2.5 AN OVERVIEW OF SIGNAL-FLOW GRAPHS

Signal-flow graphs have long been used in many areas of engineering. Originally devised by Mason for linear networks [Mason,1953], they are a mainstay of network theory and are commonly applied to areas as diverse as automatic control and data communications. This section provides an overview of linear signal-flow graphs, largely for the benefit of today's reader who may not have had much exposure to network and graph theory. Much of the following material is derived from [Haykin,1970] and the reader is also referred to [Mason,1960] and [Chen, 1991 and 1997] for a more thorough treatment of this fascinating area.

A graph is a collection of points and lines, respectively referred to as *nodes* and *branches*. Each end of a branch is connected to a node and both ends of a branch may be connected to the same node. A signal-flow graph is a diagram which depicts the cause and effect relationship among a number of variables. The variables are represented by the nodes of the graph, while the connecting branches define the relationship. A typical signal-flow graph is shown in Figure 2.16. The figure has four nodes, each representing a node signal x_j. Between a pair of nodes j and k lies a branch with a quantity called the branch transmittance t_{jk}, represented here by the letters a to f.

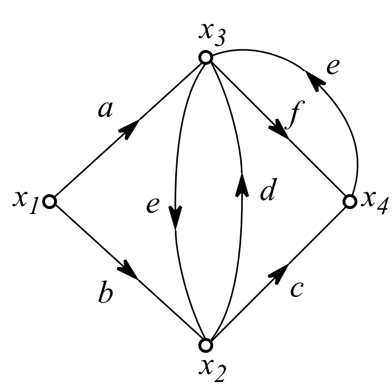

Figure 2.16 A linear signal-flow graph.

The flow of signals in the various parts of the graph is dictated by the following three basic rules which are illustrated in Figure 2.17:

1. Figure 2.17a: A signal flows along a branch only in the direction defined by the arrow and is multiplied by the transmittance of that branch.

2. Figure 2.17b: A node signal is equal to the algebraic sum of all signals entering the pertinent node via the incoming branches.

3. Figure 2.17c: The signal at a node is applied to each outgoing branch which originates from that node.

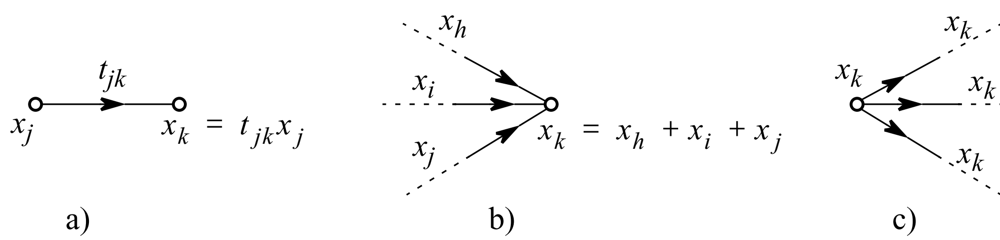

Figure 2.17 Illustrating three basic properties of signal-flow graphs.

From these basic rules are derived the four elementary equivalences shown in Figure 2.18 which guide one in the manipulation of signal-flow graphs. These equivalences are sufficient for the complete reduction of a graph containing no feedback loops. To handle graphs that incorporate feedback loops, there are two additional equivalence relations. Consider a self-loop in which a node signal is fed back to itself as illustrated on the left-hand side of Figure 2.19a. The signal-flow graph represents the relation

$$x_3 = x_2 = Lx_2 + x_1 \tag{2.7}$$

from which x_3 can be expressed exclusively in terms of x_1 as

$$x_3 = \frac{1}{1-L}x_1 \tag{2.8}$$

and represented by the right-hand side of Figure 2.19a. Figure 2.19b illustrates the classic feedback structure comprised of a gain stage A, surrounded by a feedback network β. The signal-flow graph represents the pair of equations

$$x_3 = Ax_2 \tag{2.9}$$

$$x_2 = \beta x_3 + kx_1 \tag{2.10}$$

from which we obtain the familiar expression

$$\frac{x_3}{x_1} = \frac{kA}{1 - A\beta} \tag{2.11}$$

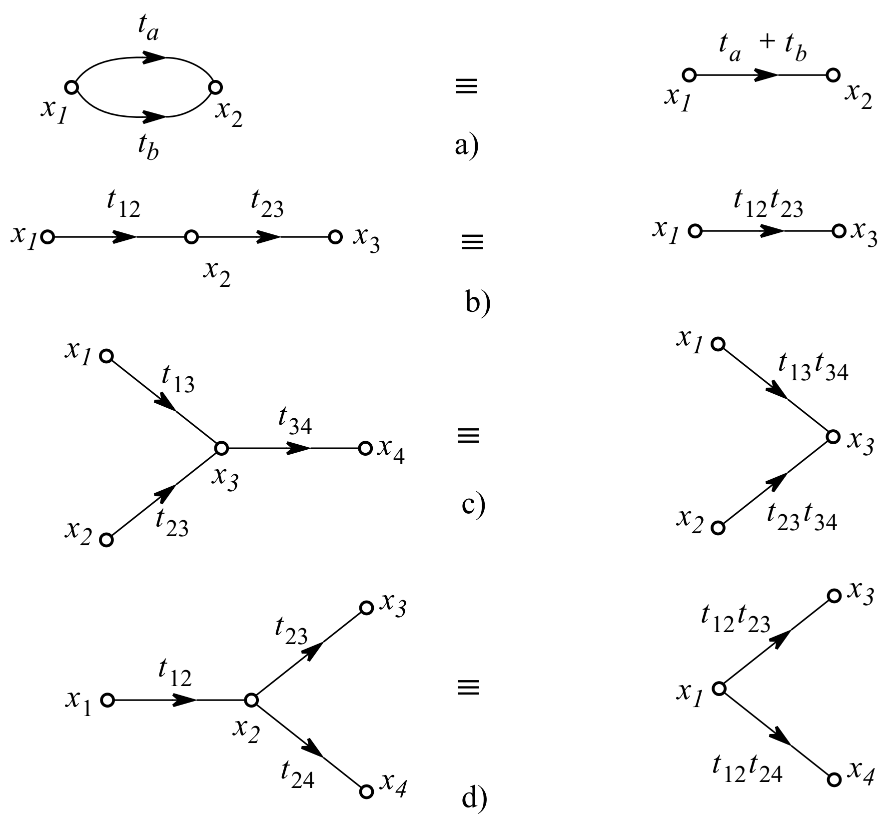

Figure 2.18 Four elementary equivalences of signal-flow graphs.

In contrast to Equations (2.8) and (2.11), most textbooks have a plus rather than minus sign, a result of adopting a convention whereby the feedback signal is subtracted rather than added back to the input node. Since the difference is only one of convention, we will continue with our existing convention in order to remain consistent with the signal-flow graph algebra. The quantities L and $A\beta$ are commonly known as the loop gain.

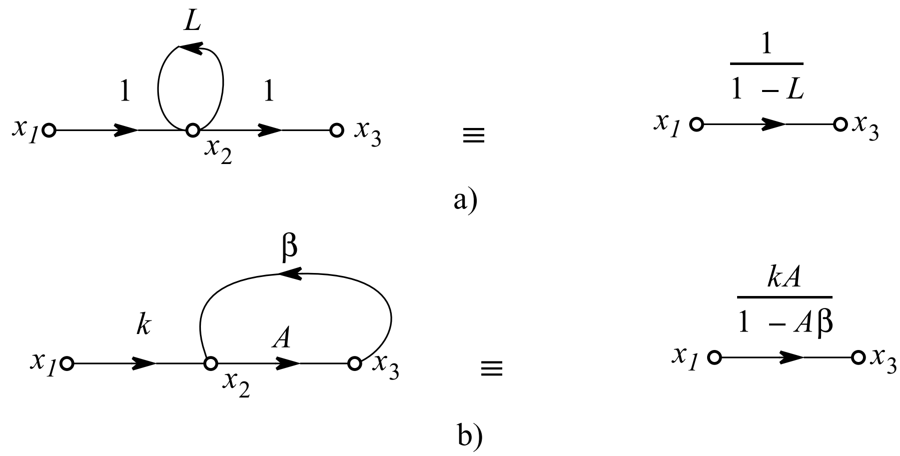

Figure 2.19 Collapsing feedback loops a) self-loop, b) general feedback loop.

By using the elementary equivalences in Figures 2.18 and 2.19, any transfer function can be derived from a signal-flow graph by successively collapsing internal nodes until only the input and output nodes remain. Figure 2.20 illustrates this process. The resulting transfer function is

$$\frac{x_{out}}{x_{in}} = \frac{abc}{1 - cd - bce}. \tag{2.12}$$

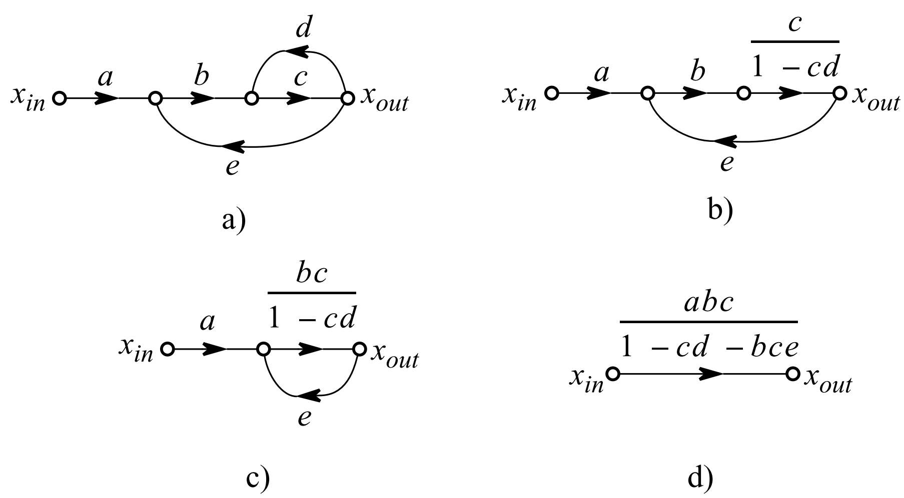

Figure 2.20 Determining a transfer function through collapsing of signal-flow graph.

2.5.1 Mason's Direct Rule

The manipulation of signal-flow graphs is an effective and straightforward means of determining transfer functions for relatively small graphs. However, such manipulations quickly become unwieldy for larger graphs, and for such situations the transfer function can be computed directly. Comparing Equation (2.12) to the original signal-flow graph in Figure 2.20a, we notice that the transfer function can be expressed as

$$\frac{x_{out}}{x_{in}} = \frac{P_1}{1 - (L_1 + L_2)} \tag{2.13}$$

where $P_1 = abc$ represents the forward transmission path from input to output, and $L_1 = cd$ and $L_2 = bce$ represent the loop gains of the two feedback loops found in the graph. In general, the transfer function of a signal-flow graph can be derived using the following expression, commonly known as Mason's Direct Rule [Mason,1960]:

$$\frac{x_{out}}{x_{in}} = \frac{1}{\Delta}\sum_{k=1}^{n} P_k \Delta_k \tag{2.14}$$

where

- P_k = transmittance of the kth forward path from input $x_{in,}$ to output, x_{out}
- Δ = 1 - (sum of all individual loop gains)

  + (sum of loop gain products of all possible sets of nontouching

  loops taken two at a time)

  - (sum of loop gain products of all possible sets of nontouching

  loops taken three at a time)

  +...

and

- Δ_k = the value of Δ for that portion of the graph not touching the *k*th forward path.

2.6 SUMMARY

In this chapter, we discussed the photodetector and optical preamplifier that make up the front-end of an optical receiver. The transimpedance amplifier is the most common preamplifier structure, and we described the three principal new requirements that we wish to address in this thesis: a wide dynamic range, ambient light rejection, and low-voltage operation. In preparation for our discussion of a graphical circuit analysis technique, we reviewed existing analysis techniques such as nodal analysis, and feedback analysis based on amplifier topology and return ratios. In addition, we reviewed the basic conventions of signal-flow graphs and outlined Mason's Direct Rule.

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