Chapter 11 - Problems

11.1) Largest voltage output $= (2^N - 1)\, V_{LSB}$

$$= (2^{10}-1)\, 1\,mV$$

$$= \underline{1.023\ V}$$

11.2) Calculate SNR.

Maximum possible SNR $= 6.02N + 1.76$ dB

$$= 6.02 \times 12 + 1.76 \text{ dB}$$

$$= \underline{74 \text{ dB}}$$

This is provided the input $V_{pp} = V_{ref}$. In our case,

$$V_{ppin} = 1/3\ V_{ref}$$

$\therefore$ we lose $20\log(1/3) = 9.5$ dB in SNR

$\therefore$ $\underline{SNR = 74 - 9.5 \text{ dB} = 64.5 \text{ dB}}$

For a SNR of 0 dB, the input signal needs to be 74 dB below its full scale level.

$\therefore$ $V_{ppin} = 3V/10^{(74/20)} = 3/5012 = \underline{0.6\ mV_{pp}}$

For comparison, $V_{LSB} = 3V/2^{12} = \underline{0.73\ mV}$

$\therefore$ The first quantization threshold is at $\frac{1}{2} V_{LSB} = \underline{0.36\ mV}$ and the signal will only span two quantization levels.

11.3) For 2's complement coding, we recognize that it is obtained from offset-binary coding by complementing the MSB.

$\therefore$ the output is represented by

$$V_{out} = V_{ref}\left[(1-b_1)2^{-1} + b_2 2^{-2} + \ldots + b_n 2^{-N}\right] - 0.5\,V_{ref}$$

$$V_{out} = V_{ref}\left[-b_1 2^{-1} + b_2 2^{-2} + \ldots + b_n 2^{-N}\right]$$

11.4)

Decimal		Two's Complement	
+5	$\rightarrow$	0101	
+5	$\rightarrow$	0101	
10	=	1010	$\leftarrow$ represents −6 (temporalily incorrect)
−7	$\rightarrow$	1001	
+3	=	0011	✓

$\therefore$ summation of 2's complement words results in the correct answer.

11.5)

	Decimal	Two's Complement
Example 1	+5	$\boxed{0}$ 0101
	+7	$\boxed{0}$ 0111
	+12	$\boxed{0}$ 1100
Example 2	−5	$\boxed{1}$ 1011
	−7	$\boxed{1}$ 1001
	−12	$\boxed{1}$ 0100
Example 3	−5	$\boxed{1}$ 1011
	+7	$\boxed{0}$ 0111
	+2	$\boxed{0}$ 0010

From the three examples, we see that we simply need to copy the MSB to increase the word size of two's complement numbers. Math is performed as before.

11.6)

Decimal	Two's Complement
+8	01000
-8	11000

$\therefore$ An adder that only added one of these to another number would do the following:

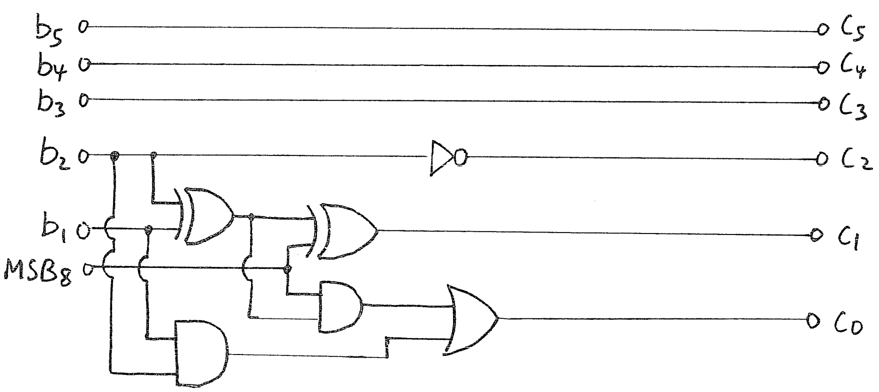
X	b_1	b_2	b_3	b_4	b_5
± 8	$\mathrm{MSB}_8 \rightarrow \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$	1	0	0	0
SUM	$C_0 C_1$	C_2	C_3	C_4	C_5

It is clear that

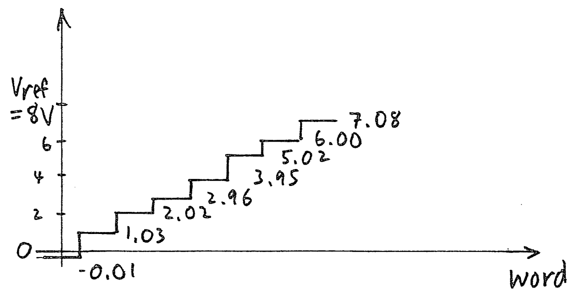
$$C_5 = b_5$$
$$C_4 = b_4$$
$$C_3 = b_3$$
$$C_2 = \overline{b_2}$$
$$C_0 C_1 = b_1 + b_2 + \mathrm{MSB}_8 \Rightarrow$$

this operation requires a full-adder circuit.

Thus the final circuit is:



11.7)



$V_{LSB} = \frac{8V}{2^3} = 1V$

$Err_{off} = -0.01$ LSB

$Err_{gain} =$ real full scale (in LSB) − ideal full scale value (in LSB)

$$= \frac{7.08V - (-0.01)}{1V} - 7 \text{ LSB}$$

$Err_{gain} = 0.09$ LSB

Compensating for offset and gain errors, the new steps in LSB's are given by

$$L_i = \frac{V_i}{V_{LSB}} - Err_{off} - E_{gain} \times \frac{i}{2^N-1}, \quad i = 0, \dots, 2^N-1$$
$$= 0, \dots, 7$$

$\therefore$

$L_0 = 0$ LSB	$L_7 = 7$ LSB	guaranteed
$L_1 = 1.027$ LSB	$L_6 = 5.933$ LSB	data-dependent
$L_2 = 2.004$ LSB	$L_5 = 4.966$ LSB	
$L_3 = 2.931$ LSB	$L_4 = 3.909$ LSB	

From these results, the integral non-linearity errors (INL) are

$\{0 : +0.027 : +0.004 : -0.069 : -0.091 : -0.034 : -0.067 : 0\}$ LSB

maximum INL is −0.091 LSB

and the differential Non-linearity errors (DNL) are

$\{+0.027 : -0.023 : -0.073 : -0.022 : +0.057 : -0.033 : 0.067\}$ LSB

maximum DNL is −0.073 LSB

11.8) Find absolute and relative accuracies.

The absolute errors are :

$\{-0.01, +0.03, +0.02, -0.04, -0.05, +0.02, +0.00, \underline{0.08}\}$ V

maximum

Thus, the largest deviation is 80mV and this corresponds to 1 LSB when $V_{ref} = 8\,V$

i.e.,

$$\frac{8V}{2^{N_{effabs}}} = 80mV$$

$$2^{N_{effabs}} = 100$$

$$\underline{N_{effabs} = 6.6 \text{ bits}}$$

For relative accuracy, the maximum INL error is -0.091 LSB or -91 mV

$$\therefore \quad \frac{8V}{2^{N_{effrel}}} = 91mV$$

$$2^{N_{effrel}} = 87.9$$

$$\underline{N_{effrel} = 6.5 \text{ bits}}$$

Thus, the converter has an absolute and relative accuracy of 6.6 bits and 6.5 bits respectively.

11.9) $V_{LSB} = \frac{V_{ref}}{2^N} = \frac{10.24V}{2^{10}} = 10\,mV$

$\therefore$ We need to keep errors within $\pm \frac{1}{2} V_{LSB} = \pm 5mV$

Now a change in the reference voltage causes the greatest error at the full range level

$$V_{full\ range} = (1 - 2^{-10})\, V_{ref}$$

$$\Rightarrow \quad Err_{full\ range} = (1 - 2^{-10})\, Err_{ref}$$

$$\therefore \underline{Err_{ref}} = \frac{Err_{full\ range}}{1 - 2^{-10}} = \frac{\pm 5mV}{0.999} = \underline{\pm 5.005\,mV}$$

Maximum temperature coefficient

$$= \frac{Err_{max} - Err_{min}}{Temp_{max} - Temp_{min}} = \frac{2 \times 5.005\,mV}{50°C} = \underline{200\,\mu V/°C}$$

11.10) $V_{LSB} = \frac{V_{ref}}{2^N} = \frac{4V}{2^2} = 1V$

$\underline{Err_{offset} = 0.01\ LSB}$

$$Err_{gain} = \frac{V_{11} - V_{10}}{V_{LSB}} - V_{11\,ideal} = \frac{3.02 - 0.01}{1} - 3\ LSB$$

$$= \underline{0.01\ LSB}$$

Absolute errors : $\{+0.01, +0.02, \underline{-0.03}, +0.02\}$

$\therefore$ Maximum absolute error is $\underline{-0.03V = -0.03\,LSB}$

For worse relative accuracy error, find INL errors using

$$L_i = \frac{V_{out}}{V_{LSB}} - Err_{offset} - Err_{gain} \times 1/3$$

$L_{00} = 0\,LSB$ $\qquad$ $L_{11} = 3\ LSB$

$L_{01} = 1.007\,LSB$ $\qquad$ $L_{10} = 1.953\,LSB$

$\therefore$ INL errors $= \{0 : +0.007 : \underline{-0.047} : 0\}$

$\therefore$ Max. relative error is $\underline{-0.047\,LSB}$ which corresponds to $\underline{6.4\ bits}$ accuracy.

11.11) With an ideal converter, the maximum quantization error is

$$\tfrac{1}{2} V_{LSB} = \tfrac{1}{2} \, \frac{5V}{2^{12}} = 0.61 \text{ mV}.$$

However, given an absolute accuracy of 0.5 LSB, we add $0.5 V_{LSB}$ to the quantization error

$\therefore$ the maximum error is now $1 V_{LSB} = 1.22 \text{ mV}$

11.12) From Eq. (11.28),

$$\Delta t < \frac{1}{2^N \pi f_{in}} = \frac{1}{2^{16} \pi \, 20\text{kHz}}$$

$$\Delta t < 0.24 \text{ nsec}$$

$\therefore$ the sampling time uncertainty should be less than 0.24 nsec.