4.1) \[ \text{dBm}(50\Omega) = 10 \log \frac{V_{\text{rms}}^2}{50 \text{ m}} \]
\[ \text{dBm}(75\Omega) = 10 \log \frac{V_{\text{rms}}^2}{75 \text{ m}} \]
\[ \therefore \text{dBm}(50\Omega) - \text{dBm}(75\Omega) = 10 \log \frac{75}{50} \approx 1.76 \]
\[ \therefore \text{dBm}(50\Omega) = \text{dBm}(75\Omega) + 1.76 \]

4.2) \[ 10 \log \frac{P_1}{1 \text{ mW}} = -20 \Rightarrow P_1 = 10 \mu \text{W} \]
\[ 10 \log \frac{P_2}{1 \text{ mW}} = -23 \Rightarrow P_2 = 5 \mu \text{W} \]
\[ a) \quad P_{\text{total}} = P_1 + P_2 = 15 \mu \text{W} \Rightarrow P_{\text{total}} = -18.24 \text{ dBm} \]
\[ b) \quad P_{\text{total}} = P_1 + P_2 + 2\sqrt{P_1 P_2} = 19.24 \mu \text{W} \]
\[ \Rightarrow P_{\text{total}} = -17.16 \text{ dBm} \]
\[ c) \quad P_{\text{total}} = -15.35 \text{ dBm} \]
\[ d) \quad P_{\text{total}} = -30.67 \text{ dBm} \]

4.3) Expected dBm for 10 Hz bandwidth = \(-40 - 10 \log \frac{30}{10} \)
\[ = -44.77 \text{ dBm} \]
\[ -40 = 10 \log \frac{P}{1 \text{ mW}} \Rightarrow P = 0.1 \mu \text{W} \]

(cont.)
4.3) \(\text{Spectral density} = \frac{0.1 \mu W}{30 \text{Hz}} = \frac{3.3}{1000} (\text{mV})^2/\text{Hz}\)

\[\Rightarrow \text{root spectral density} = 0.058 \frac{\text{mV}}{\sqrt{\text{Hz}}}\]

4.4) \(10 \log \frac{P}{1\text{mW}} = -60 \Rightarrow P = 1\text{mW}\)

\[V_n^2(f)\bigg|_{f=0.1\text{Hz}} = \frac{1\text{mW}}{1\text{mHz}} = 1 \frac{(\text{mV})^2}{\text{Hz}}\]

Assuming \(V_n^2(f) = \frac{Kv^2}{f}\) and \(V_n^2(f)\bigg|_{f=0.1\text{Hz}} = 1 \frac{(\text{mV})^2}{\text{Hz}}\),

\[V_n^2(f) = \frac{0.1 (\text{mV})^2}{f}\]

Total noise power in \((1\text{mHz}, 1\text{Hz})\) is:

\[\int_{1\text{mHz}}^{1\text{Hz}} V_n^2(f) df = 0.1 (\text{mV})^2 \left[ \ln 1 - \ln 1\text{mHz} \right] = 0.69 \mu \text{W}\]

\[10 \log \frac{0.69 \mu \text{W}}{1\text{mHz}} = -31.6 \text{ dBm}\]

4.5) \(V_{n1}^2(f) = 4KTR_1 \Rightarrow V_{n2}^2(f) = 4KTR_2\)

The two noise sources are uncorrelated.

\[V_n^2(f) = V_{n1}^2(f) + V_{n2}^2(f) = 4KTR_1 + 4KTR_2\]

\[= 4KT (R_1 + R_2)\]

which is equal to the noise spectral density of the series combination of \(R_1 \& R_2\).
4.6) \( V_{no}^2(f) = \frac{4KTR}{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2} \)  
where \( \omega_{3dB} = \frac{1}{RC} \)

For \( C = 100 \text{ pF} \) and \( R = 1 \text{ k} \Omega \), \( \omega_{3dB} = 10^7 \text{ rad/sec} \)

and \( V_{no}^2(0) = (4.08 \text{ nV})^2 = 1.66 \times 10^{-17} \text{ V}^2 \)

For \( C = 100 \text{ pF} \) and \( R = 1 \text{ M} \Omega \), \( \omega_{3dB} = 10^4 \text{ rad/sec} \)

and \( V_{no}^2(0) = (129 \text{ nV})^2 = 1.66 \times 10^{-14} \text{ V}^2 \)

\[ V_R(f) = \sqrt{4KTR} \]

\[ V_{no}(f) \]

\[ V_{no}^2(f) \]

\[ V_{no}^2(f) \mid_{R = 1 \text{ M} \Omega} \]

\[ V_{no}^2(f) \mid_{R = 1 \text{ k} \Omega} \]

The area underneath both curves are the same and is equal to \( \frac{KT}{C} = (6.45 \text{ mV})^2 \)

4.7) \( I_{nf}^2 = 8.28 \times 10^{-25} \left( \frac{A^2}{\text{Hz}} \right) \);
\( I_{n1}^2 = 1.656 \times 10^{-24} \left( \frac{A^2}{\text{Hz}} \right) \)

\( I_{n-}^2(f) = 100 \times 10^{-24} \left( \frac{A^2}{\text{Hz}} \right) \)

\( V_{n01}^2(0) = (I_{nf}^2(0) + I_{n1}^2(0) + I_{n-}^2(0)) R_f \)

\( \approx I_{n-}^2(0) R_f^2 = (0.2 \frac{\text{ mV}}{\text{sqrt Hz}})^2 \)

(cont.)
4.7) (cont.) \[ V_{n_{o1}}^2 (\text{rms}) = (0.2 \, \text{mV} \, \sqrt{\text{Hz}}) \cdot \frac{1}{4 (20 \, \text{k} \, \Omega)(1\pi)} = (22.4 \, \text{mV})^2 \]

\[ V_{n_{o2}}^2 (0) = V_n^2 (f) (1 + \frac{R_f}{R_1}) = (60 \, \text{mV} \, \sqrt{\text{Hz}})^2 \]

The noise is low pass filtered at \( f_0 = \frac{1}{2\pi R_F C_f} \approx 8 \, \text{KHz} \)
until \( f_1 = \frac{R_F}{R_1} f_0 = 16 \, \text{KHz} \).

\[ \therefore V_{n_{o2}}^2 (\text{rms}) = (60 \times 10^{-9})^2 \pi x 8K \cdot 12 \times 10^{-9} \pi \cdot 1M - 16K \]
\[ \approx (25.9 \, \text{mV})^2 \]

The total output noise is estimated to be:

\[ V_{n_{o}}^2 (\text{rms}) = V_{n_{o1}}^2 (\text{rms}) + V_{n_{o2}}^2 (\text{rms}) = (34.2 \, \text{mV})^2 \]

\[ \Rightarrow V_{n_{o}} (\text{rms}) = 34.2 \, \text{mV} \, \text{rms} \]

4.8) Since the opamp is assumed to be ideal and noiseless, we have \( V_X = 0 \) at all times.

\[ \therefore V_{n_X}^2 (f) = 0 \Rightarrow V_{n_X} (\text{rms}) = 0 \]

This noise is smaller than \( \frac{KT}{1\pi F} \) since the voltage across the capacitor is fixed by another device (i.e. opamp).

For the noise at \( V_o \), we have:

\[ V_{n_{o}}^2 (f) = (i_{R}^2 (f) + i_{R_f}^2 (f)) \frac{R_F^2}{1 + \omega^2 R_F^2 C^2} \]
\[ = (4KT \frac{R}{R} + 4KT \frac{R}{R_F}) R_F x \frac{\pi}{2} \frac{1}{2\pi R_F C} \]

(cont.)
\[ V_{no}^2(f) = \frac{2KT}{C} = \frac{2KT}{1nF} \]

The noise at \( V_0 \) is twice as much as \( \frac{KT}{1nF} \).

The reason for this increase is the additional noise current source \( i_R(f) \). Since the voltage across this current source is fixed (i.e., it is equal to zero), it does not affect the \( \omega_{3dB} \) of the filter and acts as an ideal noise current source!

\[ I_{no}^2(f) = \left| \frac{R}{R + j\omega L} \right|^2 I_R^2(f) \]

\[ = \frac{1}{1 + \frac{L^2}{R^2} \omega^2} \frac{4KT}{R} \]

\[ I_{no}(\text{rms}) = \int_0^\infty \frac{4KT/R}{1 + \frac{L^2}{R^2} \omega^2} \, df \]

\[ = \frac{4KT}{R} \frac{1}{2\pi} \frac{R}{L} \int_0^\infty \frac{d\omega}{1 + \omega^2} = \frac{KT}{L} \]
a) For Circuit (I):

\[ \frac{V_o}{V_i} = \frac{-1}{1 + j\omega (7K)(80n)} \quad (1) \]

For Circuit (II):

\[ (\frac{V_o}{2} + j\omega (80n)) + \frac{V_i}{14K} ) 14K = -V_o \]

\[ \Rightarrow \frac{V_o}{V_i} = \frac{-1}{1 + j\omega (80n)(7K)} \quad (2) \]

(1) & (2) show identical transfer functions.

b) \( V_{n0}^2 (0) = \left[ I_{n1}^2 (f) + I_{nf}^2 (f) \right] R_f^2 = (15.2 \ \frac{mV}{\sqrt{Hz}})^2 \)

\[ \therefore V_{n0(rms)} = V_{n0}^2 (0) \frac{1}{4R_F C_F} = (0.32 \ mV)^2 \]

c) the transfer function for current source entering the negative terminal of opamp 1 is:

\[ \frac{V_o}{I_{i1}} = \frac{-R_F}{1 + j\omega \ \frac{C_F R_F}{2}} \]

\[ \Rightarrow V_{no1}^2 (f) = \left[ I_{n1}^2 (f) + I_{nf}^2 (f) \right] \frac{R_F^2}{1 + \omega^2 \ \frac{C_F R_F}{4}} \]

\[ V_{no1}^2 (0) = \left[ I_{n1}^2 (f) + I_{nf}^2 (f) \right] R_F^2 = (21.5 \ \frac{mV}{\sqrt{Hz}})^2 \]

\[ V_{no1(rms)} = V_{no1}^2 (0) \frac{1}{4R_F C_F} = (0.45 \ mV)^2 \]

the transfer function for the current source entering the negative terminal of opamp 2 is:

\[ \frac{V_o}{I_{i2}} = \frac{-R_2}{1 + \frac{1}{j\omega C_F R_F/2}} = -j\omega \ \frac{R_2 R_F C_F}{2} \quad (cont.) \]
4.10) (cont.) This is the transfer function of a high-pass filter, which means, ideally, the total output noise due to any current noise at the negative terminal of opamp 2 is $\infty$.

4.11) \[ \frac{V_o}{V_i} = \frac{-R_F/R_1}{1+j\omega C_F R_F} \]

$C_F$ is reduced by a factor of 1000. Therefore, to keep the same transfer function, $R_F$ & $R_1$ must be increased by the same factor. The parameters of the new circuit are:

$R_F = 7 \text{M} \Omega$, $R_1 = 7 \text{M} \Omega$, $C_F = 80 \text{ pF}$

\[ V_{no}(0) = [I_{n1}(f) + I_{nf}(f)] R_F^2 = 8KTR_F \]

\[ V_{no}(\text{rms}) = 8KTR_F \frac{1}{4R_F C_F} = \frac{2KT}{C_F} \]

The total noise power is just a function of $C_F$ (for the case $R_F = R_1$) and increases by decreasing $C_F$:

\[ V_{no}(\text{rms}) = \left( 10.2 \text{ mV} \right)^2 \]

4.12) The $\frac{1}{f^2}$ tangent line touches the spectral density of Fig. 4.3 at 1000 Hz. The noise power can be estimated as:

\[ V_n(\text{rms}) = 10 \left( \frac{\text{mV}}{\text{Hz}} \right)^2 \times \frac{\pi \times 1000}{2} = \left( 125 \text{ mV} \right)^2 \]
Estimate output noise by integrating from 1 to 10 kHz

\[ V_{n0(rms)}^2 = \int_{1}^{10} 400 \frac{1}{f} df + \int_{10}^{100} \frac{400^2}{f} df + \int_{100}^{1000} \frac{0.4^2(10k)^2}{f^2} df \]

\[ = 7.92 \times 10^4 (mV)^2 + 7.37 \times 10^5 (mV)^2 + 1.44 \times 10^4 (mV)^2 \]

\[ = 8.31 \times 10^5 (mV)^2 \approx (1 \mu V)^2 \text{ roughly } 1 \mu V \text{ RMS} \]

4.14) A: Graphical Approach

**Region 1:** \(0.01 < f < 1\)

\[ V_{n1}^2 = \int_{0.01}^{1} V_n^2(f) df = 4.6 \times 10^{-16} \frac{V^2}{Hz} \]

**Region 2:** \(1 < f < 100\)

\[ V_{n2}^2 = 99 \times 10^{-16} \frac{V^2}{Hz} \]

**Region 3:** \(100 < f < 1000\)

\[ V_{n3}^2 = \int_{100}^{1000} \frac{10}{f^2} df = 90 \times 10^{-16} \frac{V^2}{Hz} \]

**Region 4:** \(1000 < f < \infty\)

\[ V_{n4}^2 = 10 \times \frac{\pi^2}{2} \times 9000 = 14.1 \times 10^{-15} \frac{V^2}{Hz} \]

**\(V_n(rms)\)**

\[ V_n(rms) = V_{n1}^2 + V_{n2}^2 + V_{n3}^2 + V_{n4}^2 = (0.18 \mu V)^2 \]

(cont.)
4.14) (cont.) B: Using \( \frac{1}{f} \) tangent line

The \( \frac{1}{f} \) line touches the curve at 100 Hz \& 10 KHz simultaneously. If we approximate the noise at 100Hz and 10 KHz with two lowpass-filtered noise, we will have:

\[
V_n^2 = 10^{-16} \times \frac{\pi}{2} \times 100 + 10^{-18} \times \frac{\pi}{2} \times 9000 = (0.17 \text{ mV})^2
\]

4.15) \( A(s) = \frac{A_0}{(1 + \frac{s}{2\pi f_0})^2} \Rightarrow A(j\omega) = \frac{A_0}{(1 + j \frac{\omega}{f_0})^2} \)

\Rightarrow \text{equivalent noise bandwidth} = \frac{1}{A_0^2} \int_0^\infty |A(j\omega)|^2 \, df

\[
= f_0 \int_0^\infty \frac{df}{(1+f^2)^2} = f_0 \frac{\pi}{4}
\]

4.16) The equivalent circuits for noise calculations are shown below:

![Fig. P4.16(a)](image1)

![Fig. P4.16(b)](image2)
SNR for the current mirror of Fig. P4.16(a):

Using superposition for $V_{ni1} + V_{ni2}$, and approximating $\beta + 1 \approx \beta = 100$, we have:

$$I_{no}(f) = [V_{ni1}(f) + V_{ni2}(f)] \frac{\beta^2}{(r_b + r_{\pi})^2}$$

where $V_{ni1}(f) = V_{ni2}(f) = 4kT r_b = 5.46 \frac{(nv)^2}{Hz}$

and $r_{\pi} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = 2.6 \Omega$

$\therefore I_{no}(f) = 0.0127 \frac{(nA)^2}{Hz}$

$\therefore I_{no}(\text{rms}) = 50 \text{ MHz} \times I_{no}(f) = (0.8 \text{ mA})^2$

$\therefore \text{SNR} = 20 \log \frac{100 \mu}{0.8 \mu} = 41.94 \text{ dB}$

SNR for the current mirror of Fig. P4.16(b):

$$I_{no}(f) = [V_{ni1}(f) + V_{ni2}(f) + V_{ni3}(f) + V_{ni4}(f)] \frac{\beta^2}{(r_b + r_{\pi} + r_e)^2}$$

where $V_{ni3}(f) = V_{ni4}(f) = 4kT r_e = 3.31 \frac{(nv)^2}{Hz}$

$V_{ni1}(f) = V_{ni2}(f) = 4kT r_b = 5.46 \frac{(nv)^2}{Hz}$

$r_{\pi} = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = 2.6 \Omega$

$\therefore I_{no}(f) = 0.0003336 \frac{(nA)^2}{Hz}$

$\therefore I_{no}(\text{rms}) = 50 \text{ MHz} \times I_{no}(f) = (0.13 \text{ mA})^2$

$\therefore \text{SNR} = 20 \log \frac{100 \mu}{0.13 \mu} = 57.7 \text{ dB}$
4.17) Using (4.61): \[ V_{n01}(0) = (I^2_{n1}(0) + I^2_{n2}(0) + I^2_{n3}(0)) R_F^2 \]

where \[ I_{nf}(f) = \frac{4kT}{R_F} = 1.035 \left( \frac{PA}{V_{Hz}} \right)^2 \]

\[ V^2_{n01}(0) = (2 \times 0.6 + 1.035) (16k)^2 \left( \frac{PV}{V_{Hz}} \right) = (21.2 \frac{nv}{V_{Hz}})^2 \]

\[ V^2_{n01}(rms) = V^2_{n01}(0) \times \frac{1}{4R_F C_F} = (2.65 \mu V)^2 \]

Using (4.62): \[ V^2_{n02}(0) = V^2_n(f) \left( 1 + \frac{R_F}{R_1} \right)^2 = (220 \frac{nv}{V_{Hz}})^2 \]

This noise is lowpass filtered at \[ f_0 = \frac{1}{2\pi R_F C_F} \]

until \[ f_1 = \frac{R_F}{R_1} f_0 \]

where the noise gain reaches unity.

\[ V^2_{n02}(rms) = (220 \times 10^{-9})^2 \frac{1}{4R_F C_F} + (20 \times 10^{-9})^2 \frac{\pi}{2} (f_t - f_1) \]

\[ = (44.16 \mu V)^2 \]

Using (4.64): \[ V^2_{n0}(rms) = (44.24 \mu V)^2 \]

SNR for a 100 mV signal: \[ 20 \log \frac{100 \text{ mV}}{44.24 \mu V} \]

\[ = 67.08 \text{ dB} \]
Using (4.97):

\[
V_{neq}^2(f) = \left( \frac{16}{3} \right) KT \frac{1}{g_{m1}} + \frac{16}{3} KT \left( \frac{g_{m3}}{g_{m1}} \right)^2 \frac{1}{g_{m3}}
= \left( 5.75 \frac{nv}{\sqrt{Hz}} \right)^2
\]

Since \( g_m \propto \sqrt{I_{BIAS}} \), if \( I_{BIAS} \) is doubled, \( g_{m1} \) and \( g_{m3} \) will be increased by \( \sqrt{2} \). Therefore, \( V_{neq}^2(f) \) will be reduced by \( \sqrt{2} \), and we have:

\[
V_{neq}^2(f) = \frac{1}{\sqrt{2}} \left( 5.75 \frac{nv}{\sqrt{Hz}} \right)^2 = (4.84 \frac{nv}{\sqrt{Hz}})^2
\]