mathrm{OSR})$; $\mathrm{SNR}_1 = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$;

$\mathrm{SNR}_2 = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability;

Bipolar transistors: $I_C = I_{CS}e^{V_{\mathrm{BE}}/V_T}$; $g_m = I_C/V_T$; $r_e = \alpha/g_m = V_T/I_E$;

CMOS transistors: $K_n = 0.5\mu_n C_{\mathrm{ox}}(W/L)$; $I_D = 2K_n((V_{\mathrm{GS}} - V_{tn})V_{\mathrm{DS}} - (V_{\mathrm{DS}}^2/2))$; $r_{\mathrm{ds}} = (2K_n(V_{\mathrm{GS}} - V_{tn}))^{-1}$;

$I_D = K_n(V_{\mathrm{GS}} - V_{tn})^2$; $g_m = 2K_n(V_{\mathrm{GS}} - V_{tn}) = (2I_D)/(V_{\mathrm{GS}} - V_{tn})$; $r_s = 1/g_m$;

Ideal Transcondcutor: $i_o = G_m v_i$; **Bipolar Diff Pair:** $I_{C2} = I_1/(1 + e^{v_i/V_T})$;

CMOS Pair: $K_{\mathrm{eq}} = (K_n K_p)/(\sqrt{K_n} + \sqrt{K_p})^2$; $V_{\mathrm{t\text{-}eq}} = V_{\mathrm{tn}} - V_{\mathrm{tp}}$;

Dynamic Range: (all in dB or dBm) $ID_3 = I_{D3} - I_{D1}$; $\mathrm{OIP}_3 = I_{D1} - ID_3/2$; $\mathrm{SFDR} = (2/3)(\mathrm{OIP}_3 - N_o)$;

$\mathrm{THD} = 10\log((V_{h2}^2 + V_{h3}^2 + \ldots)/V_f^2)$;