University of Toronto

Term Test 2

Date - Nov 18, 2009

Duration: 1.5 hrs

ECE512 — Analog Signal Processing
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Both programmable and non-programmable calculators allowed.
2. Equation sheet on last page of this test.
3. **Only tests written in pen will be considered for a re-mark**.
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
Total	

(max grade = 25)

Last Name: SOLUTIONS

First Name: ____________________

Student #: ____________________

[5] Question 1: Answer the True [T] or False [F] questions below by **circling** the correct answer. Each correct answer is worth 0.5 marks.

T F If a signal/noise measurement at a node results in 3 dB SNR when measured using power levels, then the same node will have 6 dB SNR when measured using voltage levels.

T F A 1 μW signal corresponds to a -30 dBm value for that signal.

T F When 2 uncorrelated noise signals of equal power are summed together, their output noise is 3 dBm higher than each individual noise signal.

T F 1/f noise is noise that is proportional to $1/f$ in the root spectral density domain.

T F A 1 $k\Omega$ resistor has a noise voltage of approx 4 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ at room temp.

T F A sample-and-hold used at the front of an analog-to-digital converter helps somewhat as an anti-aliasing filter due to it's sinx/x response.

T F Advanced clocks are used in a switched-capacitor circuit to improve stability.

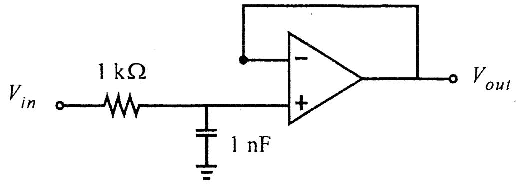
T F Fully differential circuits are used to improve thermal noise performance when using the same overall capacitor area.

T F The error feedback structure in oversampling converters is more sensitive to analog parameter errors.

T F The stability rule $|N_{TF}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability is too conservative when using multibit quantizers.

[5] Question 2:

Consider the first-order RC filter shown below



op-amp input noise

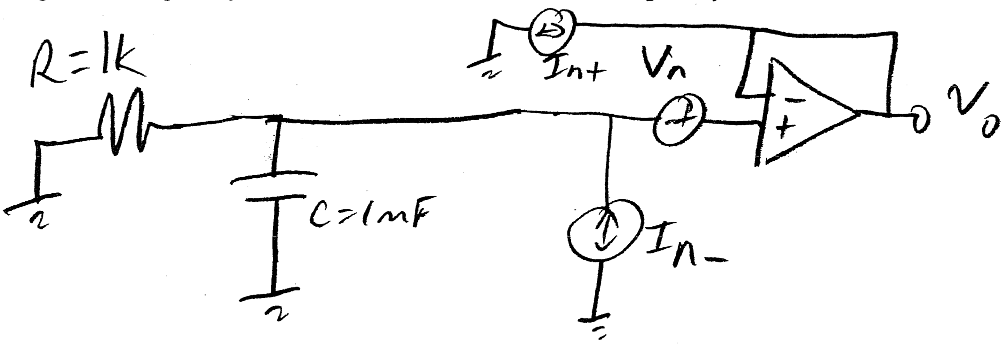
$V_n(f) = 20nV/\sqrt{Hz}$

$I_n(f) = 20pA/\sqrt{Hz}$

ft

op-amp unity gain freq = 10 MHz

Estimate the expected signal-to-noise ratio (in dB) for a 10 mVrms input signal where the input signal's frequency content is all below the 3dB frequency of the filter.



OPAMP OUTPUT IMPEDANCE = 0

$$V_o^2 = \frac{kT}{C} + (I_{n-})^2 R^2 \left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right) + V_n^2\left(\frac{\pi}{2}\right)(f_t) + (I_{n+}^2)(0)$$

$$= \frac{(1.38\times10^{-23})(300)}{(1e-9)} + (20e-12)^2(1k)^2\left(\frac{1}{4(1k)(1e-9)}\right) + (20e-9)^2\left(\frac{\pi}{2}\right)(1e7)$$

$$= (4.14e-12) + (1e-10) + (6.28e-9)$$

$$= 6.39e-9$$

$$V_o = 80\ \mu V\ RMS$$

$$SNR = 20\log_{10}\left(\frac{10\ mV}{80\ \mu V}\right) = \underline{42\ dB}$$

[5] Question 3: Using the bilinear transform, find an $H(z)$ to realize a second-order transfer-function with a dc gain of 10, a maximally-flat passband (i.e. $Q = 1/\sqrt{2}$ in cont-time domain) and a -3 dB frequency at 5 kHz when the sampling rate is 100 kHz.

$$W_{3dB} = \left(\frac{5k}{100k}\right) \times 2\pi = \frac{\pi}{10} \text{ RAD/SAMPLE}$$

$$\Omega_{3dB} = \tan\left(\frac{W_{3dB}}{2}\right) = \tan\left(\frac{\pi}{20}\right) \simeq 0.1584 \text{ RAD/S}$$

$$\Omega_0 = \Omega_{3dB}$$

$$Q = \frac{1}{\sqrt{2}}$$

$$H(p) = \frac{k_0}{p^2 + \left(\frac{\Omega_{3dB}}{Q}\right)p + \Omega_{3dB}^2}$$

$$= \frac{k_0}{p^2 + 0.2240\,p + 0.0251}$$

$$H(0) = 10 \Rightarrow k_0 = 0.251$$

$$H(p) = \frac{0.251}{p^2 + 0.2240\,p + 0.0251}$$

$$p = \frac{z-1}{z+1}$$

$$H(z) = \frac{0.251}{\left(\frac{z-1}{z+1}\right)^2 + 0.2240\left(\frac{z-1}{z+1}\right) + 0.0251}$$

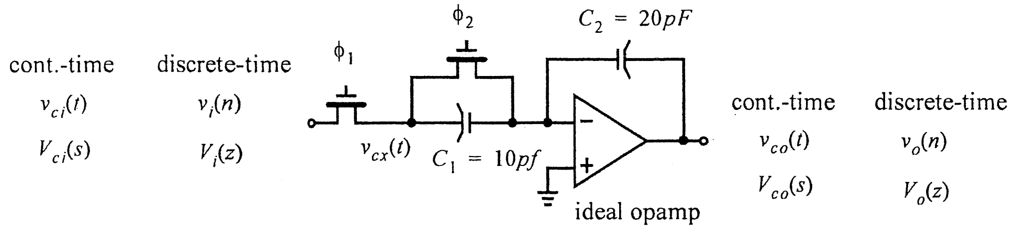
$$= \frac{0.251\,(z+1)^2}{(z-1)^2 + 0.2240\,(z-1)(z+1) + 0.0251\,(z+1)^2}$$

$$= \frac{0.251\,z^2 + 0.502\,z + 0.251}{(1+0.2240+0.0251)z^2 + (-2+0.0502)z + (1-0.2240+0.0251)}$$

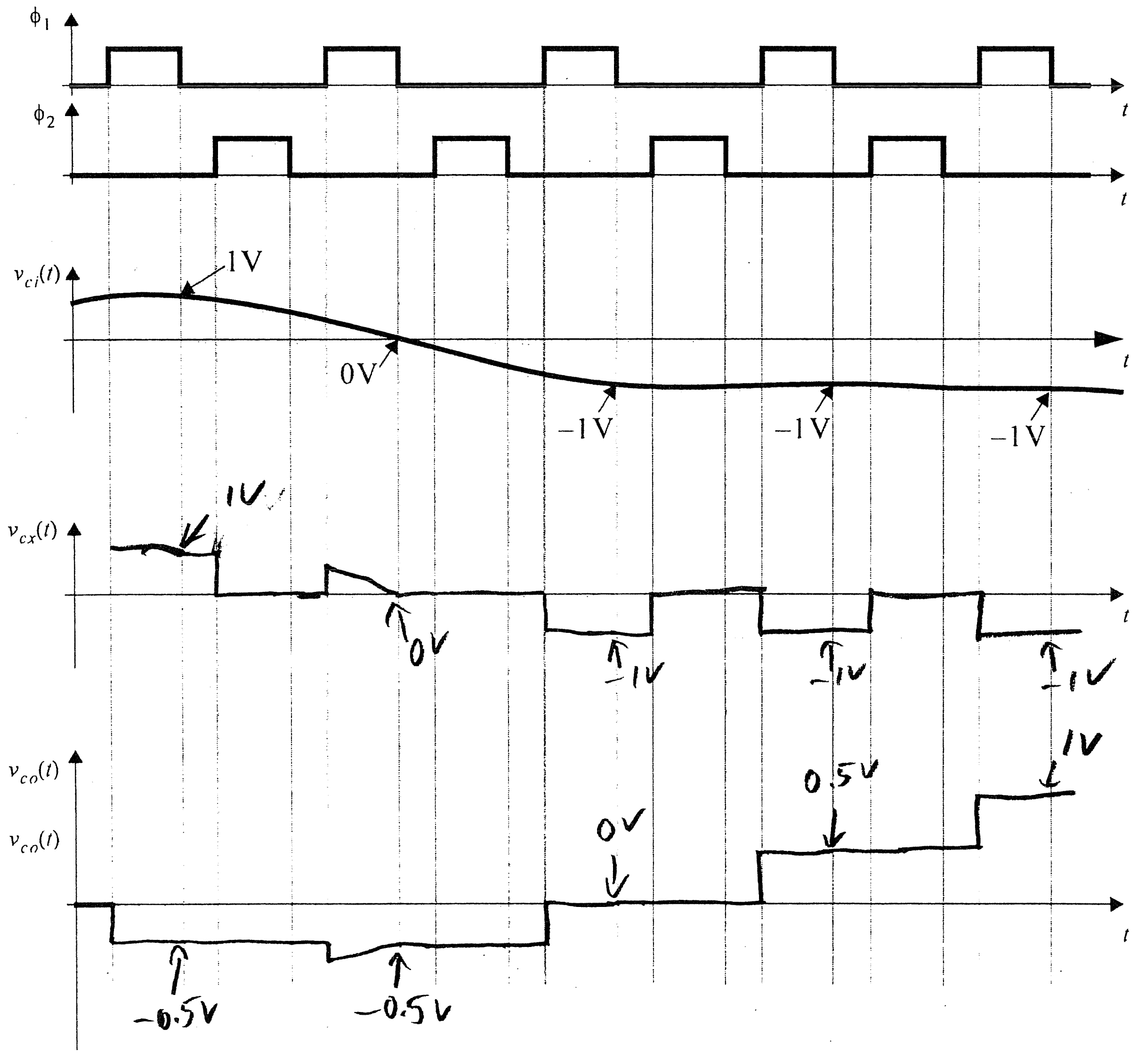
$$H(z) = \frac{0.251\,z^2 + 0.502\,z + 0.251}{1.249\,z^2 - 1.949\,z + 0.8011}$$

[5] Question 4:

Consider the following switched-capacitor circuit (ignore parasitic capacitances).



a) In the continuous-time plots below, sketch the output voltages for $v_{cx}(t)$ and $v_{co}(t)$ and show their values at the end of ϕ_1. Assume an initial $v_{co}(t)$ of 0 volts (as shown).



Question 3 (cont'd)

b) Find the discrete-time z-domain transfer-function, $V_o(z)/V_i(z)$, of the above circuit given that the sampling time of interest is at the end of ϕ_1.

$$H(z) = -\frac{C_1}{C_2}\frac{(z)}{(z-1)}$$

$$= -0.5\,\frac{z}{z-1}$$

c) Why is the above circuit not suitable for building practical SC filters? What performance would suffer?

PARASITIC CAPACITANCE AT V_{cx} NODE WOULD AFFECT INTEGRATOR GAIN AND ADD NON-LINEARITY DUE TO REVERSE BIAS JUNCTIONS.
TRANSFER-FUNCTION ACCURACY & DISTORTION WOULD BE POOR.

[5] Question 5:

a) Given an ideal 3 bit quantizer and ideal 3 bit DAC, what oversampling ratio is required to obtain an SNR equivalent to a 16 bit ADC when using a second-order delta-sigma modulator?

$$SNR = 6.02N + 1.76 - 12.9 + 50\log(OSR)$$

SNR DESIRED IS $(16)6.02 + 1.76 = 98.1\,dB$

$$98.1 = 6.02(3) + 1.76 - 12.9 + 50\log(OSR)$$

$$50\log(OSR) = 91.2\,dB$$

$$OSR = 10^{\frac{91.2}{50}} = \underline{\underline{66.6}}$$

b) Is it more important that the quantizer or the DAC is linear to the 16 bit level? Explain your reasoning.

THE DAC SHOULD BE MORE LINEAR AS THE QUANTIZER ERROR IS DIVIDED BY LOOP GAIN WHILE DAC ERROR IS NOT.

(blank sheet for scratch calculations)

ECE512 Analog Signal Processing Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}\ \mathrm{C}$; $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K;

$h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$.

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$;

$|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$; General Lowpass $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$;

Butterworth: $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}\right]$;

Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by π/N with first half angle from $j\omega$ axis

Chebyshev: (for $\omega_p = 1$) $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$ $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$

Second-order polynomial: $s^2 + (\omega_0/Q)s + \omega_0^2$; for $Q > 0.5$; poles complex at radius ω_0 and real part is $-\omega_0/(2Q)$

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/(2Q^2)}$

Bandpass: for complex poles; peak occurs at ω_0 and has 3dB bandwidth of ω_0/Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$

KHN Biquad: $RC = 1/\omega_0$; $2((R_1 \| R_2)/((R_1 \| R_2) + R_3)) = 1/Q$; $2((R_2 \| R_3)/((R_2 \| R_3) + R_1)) = k$

Tow-Thomas Biquad: $RC = 1/\omega_0$; damping resistor is QR; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$

Noise: Noise equivalent bandwidth $= (\pi/2)f_{3\mathrm{dB}}$; $V_R^2(f) = 4kTR$; $I_d^2(f) = 2qI_D$; $r_d = (kT)/(qI_D)$; $V_C^2 = (kT)/C$

Discrete-Time: $X_s(s) = \sum x_c(nT)e^{-snT}$; $X(z) = \sum x_c(nT)z^{-n}$; $p = (z-1)/(z+1)$; $z = (1+p)/(1-p)$; $\Omega = \tan(\omega/2)$

Switched-Cap: $R_{eq} = T/C$; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$

Data Converters: $B_{\mathrm{in}} = b_1 2^{-1} + \ldots + b_N 2^{-N}$; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$; $|V_Q| \le 0.5V_{\mathrm{LSB}}$;

$V_{Q(\mathrm{rms})} = V_{\mathrm{LSB}}/\sqrt{12}$; $\mathrm{SNR} = 6.02N + 1.76$; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}$; $E_{\mathrm{off(A/D)}} = V_{0\ldots01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$;

$\Delta t < 1/(2^N\pi f_{\mathrm{in}})$ $E_{\mathrm{gain(D/A)}} = (V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1\ldots1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}) - (2^N - 1)$;

$E_{\mathrm{gain(A/D)}} = (V_{1\ldots1}/V_{\mathrm{LSB}} - V_{0\ldots01}/V_{\mathrm{LSB}}) - (2^N - 2)$;

Oversampling: $\mathrm{OSR} = f_s/(2f_0)$; $\mathrm{SNR}_0 = 6.02N + 1.76 + 10\log(\mathrm{OSR})$; $\mathrm{SNR}_1 = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$;

$\mathrm{SNR}_2 = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$;

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\displaystyle\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability