University of Toronto

Term Test ~~1~~ 2

Date - Nov 17, 2010

Duration: 1.5 hrs

ECE512 — Analog Signal Processing
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Both programmable and non-programmable calculators allowed.

2. Equation sheet on last page of this test.

3. **Only tests written in pen will be considered for a re-mark.**

4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
Total	

(max grade = 25)

Last Name: SOLUTIONS

First Name: ____________________

Student #: ____________________

[5] Question 1: Answer the True [T] or False [F] questions below by **circling** the correct answer. Each correct answer is worth 0.5 marks.

T F If a random voltage signal with a **mean-squared** value of 2 is applied to a 1 ohm resistor, the power dissipated is 2 watts.

T F A random signal with a white noise spectrum could have a probability density function (pdf) which is Gaussian, Uniform or some other pdf.

T F A random signal with a Gaussian pdf will always have a white noise spectrum.

T F The 1/f noise tangent principle is used to find the components in a circuit which contribute the most amount of noise.

T F For a given filter response, to reduce noise generally requires more power dissipation.

T F When upsampling a discrete-time signal, there is no concern about aliasing.

T F The sin(x)/x response of a sample-and-hold has zero gain at fs/2, fs, 3fs/2, 2fs, ...

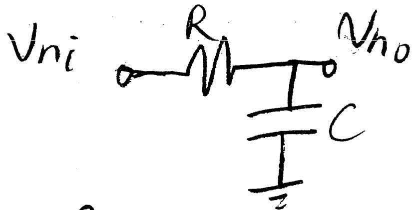
T F Advance clocks are used in switched-capacitor circuits to give more settling time.

T F In an oversampled D/A converter with a 1 bit quantizer, the quantizer is realized using a **analog** comparator.

T F In an oversampled D/A converter with a 1 bit quantizer, the quantizer is realized using a **digital** comparator.

[5] Question 2:

a) Consider a white noise signal that has a root spectral density of 10 nV/$\sqrt{\text{Hz}}$ applied to the input of a first-order lowpass RC filter with $R = 1\text{k}\Omega$ and $C = 0.159\mu\text{F}$. What is the total noise rms value at the output (include the thermal noise of the resistor). Assume $T = 300^{\circ}$.



$R = 1k$
$C = 0.159\mu F$

$$V_{n_o}^2 = \frac{KT}{C} + (10e{-}9)^2\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right)$$

$$= \frac{(1.38e{-}23)(300)}{(0.159e{-}6)} + (10e{-}9)^2\left(\frac{1}{4(1e3)(0.159e{-}6)}\right)$$

$$= 1.833e{-}13 \ V^2$$

$$V_n = 4.28e{-}7 \ V = \underline{\underline{0.428 \mu V_{RMS}}}$$

b) The output noise of a circuit is measured to be -20 dBm around 100 kHz when a resolution bandwidth of 40 Hz is used. Find the root spectral density in V/$\sqrt{Hz}$.

$1 mW, \Rightarrow 0 dBm \Rightarrow \frac{V^2}{50} = 1mW \Rightarrow V^2 = 0.05$

$0 dBm \Rightarrow V = 224 \ mV_{RMS}$

$-20 dBm \Rightarrow 0.01 mW \Rightarrow V = 22.4 \ mV_{RMS}$

MEASURED OVER 40 HZ

SO $V^2/HZ \Rightarrow (22.4e{-}3)^2$ OVER 40 HZ

OR $\frac{(22.4e{-}3)^2}{40}$ OVER 1 HZ

OR $\frac{22.4e{-}3}{\sqrt{40}} = \underline{\underline{3.54 \ mV/\sqrt{HZ}}}$

[5] Question 3: Using the bilinear transform, find an $H(z)$ to realize a second-order transfer-function with a dc gain of 5, a ~~maximally-flat~~ passband (i.e. $Q = 1$ in cont-time domain) and a -3 dB frequency at 2 kHz when the sampling rate is 50 kHz.

$$\omega_{3dB} = \left(\frac{2k}{50k}\right) \times 2\pi = \frac{4\pi}{50} \text{ RAD/SAMPLE}$$

$$\Omega_{3dB} = \tan\left(\frac{\omega_{3dB}}{2}\right) = 0.1263 \text{ RAD/S}$$

$$H(p) = \frac{k_0}{p^2 + \Omega_0 p + \Omega_0^2} \quad (\text{SINCE } Q = 1)$$

Ω_{3dB} OCCURS WHERE $|H(\Omega_{3dB})| = \frac{1}{\sqrt{2}}|H(0)|$

$$(\Omega_0 \Omega_{3dB})^2 + (\Omega_0^2 - \Omega_{3dB}^2)^2 = 2\Omega_0^4$$

$$\Omega_0 = 0.099314$$

$$H(p) = \frac{k_0}{p^2 + \Omega_0 p + \Omega_0^2} \qquad H(0) = 5 \Rightarrow k_0 = 0.04932$$

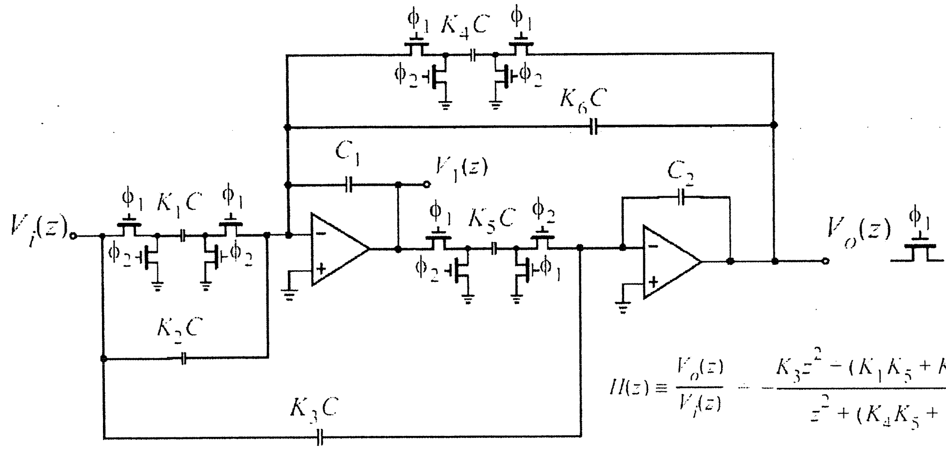
$$H(p) = \frac{0.04932}{p^2 + \Omega_0 p + \Omega_0^2} \qquad p = \frac{z-1}{z+1}$$

$$H(z) = \frac{0.04932}{\left(\frac{z-1}{z+1}\right)^2 + 0.09931\left(\frac{z-1}{z+1}\right) + 9.863e-3}$$

$$H(z) = \frac{0.04932(z+1)^2}{(z-1)^2 + 0.09931(z-1)(z+1) + (9.863e-3)(z+1)^2}$$

$$H(z) = \frac{(4.932e-2)z^2 + (9.863e-2)z + (4.932e-2)}{1.198z^2 - 1.803z + 0.9993}$$

[5] Question 4: Consider the SC biquad filter shown below.



$$H(z) \equiv \frac{V_o(z)}{V_i(z)} = -\frac{K_3z^2 - (K_1K_5 + K_2K_5 - 2K_3)z + (K_3 - K_2K_5)}{z^2 + (K_4K_5 + K_5K_6 - 2)z + (1 - K_5K_6)}$$

Given $C_1 = 1\text{pF}$ and $C_2 = 3\text{pF}$, find the values (in pF) for K_1C to K_6C so that $H(z) = -\dfrac{0.288(z-1)}{z^2 - 1.572z + 0.9429}$. Note that the subscript "j" on K_iC_j has been deleted on the above figure so that you need to determine which integrating cap to reference the value to.

$$H(z) = -\frac{0.288z - 0.288}{z^2 - 1.572z + 0.9429}$$

LET $k_4 = k_5$

$1 - k_5k_6 = 0.9429 \Rightarrow k_5k_6 = 0.0571$

$k_4^2 + 0.0571 - 2 = -1.572 \Rightarrow k_4^2 = 0.3709$

$k_4 = k_5 = 0.609 \Rightarrow k_6 = 0.0938$

$k_3 = 0$ $\quad k_3 - k_2k_5 = -0.288 \Rightarrow k_2 = 0.4729$

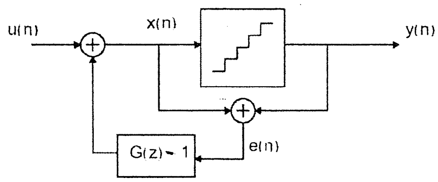
$k_1k_5 + k_2k_5 = 0.288 \Rightarrow k_1 = 0$

$C_1 = 1pF \Rightarrow k_1C_1 = 0 \quad k_2C_1 = 0.4729pF$

$k_4C_1 = 0.609\ pF \quad k_6C_1 = 0.0938\ pF$

$C_2 = 3pF \Rightarrow k_3C_2 = 0 \quad k_5C_2 = 1.827\ pF$

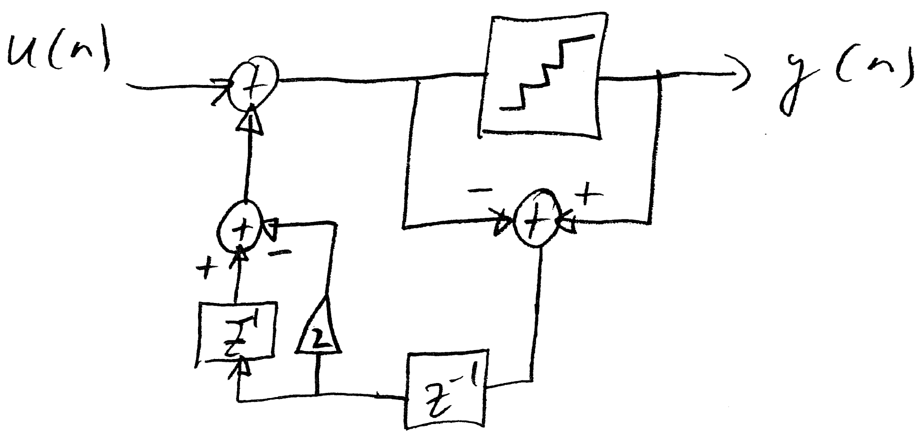
[5] Question 5: Given the error-feedback structure of a delta-sigma modulator shown below, replace the block $G(z) - 1$ by finding the appropriate transfer-function so that the noise transfer function equals $(1 - z^{-1})^2$. Show the overall implementation in terms of delay, gain, quantizer and adder blocks.



$$G(z) = NTF(z) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2}$$

$$G(z) - 1 = -2z^{-1} + z^{-2}$$

$$= z^{-1}(-2 + z^{-1})$$



(blank sheet for scratch calculations)

ECE512 Analog Signal Processing Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $V_T = kT/q \approx 26\,\mathrm{mV}$ at 300 °K ;

General: $h(t)$ is impulse response of LTI system; $H(s)$ is the Laplace transform of $h(t)$

$H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$; $T_d(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$; $|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)|$; $H(s) = \dfrac{a_m s^m + \ldots + a_0}{s^N + b_{n-1}s + \ldots + b_0}$; $|H(j\omega)|^2 = H(s)H(-s)|_{s=j\omega} = H(j\omega)H(-j\omega)$;

General Lowpass: $|H(j\omega)|^2 = A_0^2/(1 + F(\omega^2))$;

Butterworth: $F(\omega^2) = \varepsilon^2\left(\dfrac{\omega}{\omega_p}\right)^{2N}$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log\left[1 + \varepsilon^2\left(\dfrac{\omega_s}{\omega_p}\right)^{2N}\right]$; Poles lie on circle of radius $\omega_p(1/\varepsilon)^{1/N}$ spaced apart by π/N with first angle being $\pi/(2N)$

Chebyshev: (for $\omega_p = 1$); $F(\omega^2) = \varepsilon^2 C_N^2(\omega)$; $C_N(\omega) = \cos(N\cos^{-1}(\omega))$ $|\omega| \le 1$; $C_N(\omega) = \cosh(N\cosh^{-1}(\omega))$ $|\omega| \ge 1$;

$C_{N+1}(\omega) = 2\omega C_N(\omega) - C_{N-1}(\omega)$; $A_{\max} = 20\log\sqrt{1+\varepsilon^2}$; $A_{\min} \le 10\log[1 + \varepsilon^2\cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$;

Second-order polynomial: $s^2 + (\omega_0/Q)s + \omega_0^2$; for $Q > 0.5$; poles complex at radius ω_0 and real part is $-\omega_0/(2Q)$;

Lowpass and highpass: peaking occurs if $Q > 1/\sqrt{2}$ and $\omega_{\max} = \omega_0\sqrt{1 - 1/2Q^2}$

Bandpass: for complex poles; peak occurs at ω_0 and has 3 dB bandwidth of ω_0/Q

LCR: $\omega_0 = 1/\sqrt{LC}$; $Q = \omega_0 CR$;

KHN Biquad: $RC = 1/\omega_0$; $2((R_1 || R_2)/((R_1 || R_2) + R_3)) = 1/Q$; $2((R_2 || R_3)/((R_2 || R_3) + R_1)) = k$;

Tow-Thomas Biquad: $RC = 1/\omega_0$; damping resistor is QR ; numerator is $-s^2\left(\dfrac{C_1}{C}\right) - s\left(\dfrac{1}{C}\right)\left(\dfrac{1}{R_1} - \dfrac{r}{RR_3}\right) - \dfrac{1}{RR_2C^2}$;

Noise: noise equivalent bandwidth = $(\pi/2)f_{3\mathrm{dB}}$; $V_R^2(f) = 4kTR$; $I_d^2(f) = 2qI_D$; $r_d = (kT)/(qI_D)$; $V_C^2 = (kT)/C$;

Discrete-Time: $X_s(s) = \sum x_c(nT)e^{-snT}$; $X(z) = \sum x_c(nT)z^{-n}$; $p = (z-1)/(z+1)$; $z = (1+p)/(1-p)$; $\Omega = \tan(\omega/2)$;

Switched-Cap: $R_{eq} = T/C$; $Q_{\mathrm{CH}} = -WLC_{\mathrm{ox}}(V_{\mathrm{GS}} - V_t)$;

Data Converters: $B_{\mathrm{in}} = b_1 2^{-1} + \ldots + b_N 2^{-N}$; $V_{\mathrm{LSB}} = V_{\mathrm{ref}}/2^N$; $V_{\mathrm{out}} = V_{\mathrm{ref}}B_{\mathrm{in}}$; $V_{\mathrm{ref}}B_{\mathrm{out}} = V_{\mathrm{in}} + V_Q$; $|V_Q| \le 0.5V_{\mathrm{LSB}}$;

$V_{\mathrm{Q(rms)}} = V_{\mathrm{LSB}}/\sqrt{12}$; SNR $= 6.02N + 1.76$; $E_{\mathrm{off(D/A)}} = V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}$; $E_{\mathrm{off(A/D)}} = V_{0\ldots01}/V_{\mathrm{LSB}} - 0.5\mathrm{LSB}$; $\Delta t < 1/(2^N\pi f_{\mathrm{in}})$;

$E_{\mathrm{gain(D/A)}} = \left(V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{1\ldots1} - V_{\mathrm{out}}/V_{\mathrm{LSB}}|_{0\ldots0}\right) - (2^N - 1)$; $E_{\mathrm{gain(A/D)}} = (V_{1\ldots1}/V_{\mathrm{LSB}} - V_{0\ldots01}/V_{\mathrm{LSB}}) - (2^N - 2)$;

Oversampling: OSR $= f_s/(2f_0)$; $\mathrm{SNR}_0 = 6.02N + 1.76 + 10\log(\mathrm{OSR})$; $\mathrm{SNR}_1 = 6.02N + 1.76 - 5.17 + 30\log(\mathrm{OSR})$;

$\mathrm{SNR}_2 = 6.02N + 1.76 - 12.9 + 50\log(\mathrm{OSR})$; $S_{\mathrm{TF}}(z) = H(z)/(1 + H(z))$; $N_{\mathrm{TF}}(z) = 1/(1 + H(z))$

$T_{\mathrm{avg}}(z) = \dfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} = \dfrac{1}{M}\left(\dfrac{1 - z^{-M}}{1 - z^{-1}}\right)$; $T_{\mathrm{avg}}(e^{j\omega}) = \dfrac{\mathrm{sinc}((\omega M)/2)}{\mathrm{sinc}(\omega/2)}$; $|N_{\mathrm{TF}}(e^{j\omega})| \le 1.5$ for 1-bit quantizer stability;

Bipolar transistors: $I_C = I_{\mathrm{CS}}e^{V_{\mathrm{BE}}/V_T}$; $g_m = I_C/V_T$; $r_e = \alpha/g_m = V_T/I_E$;

CMOS transistors: $K_n = 0.5\mu_n C_{\mathrm{ox}}(W/L)$; $I_D = 2K_n((V_{\mathrm{GS}} - V_{tn})V_{\mathrm{DS}} - (V_{\mathrm{DS}}^2/2))$; $r_{\mathrm{ds}} = (2K_n(V_{\mathrm{GS}} - V_{tn}))^{-1}$;

$I_D = K_n(V_{\mathrm{GS}} - V_{tn})^2$; $g_m = 2K_n(V_{\mathrm{GS}} - V_{tn}) = (2I_D)/(V_{\mathrm{GS}} - V_{tn})$; $r_s = 1/g_m$;

Ideal Transcondcutor: $i_o = G_m v_i$; **Bipolar Diff Pair:** $I_{C2} = I_1/(1 + e^{v_i/V_T})$;

CMOS Pair: $K_{\mathrm{eq}} = (K_nK_p)/(\sqrt{K_n} + \sqrt{K_p})^2$; $V_{\mathrm{t\text{-}eq}} = V_{\mathrm{tn}} - V_{\mathrm{tp}}$;

Dynamic Range: (all in dB or dBm) $ID_3 = I_{D3} - I_{D1}$; $\mathrm{OIP}_3 = I_{D1} - ID_3/2$; SFDR $= (2/3)(\mathrm{OIP}_3 - N_o)$;

$\mathrm{THD} = 10\log((V_{h2}^2 + V_{h3}^2 + \ldots)/V_f^2)$;