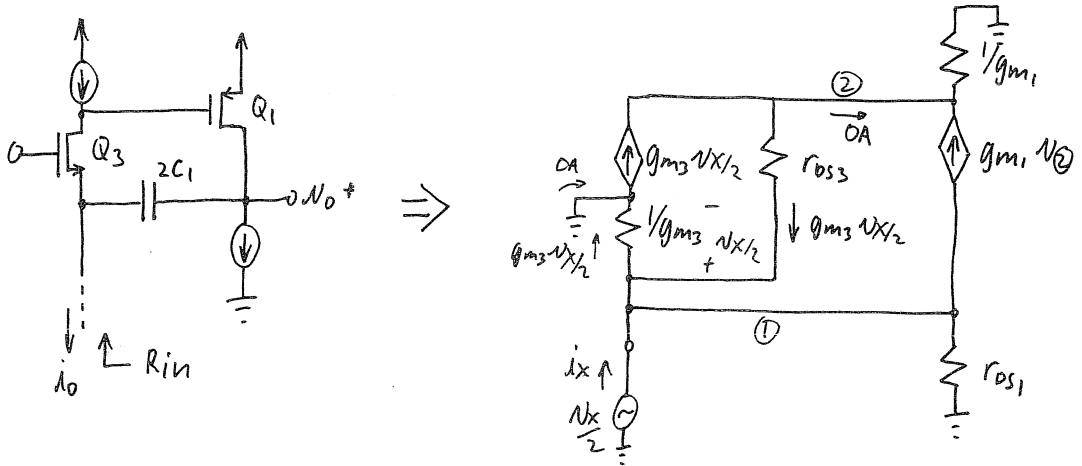


Chapter 15 - Problems

15.1) Show $R_{in} \approx \frac{2}{g_{m3}^2 r_{DS}}$

∵ circuit is perfectly symmetric, we only need to look at one half.



(capacitor $2C_1$ is modelled as a short circuit)

$$\text{KCL } \textcircled{1}: \quad i_x - g_{m3} \frac{v_x}{2} + g_{m3} \frac{v_x}{2} - g_{m1} v_{\textcircled{2}} - \frac{v_x/2}{r_{DS1}} = 0 \quad \textcircled{1}$$

$$\text{But } v_{\textcircled{2}} = \frac{v_x}{2} + g_{m3} r_{DS3} \frac{v_x}{2} \quad \textcircled{2}$$

$$\textcircled{2} \rightarrow \textcircled{1} \quad i_x - g_{m1} \left(\frac{v_x}{2} + g_{m3} r_{DS3} \frac{v_x}{2} \right) - \frac{v_x}{2 r_{DS1}} = 0$$

$$\therefore i_x = \frac{1}{2} \left(g_{m1} + g_{m1} g_{m3} r_{DS3} + \frac{1}{r_{DS1}} \right) v_x$$

$$\therefore R_{in} \triangleq \frac{v_x}{i_x} = \frac{2}{g_{m1} (1 + g_{m3} r_{DS3}) + 1/r_{DS1}} \approx \frac{2}{g_{m1} (1 + g_{m3} r_{DS3})}$$

$$\approx \frac{2}{g_{m1} g_{m3} r_{DS3}}$$

$$\therefore \underline{R_{in} \approx \frac{2}{g_{m3}^2 r_{DS}}} \quad \text{Q.E.D.}$$

(Note: g_{m3} is matched to g_{m2})

$$15.2) \quad H(s) = \frac{k_1 s + k_0}{s + \omega_0}$$

$$\therefore \text{DC gain} = \frac{k_0}{\omega_0} \equiv 10$$

$$\therefore k_0 = 10\omega_0$$

\therefore no finite zeros

$$\therefore k_1 = 0$$

$$\therefore \underline{H(s) = \frac{10\omega_0}{s + \omega_0}}$$

From the design equations found in Figure 15.9
and $\omega_0 = 15 \text{ MHz} \times 2\pi = 94.25 \times 10^6 \text{ rads/sec}$

$$\underline{C_x = C_A \frac{k_1}{1 - k_1} = 0}$$

$$C_A \equiv 5 \text{ pF}$$

$$\begin{aligned} g_{m1} &= k_0 (C_A + C_x) = 10 \times 94.25 \times 10^6 (5 \times 10^{-12}) \\ &= \underline{4.71 \text{ mA/V}} \end{aligned}$$

$$\begin{aligned} g_{m2} &= \omega_0 (C_A + C_x) = 94.25 \times 10^6 (5 \times 10^{-12}) \\ &= \underline{0.47 \text{ mA/V}} \end{aligned}$$

15.3)

$$H(s) = \frac{k_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \quad (k_1 = k_2 = 0 \text{ for low pass})$$

$$H(0) = \frac{k_0}{\omega_0^2} \equiv 5 \quad \Rightarrow \underline{k_0 = 5\omega_0^2}$$

$$\omega_0 = 2\pi \times 10 \text{ MHz} = 62.8 \times 10^6 \text{ rads/sec}$$

$$Q = 1$$

Select reasonable values for C_A and C_B

$$\text{Let } \underline{C_A = C_B \equiv 2 \text{ pF}} \quad (C_x = 0 \because \text{LP filter})$$

$$\text{Then } g_{m1} = \omega_0 C_A = 62.8 \times 10^6 \times 2 \times 10^{-12} = \underline{0.13 \text{ mA/V}}$$

$$g_{m2} = \omega_0 C_B = \underline{0.13 \text{ mA/V}}$$

$$g_{m3} = \frac{\omega_0 C_B}{Q} = \underline{0.13 \text{ mA/V}}$$

$$g_{m4} = k_0 C_A / \omega_0 = 5\omega_0 C_A = \underline{0.63 \text{ mA/V}}$$

$$g_{m5} = Q$$

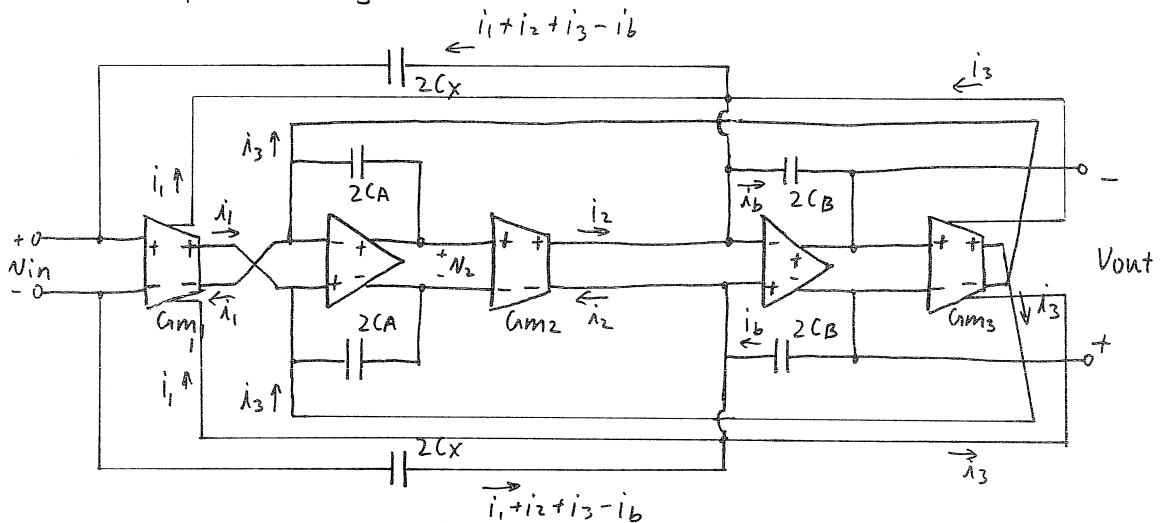
15.4) Derive design equations for Fig. 15.24.

$$i_1 = g_{m1} V_{in} \quad [1] \qquad i_3 = -g_{m3} V_{out} \quad [2]$$

$$V_2 = \frac{2(i_1 + i_3)}{2sC_A} = \frac{i_1 + i_3}{sC_A} \quad [3]$$

$$i_2 = g_{m2} V_2 \quad [4]$$

$$V_{out} = i_b / sC_B \quad [5]$$



KVL from V_{in} through $2C_X$ capacitors:

$$V_{in} + \left(\frac{i_1 + i_2 + i_3 - i_b}{2sC_X} \right) \times 2 = 0$$

$$\text{OR } i_b = i_1 + i_2 + i_3 + sC_X V_{in} \quad [6]$$

$$\therefore \text{[6]} \rightarrow \text{[5]} \quad V_{out} = \frac{i_b}{sC_B} = \frac{i_1 + i_2 + i_3 + sC_X V_{in}}{sC_B} \quad [7]$$

$$\text{[1], [2], [4]} \rightarrow \text{[7]} \quad = \frac{g_{m1} V_{in} + g_{m2} V_2 - g_{m3} V_{out} + sC_X V_{in}}{sC_B}$$

$$\therefore sC_B V_{out} = g_{m1} V_{in} + \frac{g_{m2}}{sC_A} (g_{m1} V_{in} - g_{m3} V_{out}) - g_{m3} V_{out} + sC_X V_{in}$$

$$\left(sC_B + \frac{g_{m2} g_{m3}}{sC_A} + g_{m3} \right) V_{out} = \left(g_{m1} + \frac{g_{m1} g_{m2}}{sC_A} + sC_X \right) V_{in}$$

$$(s^2 C_A C_B + g_{m2} g_{m3} + s g_{m3} C_A) V_{out} = (s g_{m1} C_A + g_{m1} g_{m2} + s^2 C_X C_A) V_{in}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{C_X C_A s^2 + C_A g_{m1} s + g_{m1} g_{m2}}{C_A C_B s^2 + C_A g_{m3} s + g_{m2} g_{m3}}$$

$$\frac{V_{out}}{V_{in}} = \frac{s^2 \frac{C_X}{C_B} + s \frac{g_{m1}}{C_B} + \frac{g_{m1} g_{m2}}{C_A C_B}}{s^2 + s \frac{g_{m3}}{C_B} + \frac{g_{m2} g_{m3}}{C_A C_B}} \quad \left[\text{which is similar to Eq. (15.24)} \right]$$

(cont.)

15.4 (cont.)

Equating our result to the generic second-order transfer function

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Gives the following design equations:

$$k_2 = \frac{C_X}{C_B} \Rightarrow \underline{C_X = k_2 C_B}$$

$$k_1 = \frac{G_{m1}}{C_B} \Rightarrow \underline{G_{m1} = k_1 C_B}$$

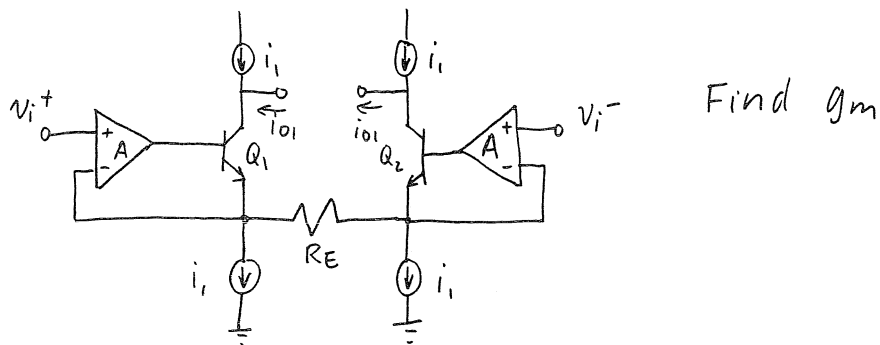
$$k_0 = \frac{G_{m1} G_{m2}}{C_A C_B} \Rightarrow \underline{G_{m2} = \frac{k_0 C_B C_A}{k_1 C_B} = \frac{k_0}{k_1} C_A} \quad (k_1 \neq 0)$$

$$\omega_0^2 = \frac{G_{m2} G_{m3}}{C_A C_B} \Rightarrow \underline{G_{m3} = \frac{C_A C_B}{G_{m2}} \omega_0^2 = \frac{C_A C_B}{\frac{k_0}{k_1} C_A} \omega_0^2 = \frac{k_1 C_B \omega_0^2}{k_0}} \quad (k_0 \neq 0)$$

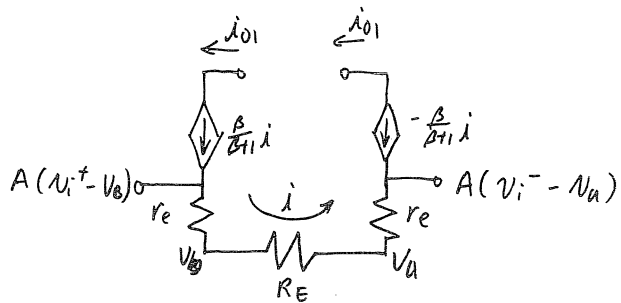
$$\frac{\omega_0}{Q} = \frac{G_{m3}}{C_B} \Rightarrow \underline{\frac{\omega_0}{Q} = \frac{k_1 C_B \omega_0^2}{k_0 C_B}}$$

$$\underline{\omega_0 Q = \frac{k_0}{k_1}}$$

15.5)



Small signal model :



$$i = \frac{A(V_{i^+} - V_b - V_{i^-} + V_a)}{2r_e + R_E}$$

But $V_b - V_a = i R_E$

$$\therefore i = \frac{A(V_{id} - i R_E)}{2r_e + R_E} \quad \text{where } V_{id} \triangleq V_{i^+} - V_{i^-}$$

$$i(2r_e + R_E + A R_E) = A V_{id}$$

$$\therefore \frac{V_{id}}{i} = R_E + \frac{2r_e + R_E}{A}$$

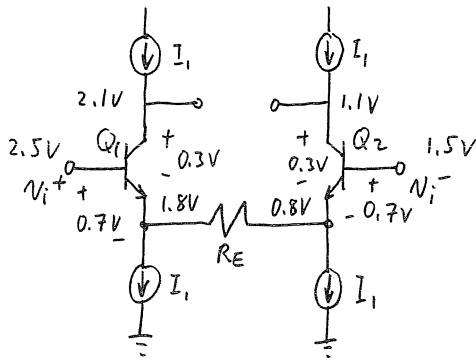
Now $i_{o1} = \frac{\beta}{\beta+1} i$

$$\therefore g_m \triangleq \frac{i_{o1}}{V_{id}} = \frac{i_{o1}}{i} \times \frac{i}{V_{id}} = \frac{\beta}{\beta+1} \times \frac{1}{R_E + \frac{2r_e + R_E}{A}}$$

Given β and $A \gg 1$

$$g_m \approx \frac{1}{R_E}$$

15.6) Find minimum collector voltages.

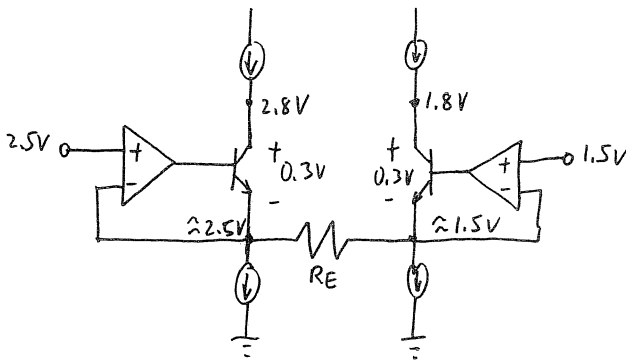


Given $V_i = \pm 1V$ and $V_{cm} = 2V$
 Consider the worst-case scenario where $V_{i+} = 2.5V$ and $V_{i-} = 1.5V$
 (Assume $V_{cesat} = 0.3V$)

From the diagram, we need $V_{C1} \geq 2.1V$

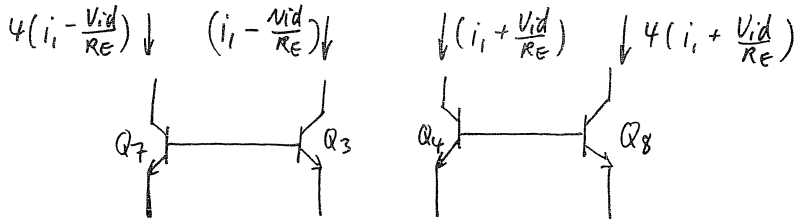
\therefore the minimum collector voltage is 2.1V

For Figure 15.13, we see that $V_{C1} \geq 2.8V$



\therefore the minimum collector voltage is 2.8V

15.7) Given the base-emitter areas of Q_7 & Q_8 are 4 times larger than those of Q_3 & Q_4 , we achieve a simple scaling of currents where



$$\therefore i_{o1} = \frac{4V_{id}}{R_E} \Rightarrow \underline{G_m = \frac{4}{R_E}}$$

15.8) Find I_1 and I_2

Q_1, Q_2, Q_5 and Q_6 have stable bias currents and only Q_3 and Q_4 have currents that vary significantly with the input signal.

∴ Ensure I_{c3} is within 20% of nominal which is I_1

Now when $V_i = 500\text{mV}$,

$$I_{c3} = I_1 - \frac{V_i}{R_E} \geq 0.8 I_1$$

$$\therefore I_1 \geq 5 \frac{V_i}{R_E}$$

Given

$$G_m = 1/R_E = 1\text{mA/V}$$

$$R_E = 1\text{k}\Omega$$

$$\text{and } I_1 \geq 5 \times 500\text{mV}/1\text{k}\Omega$$

$$\therefore \underline{I_1 \geq 2.5\text{mA}} \text{ which is rather large.}$$

We can reduce this by scaling Q_7 and Q_8 by 4 as in Problem 15.7. In this case,

$$G_m = 4/R_E \equiv 1\text{mA/V}$$

$$\therefore R_E = 4\text{k}\Omega$$

$$\text{and } \underline{I_1 \geq 5 \times 500\text{mV}/4\text{k}\Omega = 625\mu\text{A}}$$

The value of I_2 is not critical but should be small to prevent significant loading of bias current source I_1 .

$$\text{Let } \underline{I_2 = 1/4 I_1 = 156\mu\text{A}}$$

15.9) Find % error in output current.

From Eq (15.65),

$$g_m = \frac{8 I_1}{25 V_T} = \frac{8 \times 2 \text{mA}}{25 \times 26 \text{mV}} = 24.6 \text{ mA/V}$$

$$\therefore \text{Ideally, } i_o = g_m v_i = 24.6 \text{ mA/V} \cdot 0.048 \text{V}$$

$$\underline{i_o = 1.182 \text{ mA}}$$

For the actual output current, i_o

$$v_i = V_{be1} - V_{be2} \quad \text{where}$$

$$V_{be1} = V_T \ln\left(\frac{I_{C1}}{4 I_S}\right)$$

$$V_{be2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

$$\therefore v_i = V_T \left(\ln\left(\frac{I_{C1}}{4 I_S}\right) - \ln\left(\frac{I_{C2}}{I_S}\right) \right)$$

$$= V_T \ln\left(\frac{I_{C1}}{4 I_{C2}}\right)$$

$$\Rightarrow \underline{I_{C1} = 4 e^{v_i/V_T} I_{C2}}$$

$$\text{Now } I_1 = I_{e1} + I_{e2} = \frac{\beta+1}{\beta} (I_{C1} + I_{C2})$$

$$= \frac{\beta+1}{\beta} (4 e^{v_i/V_T} + 1) I_{C2}$$

Similarly

$$\therefore I_{C2} = \frac{\beta}{\beta+1} \times \frac{I_1}{1 + 4 e^{v_i/V_T}}$$

$$I_{C3} = \frac{\beta}{\beta+1} \frac{I_1}{1 + 4 e^{-v_i/V_T}}$$

$$\text{and } I_{C1} = \frac{\beta}{\beta+1} \times \frac{I_1}{1 + 1/4 e^{-v_i/V_T}}$$

$$I_{C4} = \frac{\beta}{\beta+1} \frac{I_1}{1 + 1/4 e^{v_i/V_T}}$$

$$i_o = -I_1 + I_{C1} + I_{C3} = 2 \text{mA} \left(-1 + \frac{100}{101} \left(\frac{1}{1 + 1/4 e^{-48/26}} + \frac{1}{1 + 4 e^{-48/26}} \right) \right)$$

$$= 1.119 \text{ mA}$$

$$\therefore \underline{\% \text{ error}} = \frac{i_o - i_{o \text{ ideal}}}{i_{o \text{ ideal}}} = \frac{1.119 - 1.182}{1.182} = \underline{-5.3\%}$$

(5.10)

a) Find $G_m = \mu_n C_{ox} (W/L)_q (V_{gsq} - V_{tnq})$ where

$$\begin{aligned} V_{gsq} &= V_{gq} - V_{sq} = V_c - V_{s1} = V_c - (V_{i^+} - V_{tn1}) \\ &= V_c + V_{tn1} - V_{i^+} \rightarrow \text{Assume } V_{i^+} = V_{i^-} = 2.5V \end{aligned}$$

for V_{tn1} ,

$$\begin{aligned} V_{tn1} &= V_{tn0} + \gamma (\sqrt{V_{sb1} + 2\phi_F} - \sqrt{2\phi_F}), \text{ Assume } \\ &= 0.8 + 0.5 (\sqrt{1.3 + 0.7} - \sqrt{0.7}) \quad \begin{array}{l} V_{sb1} \approx V_{i^+} - 1.2V \\ \approx 1.3V \end{array} \\ &= 1.09V \end{aligned}$$

$$\therefore V_{sb1} = 2.5 - 1.09V = 1.4V \quad \leftarrow \begin{array}{l} \text{Close enough} \\ \rightarrow \text{assumption accurate} \end{array}$$

$$\begin{aligned} \therefore V_{gsq} &= V_c + V_{tn1} - V_{i^+} = 5 + 1.09 - 2.5 \\ &= 3.59V \end{aligned}$$

$$\begin{aligned} V_{tnq} &= V_{tn0} + \gamma (\sqrt{V_{sb1} + 2\phi_F} - \sqrt{2\phi_F}) \\ &= 0.8 + 0.5 (\sqrt{1.4 + 0.7} - \sqrt{0.7}) \\ &= 1.106V \end{aligned}$$

$$\begin{aligned} \therefore G_m &= \mu_n C_{ox} (W/L)_q (V_{gsq} - V_{tnq}) \\ &= 92 \times 10^{-6} (2) (3.59 - 1.106) \end{aligned}$$

$$\therefore \underline{G_m = 0.46 \text{ mA/V}}$$

b) Find i_{o1} when $V_{i^+} = 2.6V$ & $3V$

$$\begin{aligned} i_{o1} /_{V_{i^+} = 2.6V} &= G_m \times (V_{i^+} - V_{i^-}) \\ &= 0.46 \text{ mA/V} (2.6 - 2.5) \\ &= \underline{46 \mu A} \end{aligned}$$

$$\begin{aligned} i_{o1} /_{V_{i^+} = 3.0V} &= G_m \times (V_{i^+} - V_{i^-}) \\ &= 0.46 \times (3.0 - 2.5) \\ &= \underline{230 \mu A} \end{aligned}$$

(cont.)

15.10 (cont.)

c) Find true i_{o1} when $V_{i^+} = 2.6V$ and $3.0V$

Assume that the threshold voltages of Q_1 and Q_2 remain equal to those found in part a).

$$\text{i.e., } V_{tn1} = 1.09V$$

$$V_{tn2} = 1.106V$$

$$i_{o1} = i_{o2} = \mu_n C_{ox} \left(\frac{W}{L}\right)_2 \left((V_{gs2} - V_{tn2}) V_{ds2} - \frac{V_{ds2}^2}{2} \right)$$

For $V_{i^+} = 2.6V$:

$$\begin{aligned} V_{gs2} &= V_L + V_{tn1} - V_{i^+} = 5 + 1.09 - 2.6 \\ &= 3.58V \end{aligned}$$

$$\text{and } V_{ds2} = 0.1V$$

$$\begin{aligned} \therefore i_{o1} &= 92 \times 10^{-6} \times (2) \left((3.58 - 1.106) 0.1 - \frac{(0.1)^2}{2} \right) \\ &= \underline{45 \mu A} \end{aligned}$$

$$\% \text{ error} = \frac{45 - 46}{46} \approx \underline{2\% \text{ error}}$$

For $V_{i^+} = 3.0V$:

$$\begin{aligned} V_{gs2} &= 5 + 1.09 - 3.0 \\ &= 3.09V \end{aligned}$$

$$\text{and } V_{ds2} = 0.5V$$

$$\begin{aligned} \therefore i_{o1} &= 92 \times 10^{-6} (2) \left((3.09 - 1.106) 0.5 - \frac{0.5^2}{2} \right) \\ &= \underline{160 \mu A} \end{aligned}$$

$$\therefore \% \text{ error} = \frac{160 - 230}{230} = \underline{-30\% \text{ error}}$$

15.11) Given

$$\frac{K_1}{K_3} = 6.7 \Rightarrow \frac{\frac{\mu_n C_{ox}}{2} (W/L)_1}{\frac{\mu_n C_{ox}}{2} (W/L)_3} = 6.7$$

$$\text{If } L_1 = L_2 = L,$$

$$\therefore W_1 = 6.7 W_3$$

To find W_3 ,

$$G_m = \frac{4k_1 k_3 \sqrt{I_1}}{(k_1 + 4k_3) \sqrt{K_1}} \equiv 0.3 \text{ mA/V}$$

$$\therefore \frac{4 \times 6.7 k_3 \sqrt{I_1}}{(6.7 + 4) k_3 \sqrt{6.7 k_3}} =$$

$$\sqrt{I_1 k_3} = 0.31 \times 10^{-3}$$

$$I_1 W_3 = \frac{(0.31)^2 \times 10^{-6}}{\frac{\mu_n C_{ox}}{2} \times 1/L} \quad \text{If } L = 1 \mu\text{m}$$

$$= \frac{0.31^2 \times 2 \times 10^{-6}}{92 \times 10^{-6}} = 2.09 \times 10^{-3} \text{ mA} \cdot \mu\text{m}$$

$$\text{Let } I_1 = 200 \mu\text{A}$$

$$\therefore \underline{W_3} = \frac{2.09 \times 10^{-3}}{200 \times 10^{-6}} \approx \underline{10 \mu\text{m}} = W_4$$

$$\therefore \underline{W_1} = 6.7 W_3 = \underline{67 \mu\text{m}} = W_2$$

} and $L = 1 \mu\text{m}$

15.12) Find $G_m = 4 K_{eq} (V_{c1} - V_{t_{eq}})$

Ignore body effect.

$$\text{Now } \frac{1}{\sqrt{K_{eq}}} = \frac{1}{\sqrt{K_n}} + \frac{1}{\sqrt{K_p}} \quad \text{where } K_n = \frac{\mu_n C_{ox}}{2} \frac{W}{L} = \frac{92 \times 10^{-6}}{2} \times 10/2$$

$$= 230 \times 10^{-6}$$

$$K_p = \frac{30 \times 10^{-6}}{2} \times 10/2 = 75 \times 10^{-6}$$

$$\therefore \frac{1}{\sqrt{K_{eq}}} = 181.5$$

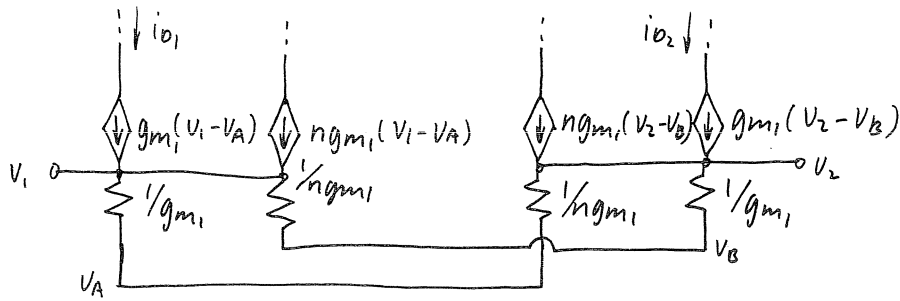
$$K_{eq} = 30 \times 10^{-6}$$

$$V_{t_{eq}} = V_{t_n} - V_{t_p} = 0.8 + 0.9 \text{ V} = 1.7 \text{ V}$$

$$\therefore G_m = 4 \times 30 \times 10^{-6} (2 - 1.7)$$

$$= \underline{36 \text{ mA/V}}$$

15.13) Show $G_m = \left(\frac{n}{n+1}\right) 4\sqrt{K I_B}$



$$i_{o1} = \frac{V_1 - V_2}{\frac{1}{g_{m1}} + \frac{1}{ng_{m1}}} \quad \text{and by symmetry, } i_{o2} = -i_{o1}$$

$$\therefore i_{o1} - i_{o2} = 2 \times \frac{V_1 - V_2}{\frac{1}{g_{m1}} + \frac{1}{ng_{m1}}}$$

\therefore the transconductance, G_m

$$G_m \triangleq \frac{i_{o1} - i_{o2}}{V_1 - V_2} = \frac{2}{\frac{1}{g_{m1}} + \frac{1}{ng_{m1}}} = \frac{2g_{m1}}{1 + 1/n}$$

$$= \frac{2n}{n+1} g_{m1} \quad \text{where } g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_B}$$

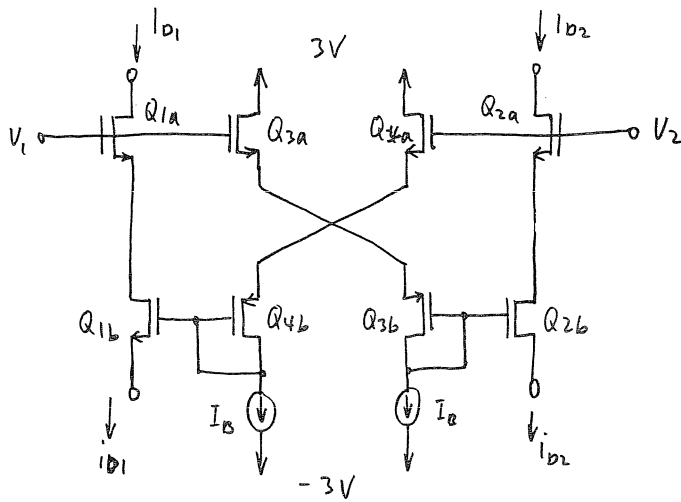
$$= \sqrt{4K I_B} \quad \text{for } K = \frac{\mu_n C_{ox} W}{2L}$$

$$\therefore G_m = \frac{2n}{n+1} \sqrt{4K I_B}$$

$$\underline{G_m = \left(\frac{n}{n+1}\right) 4\sqrt{K I_B}}$$

Q.E.D.

(15.14)



Find the maximum differential voltage centred about 0V.

$$V_{eff4b} = V_{eff3b} = \sqrt{\frac{2 I_B}{\mu_p C_{ox} W/L}} = \sqrt{\frac{2 \times 50 \times 10^{-6}}{30 \times 10^{-6} \times 10^2}} = 0.816 \text{ V}$$

$$V_{eff4a} = V_{eff3a} = \sqrt{\frac{2 I_B}{\mu_n C_{ox} W/L}} = \sqrt{\frac{2 \times 50 \times 10^{-6}}{92 \times 10^{-6} \times 10^2}} = 0.466 \text{ V}$$

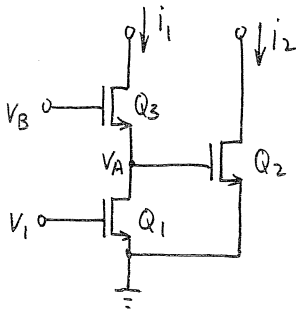
Assuming the voltage drops across current sources I_B can be as low as 0 volts, the minimum voltage level for V_1 and V_2 is

$$\begin{aligned} V_{1,2min} &= V_{SS} + V_{I_{Bmin}} - V_{tp3,4b} + V_{eff3,4b} + V_{tn3,4a} + V_{eff3,4a} \\ &= -3 + 0 + 0.9 + 0.816 + 0.8 + 0.466 \text{ V} \\ &= -18 \text{ mV} \end{aligned}$$

∴ the minimum level for V_1 and V_2 is -18 mV
and because V_1 and V_2 are centred around 0V,
∴ the maximum differential voltage is

$$V_1 - V_2 = 18 \text{ mV} - (-18 \text{ mV}) = \underline{\underline{36 \text{ mV}}}$$

15.15) Show $i_1 - i_2 = K(V_B - 2V_{tn})(2V_1 - V_B)$



$$i_{D1} = K(V_1 - V_{tn})^2$$

$$i_{D3} = K(V_B - V_A - V_{tn})^2$$

But $i_{D1} = i_{D3} \equiv i_1$

$$\therefore K(V_1 - V_{tn})^2 = K(V_B - V_A - V_{tn})^2$$

$$V_1 - V_{tn} = V_B - V_A - V_{tn}$$

OR $V_A = V_B - V_1$ \square

$$i_{D2} = K(V_A - V_{tn})^2 \quad \square$$

$$\square \rightarrow \square = K(V_B - V_1 - V_{tn})^2 \equiv i_2$$

$$\therefore i_1 - i_2 = K[(V_1 - V_{tn})^2 - (V_B - V_1 - V_{tn})^2]$$

$$= K[V_1^2 - 2V_1V_{tn} + V_{tn}^2 - (V_B^2 + V_1^2 + V_{tn}^2 - 2V_1V_B - 2V_BV_{tn} + 2V_1V_{tn})]$$

$$= K(-V_B^2 + 2V_1V_B - 4V_1V_{tn} + 2V_{tn}V_B)$$

$$\therefore \underline{i_1 - i_2 = K(V_B - 2V_{tn})(2V_1 - V_B)}$$

Q.E.D.

15.16) Refer to Figure 15.42 a) for schematic.

Given that we want a second-order low pass filter with a DC gain of 2,

$$H(s) = \frac{k_0}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} = \frac{2\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad \square$$

From Eq. (15.161)

$$\frac{V_O}{V_i} = \frac{\frac{C_1}{C_B} s^2 + \frac{G_2}{C_B} s + \frac{G_1 G_3}{C_A C_B}}{s^2 + \frac{G_5}{C_B} s + \frac{G_3 G_4}{C_A C_B}} \quad \square$$

(cont.)

15.16 (cont.)

Equating the coefficients of \square and \square ,

$$C_1 = G_2 = 0 \quad (\text{i.e., remove components})$$

$$C_A = C_B = 10 \text{ pF}$$

$$\frac{G_5}{C_B} = \omega_0 \Rightarrow G_5 = C_B \omega_0 = 10 \times 10^{-12} \times 2\pi \times 1 \text{ MHz} \\ = 63 \text{ mA/V}$$

$$\text{Now } G_1 G_3 = 2 C_A C_B \omega_0^2 = 2 (10 \times 10^{-12})^2 (2\pi \times 1 \text{ MHz})^2 \\ = 7.90 \times 10^{-9}$$

$$\text{and } G_3 G_4 = C_B C_B \omega_0^2 = 3.95 \times 10^{-9}$$

IF we let $G_3 = 100 \text{ mA/V}$,

$$G_1 = \frac{7.90 \times 10^{-9}}{100 \times 10^{-6}} = 79 \text{ mA/V} \quad \text{and}$$

$$G_4 = \frac{3.95 \times 10^{-9}}{100 \times 10^{-6}} = 40 \text{ mA/V}$$

Now transistor sizes,

$$G_i = \mu_n C_{ox} (W/L)_i (V_c - V_x - V_{tn})$$

$$\therefore G_1 = 92 \times 10^{-6} (W/L)_1 (3 - 0.8) \equiv 79 \text{ mA/V}$$

$$\therefore \underline{(W/L)_1 = 0.39}$$

Similarly,

$$G_3 = 92 \times 10^{-6} (W/L)_3 (3 - 0.8) = 100 \text{ mA/V}$$

$$\Rightarrow \underline{(W/L)_3 = 0.49}$$

$$\underline{(W/L)_4 = \frac{40 \text{ mA/V}}{92 \times 10^{-6} (3 - 0.8)} = 0.2}$$

$$\underline{(W/L)_5 = \frac{63 \text{ mA/V}}{92 \times 10^{-6} (3 - 0.8)} = 0.31}$$

and device G_2 is eliminated.

$$\begin{aligned}
 15.17) \quad \% \text{ THD} &= \left[\frac{P_{wr\ 2\text{MHz}} + P_{wr\ 3\text{MHz}} + P_{wr\ 4\text{MHz}}}{P_{wr\ \text{fundamental}}} \right]^{1/2} \\
 &= \left[\frac{V_{2\text{MHz}}^2 + V_{3\text{MHz}}^2 + V_{4\text{MHz}}^2}{V_{\text{fundamental}}^2} \right]^{1/2} \\
 &= \left[\frac{(1)^2 + (0.5)^2 + (0.3)^2}{1000^2} \right]^{1/2} = \frac{1.158}{1000}
 \end{aligned}$$

$$\therefore \% \text{ THD} = 0.12\%$$

15.18) Find output signal level.

$$\begin{aligned}
 OIP_3 &= IIP_3 + a_1 \\
 &= 10\text{dBm} + 6\text{dB}
 \end{aligned}$$

$$OIP_3 = 16\text{dBm}$$

$$\text{Let } ID_3 = -60\text{dB}$$

$$\begin{aligned}
 \therefore I_{D1} &= OIP_3 + \frac{ID_3}{2} = 16\text{dBm} - \frac{60\text{dB}}{2} \\
 &= \underline{-14\text{dBm}}
 \end{aligned}$$

Thus, an output level of -14 dBm should be used.

15.19) Given: Input at -4dBm
output at 2dBm ($= I_{D1}$)
Measured $I_{D3} = -40\text{dB}$

$$\therefore OIP_3 = I_{D1} - \frac{I_{D3}}{2} = 2\text{dBm} + \frac{40\text{dB}}{2}$$

$$\underline{OIP_3 = 22\text{dBm}} \quad \text{and}$$

$$IIP_3 = OIP_3 - a_1 = 22\text{dBm} - 6\text{dB}$$

$$\underline{IIP_3 = 16\text{dBm}}$$

For $I_{D3} = -50\text{dB}$

$$I_{D1} = OIP_3 + \frac{I_{D3}}{2} = 22\text{dBm} - \frac{50}{2}$$

$$\underline{I_{D1} = -3\text{dBm}}$$

This corresponds to an input level of $-3\text{dBm} - 6\text{dB} = \underline{-9\text{dBm}}$

If $N_0 = -60\text{dBm}$, then

$$SFDR = \frac{2}{3}(OIP_3 - N_0)$$

$$= \frac{2}{3}(22\text{dBm} + 60\text{dBm})$$

$$= \underline{55\text{dB}}$$

$$\text{and } I_{D1}^* = SFDR + N_0 = 55\text{dBm} - 60\text{dBm}$$

$$= \underline{-5\text{dBm}}$$

Thus the output signal level is -5dBm and the
SFDR is 55dB.