

EQUALIZATION AND LINEARIZATION VIA LINEAR NEGATIVE FEEDBACK

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ABSTRACT

This paper presents a method for equalizing the frequency response of a weakly non-linear system while simultaneously reducing the amount of non-linear distortion. This is achieved by introducing a linear equalizer in a feedback loop with the system. A dynamic loudspeaker is used to illustrate the theory and its mechanics. Simulation results are presented which show an improvement in frequency response and a reduction in non-linear distortion.

1. INTRODUCTION AND MOTIVATION

The dynamic loudspeaker is a good candidate to motivate and illustrate the application of the principles proposed in this paper. There is considerable interest in improving the quality of dynamic (moving-coil) loudspeakers. They are prone, for physical reasons, to producing significant amounts of non-linear distortion when driven near the limits of their power handling capability; the onset of distortion is sharp, and is accompanied by a severe degradation in sound quality. Consequently, the distortion-generation mechanism in loudspeakers has been well-studied [1, 2], and from this a number of electronic equalization schemes have been proposed and implemented. The ones of relevance to this paper are those that attempt to reduce non-linear distortion, while simultaneously equalizing frequency response.

These equalization-linearization schemes can be classified as closed-loop versus open-loop, and linear versus non-linear—depending on the type of equalization employed. Closed-loop systems [3] employ some form of feedback from the loudspeaker, such as diaphragm velocity or acceleration (often measured by means of transducers mounted physically on the dynamic element). These schemes enjoy the many advantages of feedback, such as insensitivity to variations in the loudspeaker that can arise over time. In addition, a closed-loop structure permits a linear equalizer to improve the non-linear performance of the system. This

can be an advantage since linear filters are computationally and structurally simpler than non-linear filters of the same order, thus permitting the use of new IIR filter structures that operate directly on oversampled signals [8–10].

Open-loop equalizers, on the other hand, *must* build-in non-linearities to affect an overall reduction in non-linear distortion [4]. However, since open-loop-equalized stable systems are stable if the equalizer is stable, stability is more easily established.

In this work, a linear equalizer—employed in the feedback path with the plant—is designed to optimally meet the dual criteria of frequency-response improvement and non-linearity reduction.

2. PROBLEM FORMULATION

Consider the equalization setup in fig. 1. The goal is to find

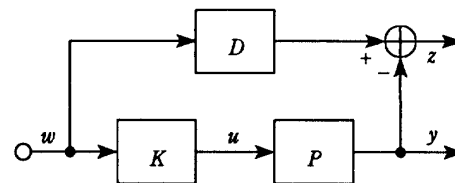


Figure 1: Equalization setup

an equalizer, K , for plant P , such that the input-output behaviour (w to y) matches some desired transfer function D . By simple manipulation of this block diagram, we obtain the diagram shown in fig. 2. If \hat{P} is equal to P , the

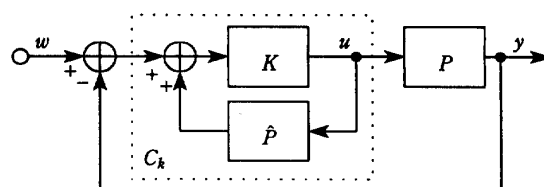


Figure 2: Modified signal-flow diagram

two systems are functionally identical. We then group the dotted block in fig. 2 to construct C_k , a linear feedback-equalizer (or internal model controller):

$$C_k = \frac{K}{1 - KP} \quad (1)$$

The equalization problem can now be posed as a Minimum Variance Model Reference Control problem [6, 7], where the object is to minimize the performance measure J (for white noise input):

$$J = \|z\|_2^2 + \eta^2 \|u\|_2^2, \eta \in \mathfrak{R}. \quad (2)$$

The scalar η penalizes large inputs, which, in turn, limit the allowable equalizer gain. This is a very useful design parameter because implementation considerations require such limits.

For a scalar, rational, continuous-time plant and controller, the Wiener-Hopf optimal solution to eq (2) can be obtained by spectral factorization as follows [5]:

$$\begin{aligned} K(s) &= \frac{\left[\frac{\Gamma(s)}{\Delta^-(s)} \right]_+}{\Delta^+(s)} \\ \Gamma(s) &= P(-s)D(s) \\ \Delta(s) &= P(s)P(-s) + \eta^2 \\ \Delta(s) &= \Delta^-(s)\Delta^+(s) \end{aligned} \quad (3)$$

The superscript + indicates a stable, minimum-phase rational function. The subscript + refers to the sum of the stable terms in the implicated partial fraction expansion.

3. EQUALIZATION AND LINEARIZATION

The application of this optimal solution to the equalization-linearization of a weakly non-linear plant P is illustrated by means of fig. 3.

First, K (fig. 1) is factored into rational functions F and G . G is turned into feedback equalizer C , while F is left outside the feedback loop to act as a cascade equalizer (fig. 3).

For some sinusoidal input w , nonlinearities in the plant will be manifested as harmonics in y (along with other prod-

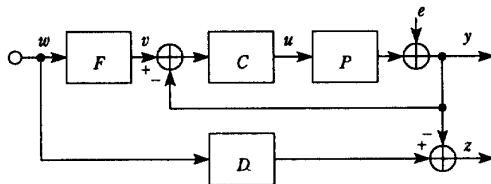


Figure 3: Linearization setup

ucts). As an approximation, the plant can be viewed as a linear system P with an additive 'noise signal' e at the output that contains the distortion products. Within this (approximate) framework, linearizing the plant is equivalent to minimizing the transmission of e into the output y — which is equivalent to minimizing the norm of the transfer function from e to y , T_{ye} :

$$T_{ye} = (1 + CP)^{-1} = 1 - PG. \quad (4)$$

This, in turn, is equivalent to minimizing the performance measure:

$$J_N = \|1 - PG\|_2^2 + \sigma^2 \|G\|_2^2, \sigma \in \mathfrak{R}. \quad (5)$$

After obtaining the optimum G that minimizes J_N , F is optimized to equalize the transmission from w to y by minimizing the performance measure:

$$J_E = \|D - PGF\|_2^2 + \rho^2 \|F\|_2^2, \rho \in \mathfrak{R}. \quad (6)$$

Both optimum solutions can be obtained by applying equation-set (3).

4. APPLICATION TO LOUDSPEAKERS

An electrical analogue model of the dynamic loudspeaker is shown in fig. 4. The terms are defined in Table 1. The dominant non-linearities are due to the displacement-dependent natures of the compliance of the suspension and the flux density in the gap [1-2].

After transforming the circuit to the secondary, the non-linear state-space representation given by eq(7) is obtained. The state-variables are displacement, velocity, and electromotive force. g is a non-linear term that captures the aforementioned nonlinearities.

In order to demonstrate the application of the theory with a numerical example, we obtain a frequency and unit-normalized loudspeaker model by substituting plausible values for the parameters in table 1.

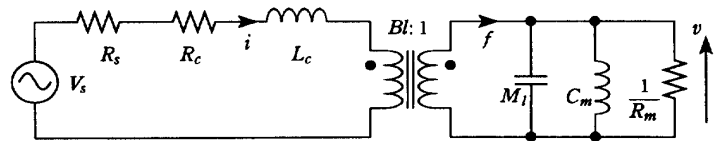


Figure 4: Equivalent circuit of loudspeaker

Table 1: Loudspeaker Model

	Description	Unit
V_s	Open-circuit voltage of amplifier	V
R_s	Amplifier output resistance	Ω
R_c	Resistance of voice-coil	Ω
R	$R_s + R_c$	Ω
L_c	Inductance of voice-coil	H
B	Magnetic flux-density in gap = $B_0 + B_1 x$	T
l	Length of voice-coil	m
M_l	Mass of diaphragm, coil, air-load	F
C_m	Compliance of suspension = $(\alpha + \beta x^2)^{-1}$	m/N
R_m	Mechanical resistance of suspension	Ns/m
x	Displacement of diaphragm	m
v	Velocity of diaphragm	m/s
u	Input= $V_s / (B l)$	m/s
f	Electromotive force	N

$$A_s = \begin{bmatrix} -\frac{R}{L_c} - \frac{(B_0 l)^2}{L_c} - \frac{2B_0 B_1 l^2}{L_c} & 0 & 0 \\ \frac{1}{M_l} & -\frac{R_m}{M_l} & -\frac{\alpha}{M_l} \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_s = \begin{bmatrix} \frac{B_0 l}{L_c} & 0 & 0 \end{bmatrix}^T$$

$$C_s = [0 \ 1 \ 0]$$

$$D_s = 0$$

$$g = \begin{bmatrix} -\frac{(B_1 l)^2 x^2 + B_1 l x u}{L_c} - \frac{\beta x^2}{M_l} & 0 \end{bmatrix}^T$$

$$z = [f \ v \ x]^T$$

$$\frac{d}{dt} z = A_s z + B_s u + g \quad (7)$$

The resulting frequency response and transfer function (ignoring the non-linear term g) are shown below.

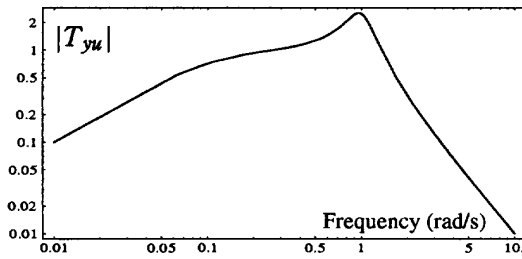


Figure 5: Nominal loudspeaker

$$P(s) = 2.5s / ((0.1 + s)(2.5s^2 + s + 2.5)). \quad (8)$$

5. RESULTS

We now apply the design procedure outlined in section 3 to obtain a linearizing feedback equalizer C . Because a loudspeaker is a band-limited system, GP need only approximate unity over the band of interest, $\omega < \omega_p$. Consequently equation (5) is modified to reflect this fact:

$$J_N = \|I - PG\|_2^2 + \sigma^2 \|G\|_2^2, \sigma \in \mathfrak{R}$$

$$|I(\omega)| \equiv \begin{cases} 1, & \omega < \omega_p \\ 0, & \omega > \omega_s \end{cases} \quad (9)$$

The choice of $I(s)$ and σ^2 influence the solution, so they must be chosen judiciously. An important consideration is that the order of C will be twice the order of the plant, plus the order of $I(s)$.

Figs. 7A–B show the result of two choices for $I(s)$ and σ^2 , and the resulting T_{ye} and T_{yv} . In the first case, design \mathcal{A} , $I(s)$ and σ are given by

$$I(s) = 4 / (s^2 + 4s + 4), \sigma = 0.01. \quad (10)$$

In the second case, design \mathcal{B} , they are given by

$$I(s) = 25 / (s^2 + 5s + 25), \sigma = 0.001. \quad (11)$$

Some comments on the designs are in order. Both choices of $I(s)$ eq(10–11) have the same order; however, because

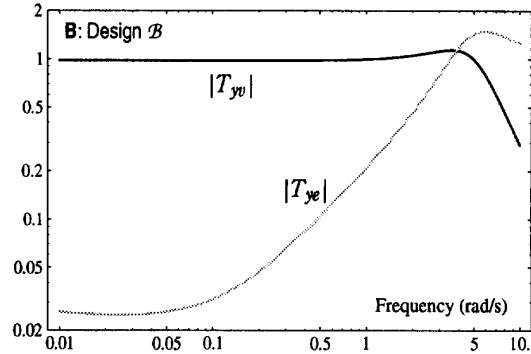
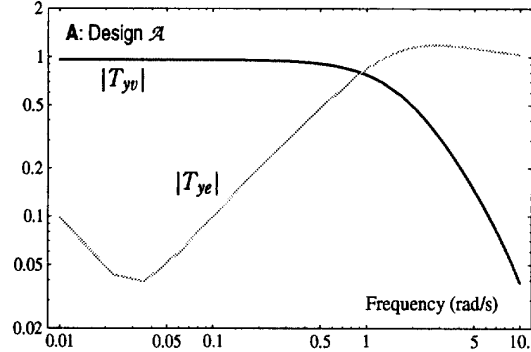


Figure 7: Designs \mathcal{A} and \mathcal{B}

the cut-off frequency for $I(s)$ is higher in design \mathcal{B} , the resulting T_{ye} has a smaller norm. (This is expected because the phase rolls-off more slowly in this case). It also follows that design \mathcal{B} would have a higher cut-off frequency as is evident from fig. 7B. However, because the nominal bandwidth of the system is specified to be 1 rad/s, this increased bandwidth is undesirable and necessitates that F be designed to roll-off the aggregate frequency response (from w to y). In the case of design \mathcal{A} , an F may only provide the high-pass roll-off near DC, which is required for loudspeaker protection.

A discrete-time simulation was run on design \mathcal{A} to examine the effect of the equalizer on non-linear distortion reduction. For this purpose, both the loudspeaker model and the feedback equalizer were converted to discrete-time state-space representations by sampling at 200 rad/s. A unit-amplitude sinusoid at approximately 0.11 rad/s was employed as the excitation signal, and the analyses were performed by taking 65536-point Fast Fourier Transforms.

For the purpose of evaluation, three numerical experiments were conducted: In the first experiment, no equalizer of any kind was employed; in the second, the optimal (design \mathcal{A}) cascade equalizer G was used; in the third the equivalent feedback equalizer was used. In all cases F was set to unity.

The log-magnitude FFTs are plotted for comparison in fig. 8. From these it is clear that the cascade equalizer had the lowest fundamental-to-first-harmonic ratio of 22.6 dB. This is expected because of the increased drive-signal amplitude into the loudspeaker. The unequalized loudspeaker, which represents the bare physical system, had a slightly better ratio of 24.82 dB. As anticipated, the feedback configuration had the best ratio at 32 dB. The effective equalizer gain at 0.1 rad/s was 2.9 dB. The linear model 'predicted' an improvement in the fundamental-to-first-harmonic ratio of about 12 dB by using the feedback arrangement over the cascade arrangement (based on the "noise-gain," $|T_{ye}|$, at 0.22 rad/s). In comparison, the simulation shows an improvement of about 10 dB. Further improvements can be had by increasing the order of the equalizer, and/or by choosing other parameters differently.

6. CONCLUSION

A simple design method for equalizing and linearizing weakly non-linear systems has been presented. Simulations bear out the claims of improved performance.

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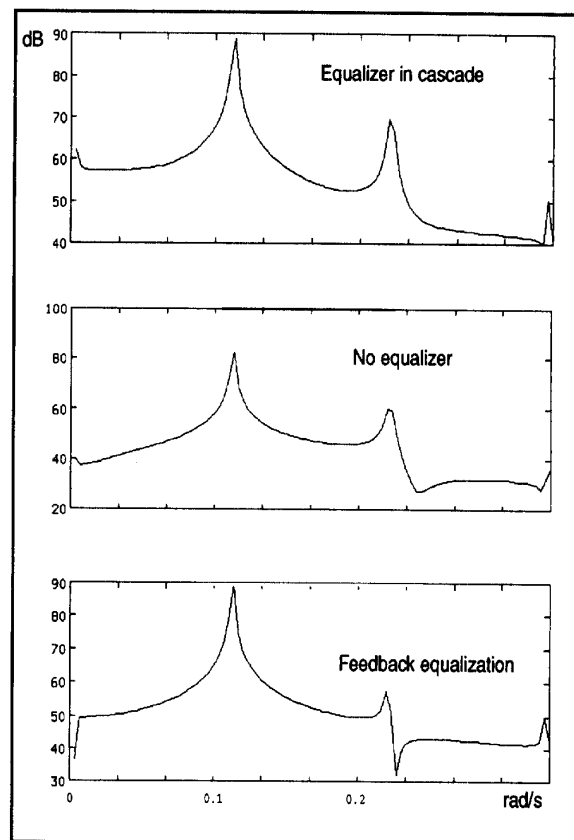


Figure 8: Linearization results