Timing Recovery

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Timing Recovery (two types)

- Timing more difficult with less excess bandwidth.

Deductive

- Timing tone detector
- PLL
- Detector
- Receive
- To reduce jitter if necessary

Inductive

- VCO
- Loop filter
- Timing error
- Estimate timing error
- Equalization
- Receive

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Deductive Timing Recovery

- Non-linear spectral line method most popular (linear spectral line method used if \(f_s\) tone present).
- Apply a non-linearity to receive signal and bandpass filter to recover \(f_s\) tone (usually with PLL).
- Works because receive signal is cyclostationary (i.e. its moments vary in time and are periodic).
- Common non-linearities used are squaring and absolute circuit (rectifier) (for low excess BW)
- \textit{Ensemble average} of non-linear circuit output is periodic in \(T\)
- Thus, a \(f_s\) component exists (scrambled data)

Example (100% excess BW)

- \begin{align*}
- \text{receive signal} \\
- \text{abs(receive signal)} \\
- \text{average(abs(receive signal))}
- \end{align*}

average NOT in time but over transmit sequences (100 sequences in this case)
**Example (20% excess BW)**

- receive signal
- \( \text{abs(receive signal)} \)
- average NOT in time but over transmit sequences (100 sequences in this case)

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**Deductive Timing**

- Can pre-filter receive signal to only non-flat portion to reduce jitter — eliminate portion that does not contribute to timing tone.

\[
P(j2\pi f) \quad H_{pf}(s) \quad f
\]

- Rx
- Clk
- \( H_{pf}(s) \) 
- non-linearity
- PLL
Inductive Timing — Early Late

- Can sample at 2X and determine if clock is early or late when a transition occurs.

<table>
<thead>
<tr>
<th>Early</th>
<th>On-time</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>(a) = (b) ≠ (c)</td>
<td>(a) ≠ (b) = (c)</td>
<td></td>
</tr>
</tbody>
</table>

Slow down clock

- If (a) = (b) = (c), do nothing
- However, (b) sample does not indicate how far away from zero crossing — can add dither to (b) to aid estimate.

Inductive Timing (MMSE)

- Commonly realized as minimum mean-square error (i.e. MMSE timing)
- Also called LMS timing.
- Assume sample times are \( kT + \tau_k \)

\[ A_k = \pm 1 \]

Correct sampling phase \( \tau_k < 0 \)

Late sampling phase \( \tau_k > 0 \)
**Inductive Timing (MMSE)**

- MMSE adjusts $\tau_k$ to minimize

$$E[E^2_k(\tau_k)] = E[(Q_k(\tau_k) - A_k)^2]$$  \hspace{1cm} (1)

where $E[\cdot]$ denotes expectation, $Q_k(\tau_k)$ is the sampled signal (it is a function of $\tau_k$) and $A_k$ is the ideal symbol.

- Stochastic gradient (as in LMS algorithm) leads to

$$\tau_{k+1} = \tau_k - \mu E_k(\tau_k) \times \frac{\partial Q_k(\tau_k)}{\partial \tau_k}$$  \hspace{1cm} (2)

- Can replace derivative wrt $\tau_k$ by derivative wrt time since sampled at $t = kT + \tau_k$

$$\frac{\partial Q_k(\tau_k)}{\partial \tau_k} = \frac{\partial Q(t)}{\partial t} \bigg|_{t = kT + \tau_k}$$  \hspace{1cm} (3)
Inductive Timing (MMSE)

- Can sample at 2X symbol-rate and perform derivative in discrete-time.

\[ Q_{k+1} - Q_{k-1} = \frac{1}{\mu} \left( 2x - 1 \right) \]

\[ DCO \]

\[ R(t) \]

\[ 2x \]

\[ Q_k \]

\[ \tau_k \]

\[ \hat{A}_k \]

\[ E_k \]

\[ \mu \]

\[ \times \]

\[ \ominus \]

\[ + \]

\[ - \]

2X Timing Example

- Sample at twice symbol-rate

\[ \tau_{k+1} = \tau_k - \mu (Q_k - \bar{A}_k) \times (Q_{k+1} - Q_{k-1}) \]  \hspace{1cm} (4)

\[ A_k = \pm 1 \]

\[ +1 \]

\[ 0 \]

\[ -1 \]

Correct sampling phase

Late sampling phase \( \tau_k > 0 \)

- At \( Q_k \), slope is neg, \( E_k \) is neg, so \( \tau_k \) is decreased.
- Use absolute values then 50% duty cycle not needed
Inductive Timing — Baud-Rate

- If all sampling done at symbol-rate, MMSE timing can still be used — base it on impulse response.

\[ h_1 \tau + h_{-1} \tau = 0 \]

- Early-late — adjust so \( h_1 - h_{-1} = 0 \)
- Zero-crossing — adjust so \( h_1 = 0 \)

Inductive Timing — Baud-Rate

- To obtain impulse response estimates, cross correlate received signals with received symbols.

\[ Q(t) = \sum_{m} A_m h(t - mT) + n(t) \quad (5) \]

- Sampled at time \( kT + \tau \), we have

\[ Q_k \equiv Q(kT + \tau) \]

\[ = \ldots + A_{k-1} h(kT + \tau - (k - 1)T) + A_k h(kT + \tau - kT) + \ldots \]

\[ = \ldots + A_{k-1} h_1(\tau) + A_k h_0(\tau) + A_{k+1} h_{-1}(\tau) + \ldots \quad (6) \]

where \( h_k(\tau) \equiv h(kT + \tau) \)
Inductive Timing — Baud-Rate

• To estimate $h_1(\tau)$, use $Q_k \times A_{k-1}$

• All other terms go to zero since $A_{k-1}$ is uncorrelated with $A_j$ when $k \neq j$

• To estimate $h_{-1}(\tau)$, we need to use a delayed version of $Q_k$

\[
Q_{k-1} = \ldots + A_{k-1}h_0(\tau) + A_kh_{-1}(\tau) + A_{k+1}h_{-2}(\tau) + \ldots \tag{7}
\]

• To estimate $h_{-1}(\tau)$, use $Q_{k-1} \times A_k$

Inductive Timing — Baud-Rate

• To build early-late scheme,

- Early-late is insensitive to amplitude distortion.
- Zero-crossing is better where phase distortion dominates
- $h_0$ factor should be known otherwise adaptation gain will vary (can divide it out in algorithm).
A Fractional-N Frequency Synthesizer

- Often need a low jitter clock that can have arbitrary frequency.
- A voltage-controlled crystal oscillator is expensive.
- Use oversampling within a PLL

![Diagram of a Fractional-N Frequency Synthesizer]

\[ f_{\text{xt}} \div M \rightarrow \text{phase detect} \rightarrow \text{loop filter} \rightarrow \text{VCO} \div P \]

\[ N = \{k-1, k, k+1\} \]

A digital controlled oscillator

Elastic Buffer

- Used to deal with low frequency input clock jitter
- Allows attenuation of clock jitter to next stage

Example

- Input clock rate — 1MHz but varies from 0.9MHz to 1.1MHz in sinusoidal fashion at 1kHz
- Output clock rate — fixed at 1MHz
- Input clock high — 16 extra bits stored in buffer
- Input clock low — 16 bits removed from buffer

- Keep elastic buffer half-full on-average through feedback