Chapter 1 - Problems

1.1) As is a pentavalent impurity

$\therefore$ n-type

$n_n = N_D = 10^{25}$ carriers/m^3

$n_p = \frac{n_i^2}{N_D}$ for $n_i\big|_{T=322K} = 4 \times n_i\big|_{T=300K} = 6 \times 10^{16} \frac{\text{carriers}}{m^3}$

$= 3.6 \times 10^8$ carriers/m^3

1.2) From Example 1.2,

$\Phi_0 = 0.88\,V$ at 300K

For $T = 311K$,

$\Phi_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 26.8\,mV \times \ln\left(\frac{10^{25} \times 10^{22}}{(2 \times 1.5 \times 10^{16})^2}\right)$

$= 0.87\,V$

$\therefore$ The built-in potential decreases when the temperature is increased.

1.3) $Q^+ = Q^- \cong [2q K_s \epsilon_0 (\Phi_0 + V_R) N_D]^{1/2}$

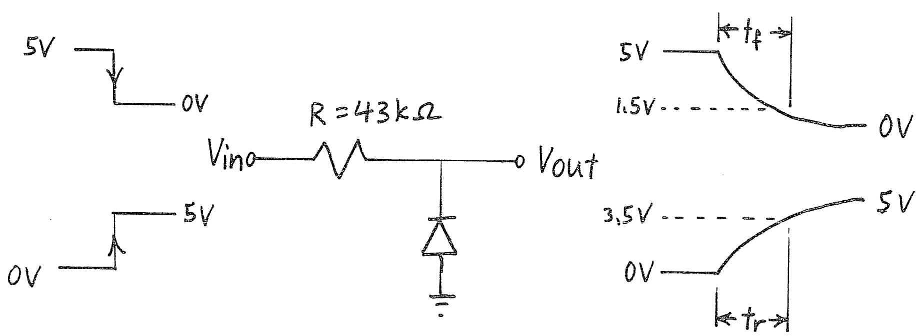
$= [2 \times 1.602 \times 10^{-19} \times 11.8 \times 8.85 \times 10^{-12} \times (0.88 + 5.0) \times 10^{22}]^{1/2}$

$= 1.4\,fC/\mu m^2$

For a $10\mu m \times 10\mu m = 100\mu m^2$ area,

140fC of charge would be present.

1.4)



Fall time, t_f :

$$C_{j\text{-}av} = 2 C_{jo} \Phi_0 \frac{\left(\sqrt{1+\frac{V_2}{\Phi_0}} - \sqrt{1+\frac{V_1}{\Phi_0}}\right)}{V_2 - V_1}$$

$$= 2 \times 15 \times 10^{-15} \times 0.9 \times \frac{\left(\sqrt{1+5/0.9} - \sqrt{1+1.5/0.9}\right)}{5 - 1.5}$$

$$= 7.2\, fF$$

$$\therefore t_f \approx 1.2\,\tau = 1.2 \times R C_{j\text{-}av}$$

$$= 1.2 \times 43 \times 10^3 \times 7.2 \times 10^{-15}$$

$$= 0.37\ nsec$$

Rise time, t_r :

$$C_{j\text{-}av} = 2 \times 15 \times 10^{-15} \times 0.9 \times \frac{\sqrt{1} - \sqrt{1+3.5/0.9}}{0 - 3.5}$$

$$= 9.3\, fF$$

$$\therefore t_r \approx 1.2\,\tau = 1.2\, R C_{j\text{-}av}$$

$$= 1.2 \times 43 \times 10^3 \times 9.3 \times 10^{-15}$$

$$= 0.48\ nsec$$

1.5) Simulation results :

t_f = 0.37 nsec

t_r = 0.44 nsec

Results are consistent with our calculations in Problem 1.4.

1.6) Active:

$$I_D = \frac{\mu_n C_{ox}}{2} \times \frac{W}{L} (V_{GS} - V_{tn})^2 \quad (1)$$

Triode:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

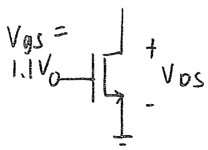
But $V_{DS} = V_{eff} = V_{GS} - V_{tn}$

$$\therefore \quad I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{tn})^2 - \frac{1}{2} (V_{GS} - V_{tn})^2 \right]$$

$$= \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{tn})^2$$

$$= (1) \qquad \text{Q.E.D.}$$

1.7)



Estimate I_D when $V_{DS} = V_{eff} + 0.3V$.

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{gs} - V_{tn})^2 (1 + \lambda (V_{DS} - V_{eff}))$$

Find $\lambda \triangleq \dfrac{k}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$

where $k = \sqrt{\dfrac{2 K_s \epsilon_0}{q N_A}} = \sqrt{\dfrac{2 \times 11.8 \times 8.85 \times 10^{-12}}{1.602 \times 10^{-19} \times 10^{22}}}$

$$= 3.612 \times 10^{-7} \ m/\sqrt{V}$$

For $V_{DS} = V_{eff}$,

$$\lambda = \frac{3.612 \times 10^{-7}}{2 \times 1.5 \times 10^{-6} \sqrt{0.9}} = 0.127 \ V^{-1}$$

Now

$$I_D \Big|_{V_{DS} = V_{eff}} = \frac{92}{2} \mu A/V^2 \times \frac{50}{1.5} \times (1.1 - 0.8)^2 (1 + \cancel{\lambda} \times 0)$$

$$= \underline{138 \mu A}$$

$$I_D \Big|_{V_{DS} = V_{eff} + 0.3V} = \frac{92}{2} \frac{\mu A}{V^2} \times \frac{50}{1.5} \times (1.1 - 0.8)^2 (1 + 0.127 \times 0.3)$$

$$= \underline{143 \mu A}$$

which translates into a 4% increase in current.

1.8) Find r_{DS} and λ.

Given: $I_D|_{V_{DS}=V_{eff}} = 20\mu A$

$I_D|_{V_{DS}=V_{eff}+0.5V} = 23\mu A$

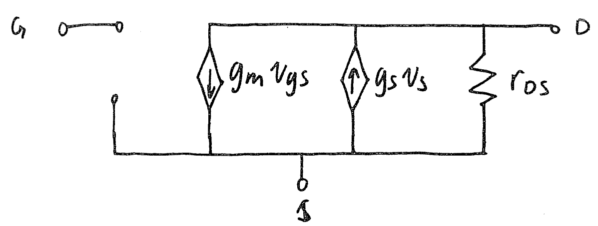
Now $I_D|_{V_{DS}=V_{eff}+0.5V} = I_D|_{V_{DS}=V_{eff}} \times (1+\lambda\times 0.5V)$

$$\lambda = 2\left(\frac{23\mu A}{20\mu A} - 1\right)$$

$$\underline{\lambda = 0.3\ V^{-1}}$$

$$r_{DS} = \frac{\Delta V}{\Delta I} = \frac{0.5V}{3\mu A} = \underline{167k\Omega}$$

1.9) Derive model parameters g_m, g_s and r_{DS}.



$$\lambda|_{V_{DS}=V_{eff}} = \frac{k}{2L\sqrt{\Phi_0}}$$

from Problem 1.7, $k = 3.61\times10^{-7}\ m/\sqrt{V}$

$$\therefore \lambda|_{V_{DS}=V_{eff}} = \frac{3.61\times10^{-7}}{2\times1.2\times10^{-6}\sqrt{0.9}} = \underline{0.159\ V^{-1}}$$

$$\text{Now } I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{tn})^2$$

$$= \frac{92}{2}\frac{\mu A}{V^2}\times\frac{10}{1.2}(1.1-0.8)^2$$

$$= \underline{34.5\mu A}$$

$$\therefore r_{DS} = \frac{1}{\lambda I_D} = \frac{1}{0.159\times34.5\times10^{-6}} = \underline{182k\Omega}$$

(cont.)

1.9) (cont.)

$$g_m = \sqrt{2 \mu_n C_{ox} W/L \cdot I_D} = \sqrt{2 \times 92 \frac{\mu A}{V^2} \times \frac{10}{1.2} \times 34.5 \times 10^{-6}} = 230 \ \mu A/V$$

$$g_s = \frac{\gamma g_m}{2\sqrt{V_{SB} + 2|\phi_F|}}$$

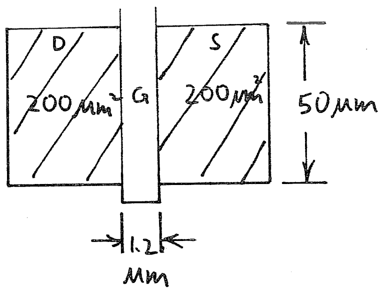
Assume $\gamma = 0.5$

$$\phi_F = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) = \frac{k\,300}{q} \ln\left(\frac{10^{22}}{1.1\times 10^{16}}\right) = 0.35V$$

$$= \frac{0.5 \times 230 \times 10^{-6}}{2\sqrt{1 + 0.7}}$$

$$= 44 \ \mu A/V$$

1.10) Find C_{gs}, C_{gd}, C_{db}, C_{sb}



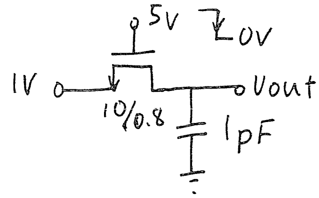
$$C_{gs} = \tfrac{2}{3} WL C_{ox} + C_{ov} W$$
$$= \tfrac{2}{3} \times 50 \times 1.2 \times 1.9 \times 10^{-3} + 2 \times 10^{-4} \times 50$$
$$= 86 \ fF$$

$$C_{gd} = C_{gd\text{-}ov} \cdot W = 2 \times 10^{-4} \times 50$$
$$= 10 \ fF$$

$$C_{db} = A_d C_j + P_d C_{jsw}$$
$$= 200 \ \mu m^2 \cdot 2.4 \times 10^{-4} \frac{pF}{\mu m^2} + 58 \times 2 \times 10^{-4}$$
$$= 60 \ fF$$

$$C_{sb} = C_j(A_s + WL) + C_{jsw} P_s$$
$$= 2.4 \times 10^{-4}(200 + 50 \times 1.2) + 2 \times 10^{-4} \times 58$$
$$= 74 \ fF$$

1.11)



Find Vout

channel charge $Q_{ch} = WL\,C_{ox}\,(V_{GS} - V_{tn})$

$= 10 \times 0.8 \times 0.0019\,((5-1)-0.8)$

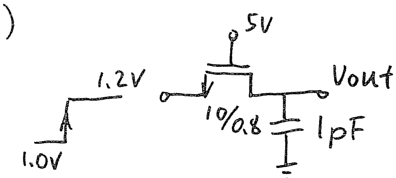
$= \underline{48.6\ fC}$

Assuming $\frac{1}{2} \times Q_{ch}$ is injected into the storage capacitor,

$$\Delta V = -\frac{\frac{1}{2} Q_{ch}}{C_L} = -\frac{1}{2} \times 48.6\,fC / 1\,pF = -24\,mV$$

$\therefore$ $V_{out} = 1V - \Delta V = \underline{0.976\ V}$

1.12)



Find settling time.

$\tau = r_{DS} C_L$

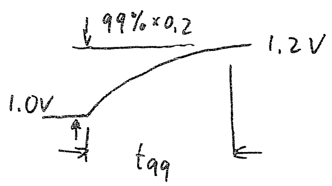
where $r_{DS} = \dfrac{1}{\mu_n C_{ox} \frac{W}{L} V_{eff}} = \dfrac{1}{92 \times 10^{-6} \times \frac{10}{0.8} \times 3.2V}$

$= 272\,\Omega$

$\therefore$ $\tau = 272\,\Omega \times 1\,pF$

$= \underline{0.27\ nsec}$

For a 99% settling time,



$V_{out}(t) = 1V + 0.2\,(1 - e^{-t/\tau})$

$\therefore$ find t_{99} where $1 - e^{-t_{99}/\tau} = 0.99$

$e^{-t_{99}/\tau} = 0.01$

$t_{99} = 4.6\,\tau$

$= \underline{1.25\,nsec}$

$\therefore$ the settling time is 1.25 nsec.

1.12) (cont.)

Repeat for V_{in} 3.0V → 3.1V

$$r_{DS} = \frac{1}{92\times10^{-6}\times\frac{10}{0.8}(5-3-0.8)} = 725\,\Omega$$

$\therefore\ \tau = r_{DS}C_L = 725\,\Omega \times 1\,pF = 0.725\ nsec$

and $t_{99\%} = 4.6\tau = \underline{3.33\ nsec}$

1.13) Repeat Problem 1.12, this time accounting for the body effect.

changes the threshold voltage

$$V_{tn} = V_{tn0} + \gamma\left(\sqrt{V_{SB}+2|\phi_F|} - \sqrt{2|\phi_F|}\right)$$

For $V_{in} = 1V$,

$$V_{tn} = 0.8V + 0.5\left(\sqrt{1+0.7} - \sqrt{0.7}\right)$$
$$= 1.03V$$

$$\therefore\ r_{ds} = \frac{1}{\mu_n C_{ox}\frac{W}{L}V_{eff}} = \frac{1}{92\times10^{-6}\times\frac{10}{0.8}\times(4-1.03)}$$

$$= 293\,\Omega$$

$\therefore\ \tau = 293\times1\,pF = 293\,ps$

leading to $t_{99\%} = \underline{1.35\ ns}$

FOR $V_{in} = 3V \Rightarrow V_{tn} = 1.34V$

$r_{ds} = 1.32\,k\Omega \qquad \tau = 1.32\ ns$

$t_{99\%} = \underline{6.1\ ns}$

1.14)

$$g_m = \frac{I_C}{V_T} = \frac{0.1mA}{26mV} = 3.8\ mA/V$$

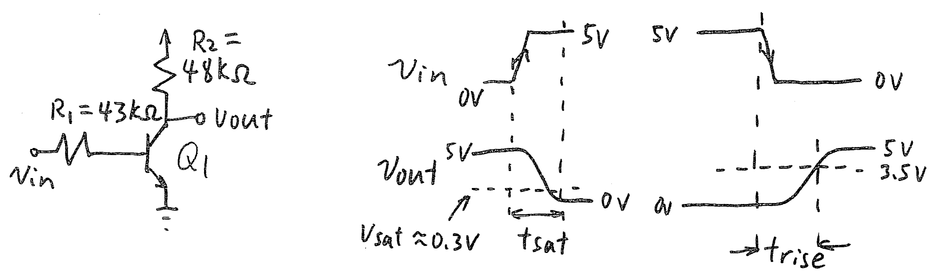
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{3.8\ mA/V} = 26\ k\Omega$$

$$r_e = \alpha / g_m = \frac{100}{101} \times \frac{1}{3.8\ mA/V} = 260\ \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{80V}{0.1mA} = 800\ k\Omega$$

$$g_m r_o = 3.8\ mA/V \times 800k\Omega = 3 \times 10^3$$

1.15)



Find tsat and trise.

Part 1 : Estimating tsat

The process of saturating the output is comprised of two phases :

① The transistor must first require time, ton, to turn on.

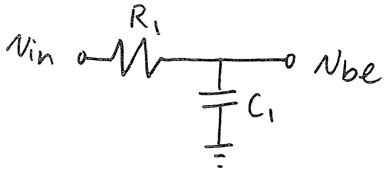
② The transistor then requires time, tfall, to drive the output low.

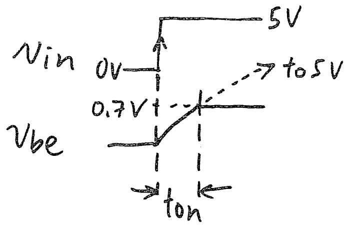
(cont.)

1.15) (cont.) [Part 1, phase 1]

Phase ① - turning transistor on

The transistor is initially off and will remain so until $v_{be} \approx 0.7V$. Before then, the following circuit model can be used:





for a simple RC network

$$v_{be} = 5V \times \left(1 - e^{-t/\tau_{in}}\right)$$

where $\tau_{in} = R_1 C_1$

$$R_1 = 43k\Omega$$

$$C_1 = C_{be}\big|_{v_{in}=0V} + C_{cb}\big|_{V_{cb}=5V}$$

$$= C_{je0} + \frac{C_{jc0}}{\left(1+V_{cb}/\phi_{c0}\right)^{1/3}} = 15 + \frac{18}{\left(1+\frac{5V}{0.7}\right)^{1/3}} fF$$

$$= 24 fF$$

$$\therefore \quad \tau_{in} = 43k\Omega \times 24fF = 1.0\,nsec$$

To determine the time for v_{be} to reach 0.7V, t_{on},

$$v_{be}(t) = 5\left(1 - e^{-t_{on}/\tau_{in}}\right)V \equiv 0.7\,V$$

$$e^{-t_{on}/\tau_{in}} = 0.86$$

$$t_{on} = 0.15\,\tau_{in}$$

$$t_{on} = 150\,psec$$

(cont.)

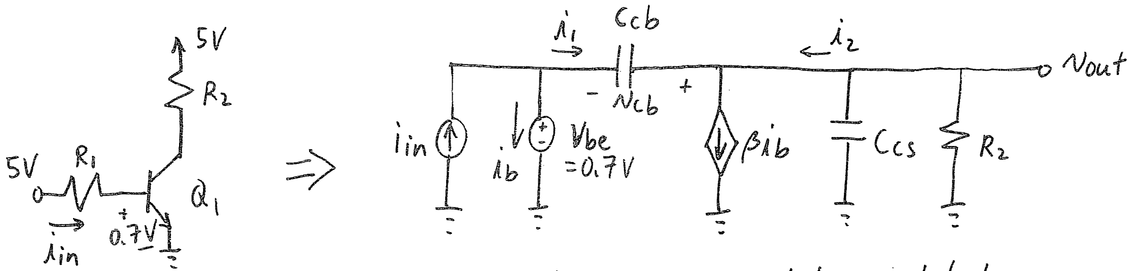
1.15 (cont.) [Part 1, phase 2]

Phase ② - Pulling the output, V_{out}, Low

During this phase, the transistor is on and in the active mode. To analyze the large signal behaviour, we take note of two properties of BJT devices in order to create the model below:

1) V_{be} is approximately constant and equal to 0.7 V.

2) $i_c = \beta i_b$



$$i_{in} = \frac{5V - 0.7V}{43k\Omega}$$

$$i_{in} = 0.1\,mA$$

Schematic used to model large signal transients

Note that C_{be} is no longer significant as V_{be} is now constant.

$$C_{cb} = \frac{C_{jc0}}{(1 + V_{CB}/\phi_{co})^{1/3}}$$

Let $V_{CB} = 2.5V$ to represent the average value during t_{fall}.

$$= \frac{18\,fF}{(1 + 2.5/0.7)^{1/3}}$$

$$C_{cb} = 11\,fF$$

$$C_{cs} = \frac{C_{js}}{(1 + V_{cs}/\phi_{s0})^{1/2}} = \frac{40\,fF}{(1 + 2.5/0.64)^{1/2}} = 18\,fF$$

Now $\quad i_b = i_{in} - i_1 \quad \boxed{1}$

$$i_1 = -C_{cb}\frac{dV_{cb}}{dt} = -C_{cb}\frac{dV_{out}}{dt} \quad \boxed{2}$$

$$i_2 = -C_{cs}\frac{dV_{out}}{dt} \quad \boxed{3}$$

(assume effectively all of i_2 passes through C_{cs})

and $\quad \beta i_b = i_1 + i_2 \quad \boxed{4}$

1.15 (cont.) [Part 1, phase 2]

$\boxed{1} \rightarrow \boxed{4}$ $\qquad \beta(i_{in} - i_1) = i_1 + i_2$

$$\beta i_{in} = (\beta+1) i_1 + i_2$$

sub $\boxed{2}, \boxed{3}$

$$= -((\beta+1) C_{cb} - C_{cs}) \frac{dV_{out}}{dt}$$

$$\therefore \underbrace{\frac{dV_{out}}{dt} = \frac{-\beta i_{in}}{(\beta+1)C_{cb} + C_{cs}}}$$

$$= \frac{-100 \times 10^{-4}}{(101 \times 11 + 18) \times 10^{-15}}$$

$$\underbrace{\frac{dV_{out}}{dt} = -8.86 \text{ V/nsec}}$$

$\therefore$ For the output to drop $(5 - 0.2)\text{V} = 4.8\text{V}$,

$$t_{fall} = \Delta V / \frac{dV_{out}}{dt} = -4.8\text{V} / -8.86 \text{ V/nsec}$$

$$\underbrace{t_{fall} = 540 \text{ psec}}$$

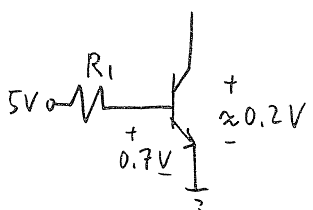
$\therefore$ The total time required to saturate Q_1 is

$$t_{sat} = t_{on} + t_{fall} = 150 + 540 \text{ psec}$$

$$\underbrace{t_{sat} = 690 \text{ psec}}$$

Part 2: Estimating t_{rise}

For the case where V_{in} drops from 5V to 0V, Q_1 starts out in saturation. In order to pull the output up, the saturation charge, Q_S, must first be removed. After this, the base minority charge, Q_B, is removed. The times representing these two phases are t_s and t_b.



$$\therefore \underbrace{t_{rise} = t_s + t_b}$$

(cont.)

1.15 (cont.) [Part 2]

Now $t_s = \tau_R \ln\left[\dfrac{I_{BR} + I_B}{I_{BR} + I_C/\beta}\right]$ (1.140)

where $I_{BR} = \dfrac{V_{fwd\text{-}diode}}{R_1} = \dfrac{0.7\,V}{43\,k\Omega} = 16\,\mu A$

$I_B = \dfrac{V_{in_0} - V_{fwd\text{-}diode}}{R_1} = \dfrac{5-0.7\,V}{43\,k\Omega} = 0.1\,mA$

$I_C = \dfrac{V_{DD} - V_{sat}}{R_2} = \dfrac{5-0.2\,V}{48\,k\Omega} = 0.1\,mA$

$\tau_R = 4\,nsec$

$\therefore\ t_s = 4\times10^{-9} \ln\left[\dfrac{16\times10^{-6} + 10^{-4}}{16\times10^{-6} + 10^{-4}/100}\right] = 7.7\,nsec$

$t_b = \dfrac{Q_b}{I_{BR}} = \dfrac{\tau_b I_C}{I_{BR}} = \dfrac{12\times10^{-12}\times100\times10^{-6}}{16\times10^{-6}} = 0.08\,nsec$

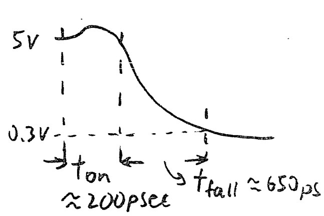
$\left\{\begin{array}{l} \therefore\ t_{rise} = 7.7 + 0.08 = 7.8\,nsec \\ \text{and } t_{sat} = 690\,psec \end{array}\right\}$

1.16) SPICE results:

$\left\{\begin{array}{l} t_{rise} = 10.3\,nsec \\ t_{sat} = 850\,psec \end{array}\right\}$

Thus our estimates are accurate to within about 25 percent. It is interesting to note that the different phases of each of the rise and fall times are clearly visible in the simulation results.

tsat



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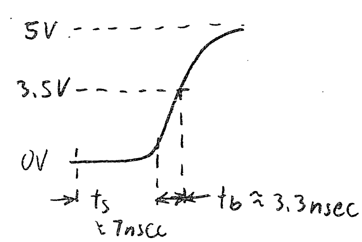

1.17) Verify Eq. (1.140) $\rightarrow$ Eq. (1.139) for $I_{BR} \gg (I_B$ & $I_C/\beta)$

Eq. (1.140): $$t_s = \tau_s \ln\left[\frac{I_{BR} + I_B}{I_{BR} + I_C/\beta}\right]$$

$$= \tau_s \ln\left[1 + \frac{I_B - I_C/\beta}{I_{BR} + I_C/\beta}\right]$$

$$\approx \tau_s \ln\left[1 + \underbrace{\frac{I_B - I_C/\beta}{I_{BR}}}_{\approx 0 \;\because\; I_{BR} \gg I_B \text{ or } I_C/\beta}\right]$$

$\therefore$ We can approximate this function with a first order Taylor Series Expansion

$$f(x)\Big|_{x \approx 0} \approx f(0) + f'(0)\,x$$

For $f(x) = \tau_s \ln[1 + x]$

$$f(x)\Big|_{x \approx 0} \approx \tau_s\left[\cancel{\ln[1+0]} + \frac{1}{1+0} \times x\right]$$

$$= \tau_s x$$

$$\therefore t_s = f\left(\frac{I_B - I_C/\beta}{I_{BR}}\right)$$

$$\therefore \underbrace{t_s \cong \tau_s \times \frac{I_B - I_C/\beta}{I_{BR}}}$$ which is Eq. (1.139)

Q.E.D.