Chapter 14 – Problems

14.1) Using (14.14): $80 = 6.02 + 1.76 + 10 \log OSR$

$\Rightarrow OSR = 16672472$!

$f_s = (OSR) \times 2 f_0 = \underline{33.345 \text{ GHz}}$!

14.2)

n	$x(n)$	$x(n+1)$	$y(n)$	$e(n)$
0	0.1	−0.5	1	0.9
1	−0.5	0.9	−1	−0.5
2	0.9	0.3	1	0.1
3	0.3	−0.3	1	0.7
4	−0.3	1.1	−1	−0.7
5	1.1	0.5	1	−0.1
6	0.5	−0.1	1	0.5
7	−0.1	1.3	−1	−0.9
8	1.3	0.7	1	−0.3
9	0.7	0.1	1	0.3
10	0.1	−0.5	1	0.9 ⇐ repeat !

The state, $x(n)$, repeats every 10 cycle. Therefore, there is a tone in the output stream with the frequency of $\underline{\frac{f_s}{10}}$!

14.3) Since $x(n+1) = 1.1 + x(n) - y(n)$, if $x(0) = 0.1$, $y(n)$ will always remain 1, and $x(n)$ increments by 0.1 at each cycle until it saturates!

n	$x(n)$	$x(n+1)$	$y(n)$
0	0.1	0.2	1
1	0.2	0.3	1
2	0.3	0.4	1
..	..	..	..

14.4)

n	$u(n)$	$x(n)$	$x(n+1)$	$y(n)$
0	10	0.1	9.1	1
1	−10	9.1	−1.9	1
2	10	−1.9	9.1	−1
3	−10	9.1	−1.9	1 ⇐ repeat !

14.5) Doubling OSR improves the accuracy by 0.5 bit. For a 4-bit improvement, OSR will be

$$OSR = 2^{4/0.5} = 256 \Rightarrow f_s = 2f_o \times OSR = \underline{512 \text{ MHz}}$$

14.6) For a 4-bit improvement in accuracy using a 1st-order $\Delta\Sigma$, we must have: $6.02 \times 4 + 5.17 = 30 \log(OSR)$

(i.e. $6.02(8) + 1.76 - 5.17 + 30 \log(OSR) = 6.02(12) + 1.76$)

$$\Rightarrow OSR = 9.44 \Rightarrow f_s = 2f_o \times OSR = \underline{18.88 \text{ MHz}}$$

Using a 2nd-order $\Delta\Sigma$, we must have:

$$6.02 \times 4 + 12.9 = 50 \log(OSR) \Rightarrow OSR = 5.49$$

$$f_s = 2f_o \times OSR = \underline{10.98 \text{ MHz}}$$

14.7) Using a 1st-order $\Delta\Sigma \Rightarrow 80 = 6.02 + 1.76 - 5.17 + 30 \log(OSR)$

$$\Rightarrow \underline{OSR = 380} \Rightarrow \underline{f_s = 760 \text{ KHz}}$$

Using a 2nd-order $\Delta\Sigma \Rightarrow 80 = 6.02 + 1.76 - 12.9 + 50 \log(OSR)$

$$\Rightarrow \underline{OSR = 50.4} \Rightarrow \underline{f_s = 100.8 \text{ kHz}}$$

14.8) Assuming quantization noise as an independent noise signal:

output signal power = input signal power + quant. noise power

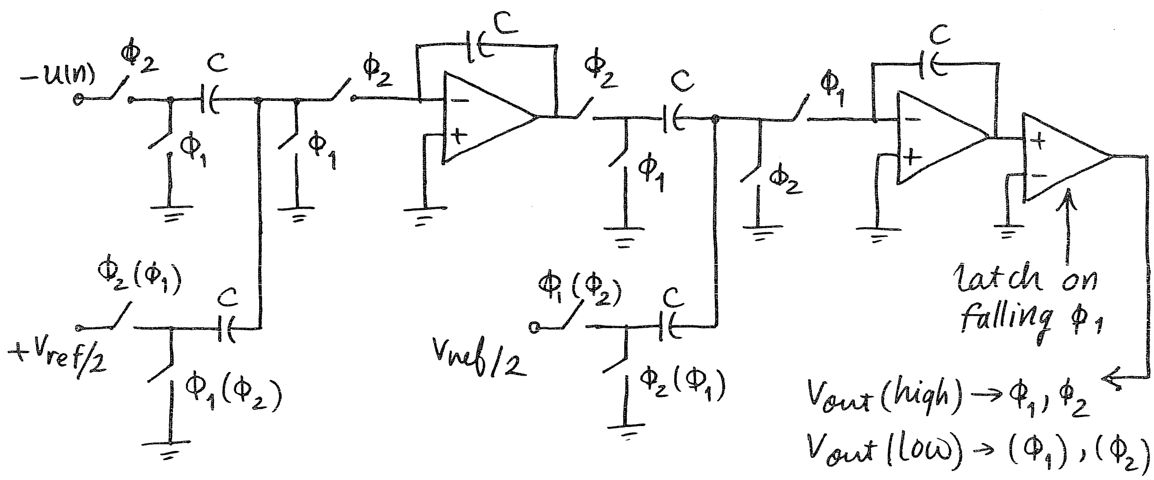
$$\Rightarrow 1W = \frac{(0.5)^2}{2} + P_e \Rightarrow P_e = 0.875\,W$$

$$\Rightarrow \frac{P_s}{P_e} = \frac{0.125}{0.875} = \frac{1}{7} = -8.5\ dB$$

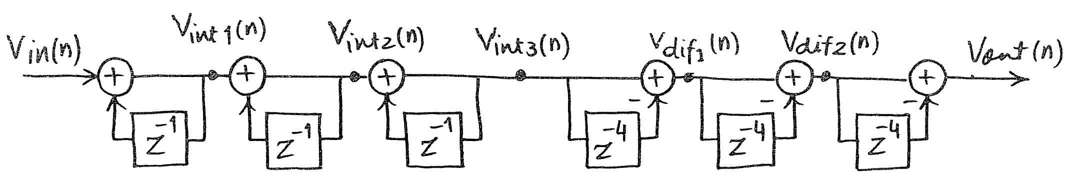
14.9) $G(z) = (1-z^{-1})^2 \Rightarrow G(z)-1 = (1-z^{-1})^2 - 1$

$$\therefore G(z)-1 = (1-z^{-1}-1)(1-z^{-1}+1) = z^{-1}(z^{-1}-2)$$

14.10)



14.11)

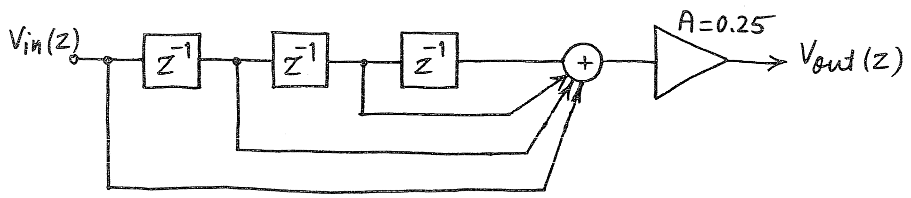


14.11) (cont.)

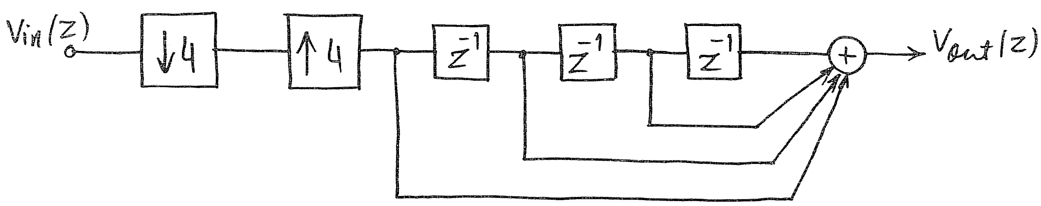
	0	1	2	3	4	5	6	7	8	9	10	11	12
$V_{in}(n)$	0	1	1	−1	1	1	−1	1	1	−1	1	1	−1
$V_{int1}(n)$	0	1	2	1	2	3	2	3	4	3	4	5	4
$V_{int2}(n)$	0	1	3	4	6	9	11	14	18	21	25	30	34
$V_{int3}(n)$	0	1	4	8	14	23	34	48	66	87	112	142	176
$V_{dif1}(n)$	0	1	4	8	14	22	30	40	52	64	78	94	110
$V_{dif2}(n)$	0	1	4	8	14	21	26	32	38	42	48	54	58
$V_{out}(n)$	0	1	4	8	14	20	22	24	24	21	22	22	20
$\frac{1}{64}V_{out}(n)$						0.31	0.34	0.37	0.37	0.33	0.34	0.34	0.31

The steady-state value of $\frac{1}{64}V_{out}(n)$ is close to $\frac{1}{3}$.

14.12)



Block diagram of a running-average filter of length 4.



Block diagram of a hold system with length 4.

While the first system is a time-invariant system, the second system is time-varying due to the down-sampler!

14.13) $Y(z) = z^{-1}\left[z^{-1}U(z) + (1-z^{-1})E_1(z)\right]$
$\qquad - (1-z^{-1})\left[z^{-1}E_1(z) + (1-z^{-1})E_2(z)\right]$

$$= \underline{z^{-2}U(z) - (1-z^{-1})^2 E_2(z)}$$

14.14) Using (14.27) & (14.28), for a 2nd-order $\Delta\Sigma$ mod. we have:

$$y_1(n) = z^{-1}u(n) + (1-z^{-1})^2 e_1(n)$$

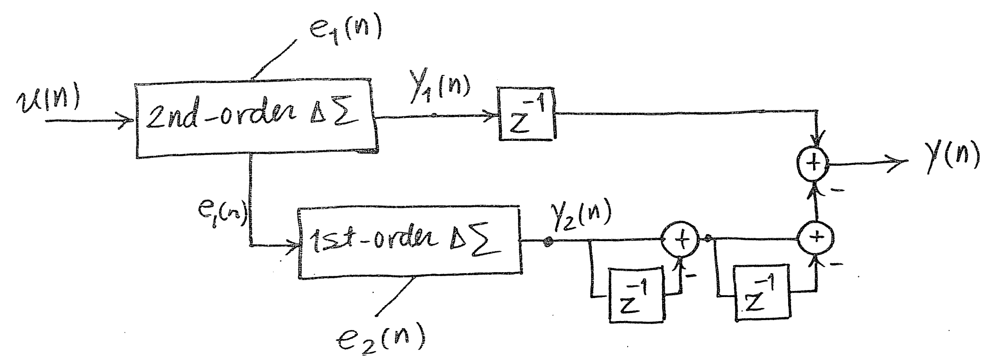
If $e_1(n)$ is fed into a 1st-order $\Delta\Sigma$ mod., the output, $y_2(n)$, will be:

$$y_2(n) = z^{-1}e_1(n) + (1-z^{-1})e_2(n)$$

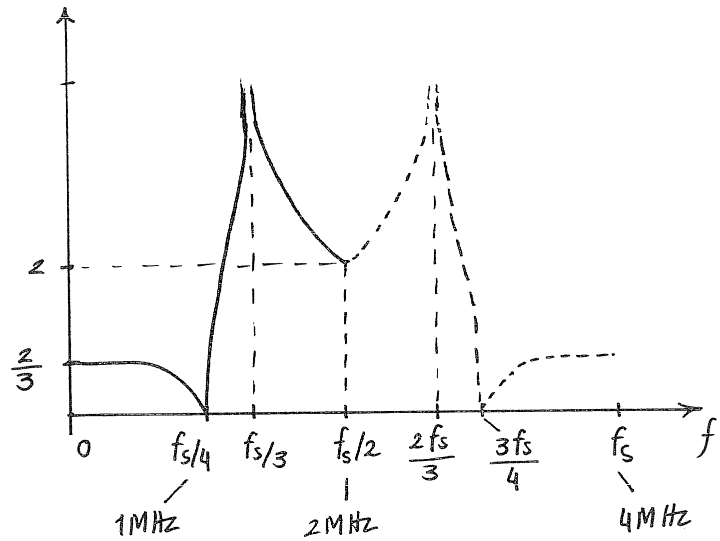
In order to eliminate $e_1(n)$ in a combination of $y_1(n)$ & $y_2(n)$, $y_1(n)$ must be multiplied by z^{-1} and $y_2(n)$ by $(1-z^{-1})^2$. The final output, $y(n)$, will be:

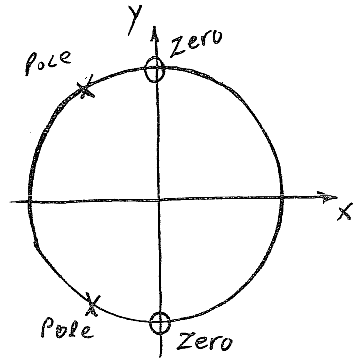
$$y(n) = z^{-1}y_1(n) - (1-z^{-1})^2 y_2(n) = z^{-1}u(n) - (1-z^{-1})^3 e_2(n)$$

The block diagram for this system is shown below:



14.15) $N_{TF}(z) = \dfrac{1}{1+H(z)} = \dfrac{1+z^{-2}}{1+z^{-1}+z^{-2}}$





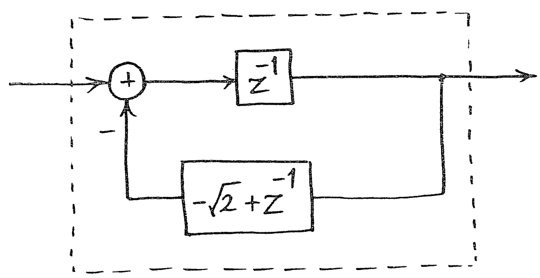
Z-plane

Poles should be adjusted for better stability.

14.16) $H(z)$ is required to have its pole at $e^{\pm j\pi/4}$. Therefore,

$$(z-e^{j\pi/4})(z-e^{-j\pi/4}) = z^2-\sqrt{2}z+1$$

$$\Rightarrow H(z) = \frac{z}{z^2-\sqrt{2}z+1} = \frac{z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$



$H(z)$

14.17) $0 : \{1, -1, 1, -1, 1, -1, \cdots\}$

$$\Rightarrow V_{ave} = \frac{A_1 + A_0}{2} + \frac{\delta_1 + \delta_2}{2} = \underbrace{V_{ave\text{-}ideal}\Big|_0 + \frac{\delta_1 + \delta_2}{2}}$$

$\frac{1}{2} : \{1, 1, 1, -1, 1, 1, 1, -1, \cdots\}$

$$\Rightarrow V_{ave} = \frac{3A_1 + A_0}{4} + \frac{\delta_1 + \delta_2}{4} = \underbrace{V_{ave\text{-}ideal}\Big|_{1/2} + \frac{\delta_1 + \delta_2}{4}}$$

$-\frac{1}{2} : \{-1, -1, -1, 1, -1, -1, -1, 1, \cdots\}$

$$\Rightarrow V_{ave} = \frac{3A_0 + A_1}{4} + \frac{\delta_1 + \delta_2}{4} = V_{ave\text{-}ideal}\Big|_{-1/2} + \frac{\delta_1 + \delta_2}{4}$$

The averages lie on a straight line if $\delta_1 = -\delta_2$!