The Volterra Series and The Direct Method of Distortion Analysis

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ABSTRACT

In this paper, two methods of distortion analysis are described. The Volterra series is usually used to determine nonlinear behaviour. The direct method is an extension of the Volterra series method to circuits with multiple inputs. The Volterra series method of distortion analysis is presented in the analysis of a common emitter circuit. Next the direct method is used to analyze a mixer circuit with two inputs. Distortion components are calculated for both circuits.

INTRODUCTION

Distortion is a key issue in the design of many types of circuits. The Volterra series has long been used to analyze distortion in analog circuits.[2] Unlike numerical simulations which give no information about the source of the distortion, closed form expressions for distortion components in terms of circuit parameters can be found using Volterra series. Unfortunately, the method of presenting the Volterra series analysis is complex, confusing, and often intimidating to the uninitiated. Consequently, the Volterra series is often under-utilized by circuit designers. In this paper, the basics of distortion analysis is presented with the hope that it will help the reader to more easily familiarize themselves with distortion analysis and thus take advantage of the Volterra series method or the direct method to analyze their designs.

BASICS OF VOLTERRA SERIES

Circuit designers prefer to work with linear models of circuits but more accurate models which take into account the nonlinearities in a circuit are often required. Many practical circuits can be assumed to behave in a weakly nonlinear way, and under this condition, closed form expressions for the nonlinearity can be obtained using the Volterra series. The Volterra series is a Taylor series that simplifies to a power series when the system is memoryless. It describes a signal as a summation of the linear behaviour, the second order behaviour, the third order behaviour, and so on.   That is,

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots \quad (1)$$

Note that as the amplitude of the input x(t) increases, the magnitude of the higher order components will increase more quickly than the lower order components. For weakly nonlinear circuits that are excited by small signals, usually only the first three terms are retained. You can think of the nonlinearity as being approximated by the linear behaviour and made more accurate by a squared and cubed component. The squared and cubed products in the nonlinearity gives rise to harmonic and Intermodulation components. Using the Volterra series, closed form expressions for the different distortion components can be found. This gives the designer insight with regard to improving circuit performance.

The Volterra series for a circuit is generally represented as a summation of n^{th} order operators:

$$y(t) = H[x(t)] = H_1[x(t)] + H_2[x(t)] + H_3[x(t)] + ... + H_n[x(t)] + ... \qquad (2)$$

where

$$H_n[x(t)] = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) x_1(t-\tau_1) \dots x_n(t-\tau_n) d\tau_1 \dots d\tau_n \qquad (3)$$

The Laplace transform of the n^{th} order Volterra kernel $h_n(t_1,...,t_n)$ is represented by $H_n(s_1,...,s_n)$ and can used to calculate the magnitude of distortion components. The representation of the nonlinearity as a summation of operators of different order operating on a signal allows us to examine the contribution of each order individually. Dominant contributions can then be easily identified and analyzed. The block diagram of this summation is shown in Figure 1.

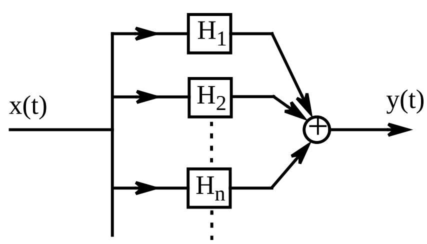

Figure 1. Block Diagram representation of the Volterra Series

DISTORTION ANALYSIS OF A COMMEN EMITTER CIRCUIT USING THE VOLTERRA SERIES METHOD

The Volterra Series method will be explained with the example of a common emitter amplifier shown in

Figure 2[5].

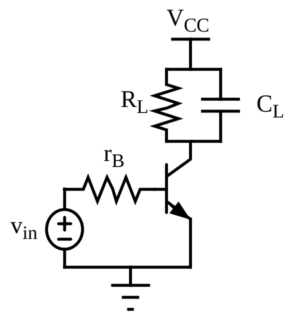

Figure 2. Schematic of the Common Emitter circuit

The small signal model of this circuit is shown in Figure 3.

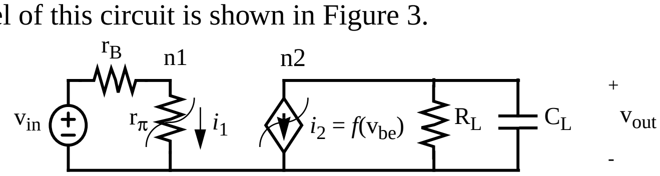

Figure 3. Small signal representation of the nonlinear circuit.

The nonlinearities considered in this circuit are due to the resistor r_π and the transconductance g_m. These nonlinearities are due to the exponential I-V characteristic of the transistor:

$$i_B = (I_s/\beta)e^{v_{BE}/V_T} \quad \text{, and} \quad i_C = \beta i_B \tag{4}$$

This characteristic can be approximated using the first three terms of a power series.

$$i_B \approx I_B + g_\pi v_{be} + K_{2g\pi} v_{be}^2 + K_{3g\pi} v_{be}^3 \tag{5}$$

$$i_C \approx \beta I_B + g_m v_{be} + K_{2gm} v_{be}^2 + K_{3gm} v_{be}^3 \tag{6}$$

where I_B is the dc bias current, and the power series coefficients are given by

$$K_{2g\pi} = K_{2gm}/\beta = I_B/V_t^2 = g_\pi/2V_t = g_m/2\beta V_t \tag{7}$$

$$K_{3g\pi} = K_{3gm}/\beta = I_B/3!V_t^3 = g_\pi/6V_t^2 = g_m/6\beta V_t^2 \tag{8}$$

Nonlinearity coefficients depend upon the complexity of the model used to model the nonlinearity. For a transconductance of order n, the nonlinearity coefficient is calculated using the following equation[5]:

$$K_{ng1} = \frac{1}{n!} \cdot \frac{\partial^n}{\partial v^n} f(v)\Big|_{v = V_{CONTR}} \tag{9}$$

where f(v) is the expression for the current in terms of a voltage and V_{CONTR} is the DC value of the controlling voltage.

THE LINEAR SYSTEM

The first-order output kernel H_{1out} is equivalent to finding the linear transfer function. Thus the first step is to analyze the linearized circuit shown in Figure 4. The SFG can be easily derived using the DPI/SFG method [3][4] and is shown in Figure 5. The resulting transfer function is

$$H_{1out}(s) = \frac{-g_m g_B}{(g_B + g_\pi)(g_L + sC_L)} \quad (10)$$

The notation we use for the transfer function $H_{1out}(s)$ is such that the subscript '1' tells us that it's a 1st order kernel transform and the subscript 'out' refers to the output node.

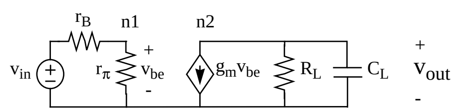

Figure 4. Linearized small signal equivalent circuit

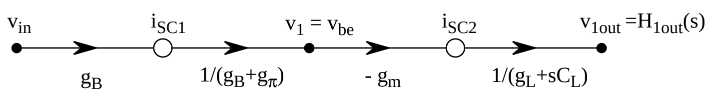

Figure 5. SFG of the linearized circuit

HIGHER ORDER SYSTEMS OF THE VOLTERRA METHOD

Kernel transforms of higher order are found by solving the same basic linear network with a couple of changes. The series expansion expressions in (5) and (6) suggest that the linear network is no longer directly excited by the input signal, but by new excitations in the form of n^{th} order nonlinear current sources that represent the nonlinear components. The sources are placed in parallel with each nonlinear element and the orientation of each source is the same as the corresponding controlled current in the original circuit. The nonlinear signals then propagate through the rest of the linear circuit. The resulting circuit to be solved is shown in Figure 6. The same network is solved for each order but at different frequencies and with different expressions for the nonlinear sources. The nonlinear signals depend on lower order kernels and the nonlinear coefficients given in Equations (7) and (8). We use the same nonlinear current source expressions as Wambacq[5].

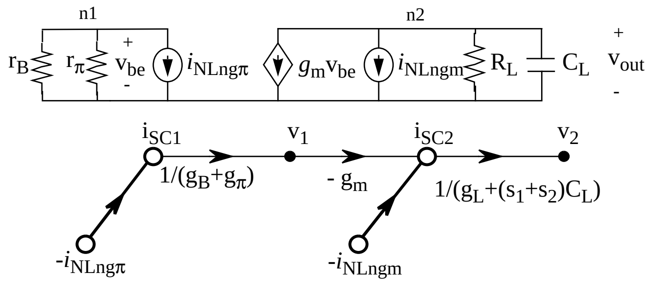

Figure 6. Circuit and the equivalent SFG to be solved for the n^{th} order kernels

2nd Order Distortion

The second-order behavior is represented by Figure 6 with the following expressions for the nonlinear current sources[5]:

$$i_{NL2g\pi} = K_{2g\pi}H_{1vbe}(s_1)H_{1vbe}(s_2) \quad (11)$$

$$i_{NL2gm} = K_{2gm}H_{1vbe}s_1)H_{1vbe}(s_2) \quad (12)$$

Thus the nonlinearity of both r_π and g_m is due to a squaring of the controlling 1^{st} order signal $H_{1vbe}(s)$ and a nonlinearity coefficient. H_{1vbe} is found by solving for the voltage v_{be} in Figure 5. To calculate the second-order intermodulation distortion ID_2 at a frequency $w_1 \underline{+} w_2$, we need to find the ratio of $Y_2(jw_1,\underline{+}jw_2)$ and $Y_1(jw_1)$ where $Y_2(jw_1,\underline{+}jw_2)$ is the output of the second-order system and $Y_1(jw_1)$ is the output of the 1^{st} order system. The kernels H_{2out} and H_{1out} are found by directly solving for the node voltages in the SFG of Figure 5 and Figure 6 respectively. The factor of A^2 in the intermodulation output results from applying (3) with $x(t) = A\cos w_1 t + A\cos w_2 t$.

$$ID_2 = \left|\frac{Y_2(jw_1, jw_2)}{Y_1(jw_1)}\right| = \frac{A^2|H_{2out}(jw_1, jw_2)/v_{1be}|}{A|H_{1out}(jw_1)/v_{1be}|} \quad (13)$$

Solving for the kernels using SFG analysis,

$$ID_2 = \left|\frac{A^2\left(\frac{g_B}{(g_B+g_\pi)^2}\right)\frac{((g_B+g_\pi)K_{2g_m} - g_m K_{2g_m})}{g_L + (jw_1 + jw_2)C_L}}{A\frac{-g_m}{g_L + jw_1 C_L}}\right| \quad (14)$$

Inserting the nonlinear coefficients of Equation (7) and simplifying gives

$$ID_2 = \left|\frac{A}{2V_T}\frac{{g_B}^2(g_L + jw_1 C_L)}{(g_B+g_\pi)^2(g_L + (jw_1 + jw_2)C_L)}\right| \quad (15)$$

If we assume $r_B << r_\pi$, the expression further simplifies to

$$ID_2 = \left|\frac{A}{2V_T}\frac{(g_L + jw_1C_L)}{(g_L + (jw_1 + jw_2)C_L)}\right| \quad (16)$$

The harmonic distortion HD_2 is simply equal to ID_2 for the case when $jw_2 = jw_1$. Consequently, we are interested only in the response to $x(t) = A\cos w_1 t$. The second-order harmonic output is scaled by 1/2 due to the squaring operation. The factor $A^2/2$ can be understood to come from the trigonometric identity $(A\cos w_1 t)^2 = (A^2/2)(1+\cos 2w_1 t)$. It also results from applying (3).

Therefore, if we assume $r_B << r_\pi$,

$$HD_2 = \left|\frac{A}{4V_T}\frac{(g_L + jw_1C_L)}{(g_L + 2jw_1C_L)}\right| \quad (17)$$

At low frequencies, HD_2 is independent of bias conditions

$$HD_2 = A/4V_t \quad (18)$$

and at high frequencies, the distortion reduces to

$$HD_2 = A/8V_t \quad (19)$$

3rd Order Distortion

The third-order harmonic and intermodulation distortion components are calculated in the same way as the second-order distortion. The Volterra kernels are found by solving for node voltages in the linearized network of the circuit shown in Figure 6 with third-order nonlinear current sources. In general, the expressions for the third-order nonlinear current sources are

$$i_{NL3gm} = \beta i_{NL3g\pi} = K_{3gm}H_{1vbe}(s_1)H_{1vbe}(s_2)H_{1vbe}(s_3) + K_{2gm}(2/3)[\, H_{1vbe}(s_1)H_{2vbe}(s_2,s_3) + H_{1vbe}(s_2)H_{2vbe}(s_1,s_3) + H_{1vbe}(s_3)H_{2vbe}(s_1,s_2)\,] \quad (20)$$

The factor of 1/3 in Equation (20) comes from averaging the combination of products and the factor of 2 is due to the fact that the Volterra kernels are symmetric, i.e. $H_{2vbe}(s_1,s_2) = H_{2vbe}(s_2,s_1)$. Equation (20) shows that the third-order nonlinearity results from a component due to the multiplication of three 1^{st} order signals, and a component due to the multiplication of a 1^{st} order signal and a second-order signal.

Making use of (7),(8) and noting that the H_{vbe} is independent of frequency we have

$$i_{NL3gm} = \beta i_{NL3g\pi} = K_{3gm}H_{1vbe}^3 + 2K_{2gm}H_{1vbe}H_{2vbe} \quad (21)$$
$$= K_{3gm}(g_B/(g_B+g_\pi))^3 - 2K_{2gm}K_{2g\pi}g_B^3/(g_B+g_\pi)^4 \quad (22)$$
$$= (g_m/6V_t^2)(g_B/(g_B+g_\pi))^3 - (2g_m^2g_B^3)/(4\beta V_t^2(g_B+g_\pi)^4) \quad (23)$$

The third order response is found by solving the third-order system of Figure 3, where the nonlinear inputs i_{NL3gm} and $i_{NL3g\pi}$ are given by Equation (23).

$$H_{3out}(s_1,s_2,s_3) = v_{3out} \quad (24)$$
$$= (i_{NL3g\pi}g_m/(g_B+g_\pi) - i_{NL3gm}) / (g_L + (s_1 + s_2 + s_3)C_L) \quad (25)$$
$$= -g_mg_B^4(g_B-2g_\pi) / [6V_t^2(g_B+g_\pi)^5(g_L + (s_1 + s_2 + s_3)C_L)] \quad (26)$$

Thus

$$HD_3 = |Y_3(jw_1,jw_1,jw_1)| / |Y_1(jw_1)| \quad (27)$$
$$= (1/4)A^3|H_{3vout}(jw_1,jw_1,jw_1)| / A|H_{1vout}(jw_1)| \quad (28)$$
$$= | -A^3g_mg_B^4(g_B-2g_\pi)/[24(g_L + 3jw_1C_L)V_t^2(g_B+g_\pi)^5] | / |Ag_Bg_m/(g_B+g_\pi)(g_L + jw_1C_{Lp})| \quad (29)$$
$$= | -A^2g_B^3(g_B-2g_\pi)(g_L + jw_1C_L) / (24(g_L + 3jw_1C_L)V_t^2(g_B+g_\pi)^4 | \quad (30)$$

If we assume $r_B << r_\pi$, the expression simplifies to

$$HD_3 = | -(A^2/24V_t^2)(g_L + jw_1C_L) / (g_L + 3jw_1C_L) | \quad (31)$$

Again, we see that at low frequencies, HD_3 is independent of bias,

$$HD_3 = A/24V_t \quad (32)$$

and at high frequencies, the distortion reduces to

$$HD_3 = A/72V_t \quad (33)$$

THE DIRECT METHOD

An alternative to the Volterra series method is what is referred to as the direct method of calculating distortion outputs.[5] The direct method of calculating nonlinear responses is most useful in dealing with multiple-input circuits. In circuits with more than one input source, Volterra kernels become tensors [Chua 79a]. For example, the second order kernel of a voltage is a matrix of size (# inputs) x (# inputs), and calculations become more involved. An alternate method is derived in [5] where the required responses are directly computed. The method is similar to the volterra series method in that the same basic linear network is solved repeatedly for different inputs. The difference is in the expressions used for the nonlinear current sources. In particular, the sources now depend on the order of response as well as the particular type of response calculated. That is, a new set of equations must be solved for each type of response (harmonic or intermodulation). The direct method will be illustrated with an example. The circuit shown in Figure 7 will be analyzed for the distortion due to the nonlinear collector current through the bipolar junction transistors. The nonlinearity of the collector current is described by (5)(7) and(8).

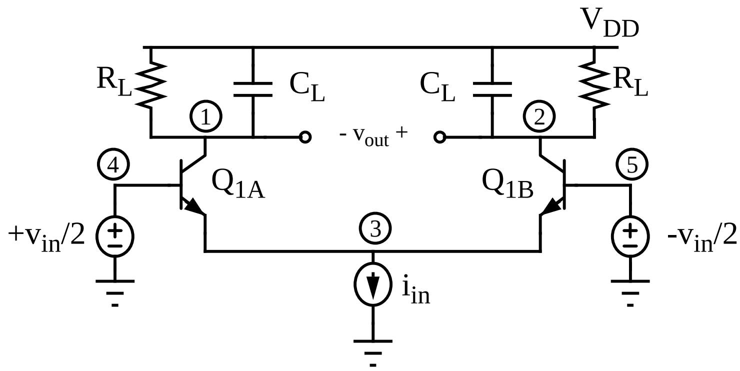

Figure 7. Differential Pair

The distortion due to the two inputs $v_{in} = Re(V_{in}e^{jw1t})$ and $i_{in} = Re(I_{in}e^{jw2t})$ will be analyzed.

FIRST ORDER RESPONSE

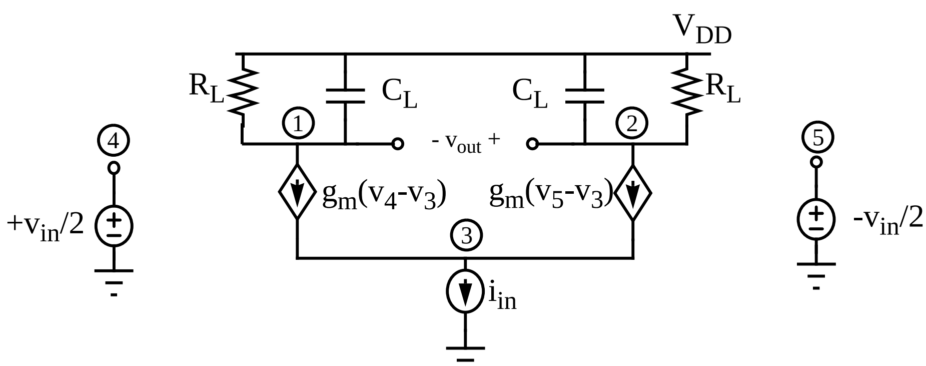

Figure 8. Linearized Small Signal Equivalent Circuit

To find the 1st order response, the linear network of Figure 8 is solved for all the node voltages due to V_{in} only. The output and controlling voltages for the nonlinearities can be written as a linear combination of the node voltages. Either a SFG analysis or the matrix method used by [5] can be used to solve the network. The notation used is such that $V_{x,m,n}$ represents the voltage at node x, due to the input with frequency mw_1+nw_2. Note that the voltages solved are actual voltages now and not Volterra kernels as in the Volterra series method.

The output due to V_{in} at frequency w_1 can be shown to be

$$V_{out,1,0} = V_{2,1,0} - V_{1,1,0} = [g_m/(g_L+jw_1C_L)]V_{in} \quad (34)$$

The controlling voltages for the nonlinearity are

$$V_{43,1,0} = Vin/2 \ ; \ V_{53,1,0} = -V_{in}/2 \quad (35)$$

Next, the linear network is solved for all node voltages due to the input I_{in} only. These voltages are denoted by $V_{x,0,1}$ for the linear response is at frequency w_2.

The output due to I_{in} only is

$$V_{out,0,1} = V_{2,0,1} - V_{1,0,1} = [1/(g_L+jw_2C_L)]I_{in} \quad (36)$$

The controlling voltages for the nonlinearity are

$$V_{43,0,1} = V_{53,0,1} = I_{in}/2g_m \quad (37)$$

HIGHER ORDER RESPONSE OF THE DIRECT METHOD

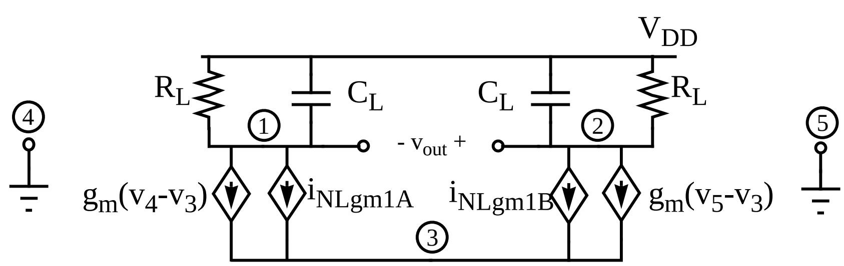

Figure 9. Circuit to be Solved for the Higher Order Responses

To find the higher order voltages, the same basic nonlinear network is solved for the response of each order. The circuit to be solved is shown in Figure 9. It is the same circuit we solved for the 1st order response except that the inputs are zeroed and excitations referred to as nonlinear current sources are placed in parallel with the nonlinear components. Only the expressions for the nonlinear current sources change depending on the order of response to be solved. These higher order currents sources model the behaviour of the nonlinear component. Like the Volterra Series method, the nonlinear sources depend on the basic type of nonlinearity modeled and are a function of n^{th} order nonlinear coefficients K_n and the lower order responses for the controlling voltage(s). Again, any linear analysis method can be used to solve this circuit. [5] uses a matrix method based on solving a set of KCL equations. SFGs can also be used and arguably gives more insight into the response.

One difference between the direct method and the Volterra series method is that solving the network for the node voltages correspond to actual voltages as opposed to Volterra kernels. This removes the intermediate step of converting the kernel to the output response. This comes at the cost of a less general solution. The direct method requires different expressions for the nonlinear sources depending on the type of response that is solved.

2^{nd} Order Response

The 2^{nd} order response is found by solving the linear circuit in Figure 9 with nonlinear sources of order 2. The 2nd order nonlinear current source for the response at $|w_1 \pm w_2|$ is

$$i_{NL2gm} = K_{2gm} V_{i,1,0} V_{i,0,\pm 1} \quad (38)$$

and for the harmonic response at $2w_1$ is

$$i_{NL2gm} = K_{2gm} (V_{i,1,0})^2 \quad (39)$$

Harmonic and intermodulation distortion as well as intercept points are directly calculated using the output voltages solved using Figure 9. The 2^{nd} order intermodulation output at frequency $w_1 + w_2$ can be shown to be

$$V_{out,1,1} = V_{2,1,1} - V_{1,1,1} \quad (40)$$
$$= (K_{2gm} V_{in} I_{in}) / [2g_m (g_L + j(w_1 + w_2) C_L)] \quad (41)$$

If the mixing circuit of Figure 7 is used as an upconverter, $V_{out,1,1}$ is the wanted mixing product. Assuming the baseband signal is applied at the bases of transistors Q_{1A} and Q_{1B} and the AC current is delivered by the current source I_{in} is proportional to the local oscillator signal. Then the conversion gain of this mixer can be found by dividing $V_{out,1,1}$ by the amplitude V_{in} of the baseband signal.

$$\text{Conversion gain} = K_{2gm} I_{in} / [2g_m (g_L + j(w_1 + w_2) C_L)] \quad (42)$$
$$= I_{in} / [4V_t (g_L + j(w_1 + w_2) C_L)] \quad (43)$$

Thus the conversion gain is proportional to the local oscillator signal. It is also seen that the conversion gain is proportional to the 2^{nd} order nonlinearity coefficient of the collector current.

For the case where the output at frequency $w_1 + w_2$ is an unwanted signal, then the intermodulation distortion ID_2 is calculated directly from the responses solved from the 1^{st} and 2^{nd} order systems.

$$ID_2 = V_{out,1,1}/V_{out,1,0} \quad (44)$$
$$= [V_{in}I_{in}/[4V_t(g_L+j(w_1+w_2)C_L)]] / [g_m/(g_L+jw_1C_L)]V_{in} \quad (45)$$
$$= I_{in}(g_L+jw_1C_L) / [4V_tg_m(g_L+j(w_1+w_2)C_L)] \quad (46)$$

3^{rd} Order Response

The 3^{rd} order kernels are evaluated and found in exactly the same way as the 2^{nd} order kernels except with 3^{rd} order nonlinear sources applied. The third order nonlinear behaviour results in responses at w_1, w_2, $|2w_1 \pm w_2|$, $|2w_2 \pm w_1|$, $3w_1$ and $3w_2$. The nonlinear current sources depend on the 1st and 2nd order controlling voltages. The equations (47) and (48) for the nonlinear current sources are given in [5].

Nonlinear current source for a transconductance g_m at frequency at $2w_1 \pm w_2$:

$$i_{NL3gm} = K_{2gm}V_{i,1,0}V_{i,1,\pm 1} + K_{2gm}V_{i,0,\pm 1}V_{i,2,0} + (3/4)K_{3gm}V^2_{i,1,0}V_{i,0,\pm 1} \quad (47)$$

Nonlinear current source for a transconductance g_m at frequency at $3w_2$:

$$i_{NL3gm} = K_{2gm}V_{i,0,1}V_{i,0,2} + K_{3gm}V^3_{i,0,1} \quad (48)$$

where V_i in our example would be the voltage v_{be} which controls the nonlinear transconductance g_m.

The value of the sources in (47) and (48) can be understood to come from considering all the possibilities to produce a third order signal. For instance, to create to third order nonlinearity at frequency $2w_1+w_2$, the second order nonlinearity combines a first order signal at frequency w_1 with a second order nonlinearity at frequency w_1+w_2. The second term corresponds to the nonlinearity which results from the combination of a first order signal at w_2 and a second order signal at $2w_1$. And lastly the third term results from the third order nonlinearity combining three first order signals: two at frequency w_1 and one at frequency w_2.

Solving the circuit of Figure 9 with the nonlinear current in (47) results in the following responses:

$$V_{out,2,1} = 0 \quad (49)$$

The intermodulation output at frequency $2w_1+w_2$ is zero because even order harmonics of the input signal are cancelled in a differential circuit in which the components are matched.

The intermodulation at frequency w_1+2w_2 is

$$V_{out,1,2} = (1/8)[1/(g_L+(jw_1+2jw_2)C_L)][-K_{2gm}^2 / g_m^3 + (3/2)K_{3gm}/g_m^2]V_{in}I_{in}^2 \quad (50)$$

$$V_{out,1,2} = (1/8)[1/(g_L+(jw_1+2jw_2)C_L)][-g_m^2 / (4V_t^2g_m^3) + (3/2)g_m/6V_t^2g_m^2]V_{in}I_{in}^2 = 0 \quad (51)$$

It is seen that this intermodulation distortion is also zero when the exponential model for the collector current holds. When the transistors Q_{1A} and Q_{1B} are implemented as MOSFETs, the drain current of the MOS satisfies the square law and thus $g_m = \beta(V_{GS} - V_T)$, $K_{2gm} = \beta/2$, and $K_{3gm} = 0$. The resulting intermodulation distortion output from (50) becomes,

$$V_{out,1,2} = -(1/32)[1/(g_L+(jw_1+2jw_2)C_L)][V_{in}I_{in}^2/\beta(V_{GS} - V_T)^3] \quad (52)$$

In the case of an upconverter, the response at w_1+w_2 is the wanted signal, and using (41) with the appropriate nonlinearity coefficients we obtain

$$V_{out,1,1} =(1/4)[1/(V_{GS} - V_T)][1/(g_L+j(w_1+w_2)C_L)]V_{in}I_{in} \quad (53)$$

The ratio of the third order intermodulation product and the wanted response is

$$V_{out,1,2}/V_{out,1,1} = -(1/8) [1/\beta(V_{GS} - V_T)] [(g_L+j(w_1+w_2)C_L)/(g_L+(jw_1+2jw_2)C_L)] I_{in} \quad (54)$$

When this ratio is set to 1 and solved for I_{in}, then we have an expression for the third-order intercept point:

$$IP_3 = 8\beta(V_{GS} - V_T)^2 |(g_L+j(w_1+2w_2)C_L)/(g_L+(jw_1+jw_2)C_L)| \quad (55)$$

CONCLUSIONS

In this paper, the Volterra series and the direct method of distortion analysis were presented. The advantage of the direct method is that circuits with multiple inputs can be analyzed. Recent work has shown how SFGs can be applied to the Volterra Series Method to simplify the method of analysis. Since the Direct Method also finds higher order responses by solving a linear network, the SFG can also be used to solve for distortion components. SFGs presents a graphical analysis which is often more intuitive and insightful than resorting to a KCL analysis. A recent paper we submitted to the ISCAS 2002 conference outlines this graphical method of distortion analysis. In it, the first 3 orders of response are represented by a single SFG. This graphical representation more clearly illustrates how nonlinearities are generated in the system and how various parameters affect the response. Conversely, the usual method of solving a matrix of nodal equations gives us little information about the circuit and even less insight into how expressions for the kernels were derived. Future work could include integrating the direct method with SFGs such that systems with multiple inputs can be analyzed more easily. Multiple inputs introduce complexity into the system, and it may be more difficult to be possible to generalize a solution where the distortion components of different orders can be found from a single graph. Nevertheless, this approach is worth pursuing for the SFG representation allows us to easily modify the circuit, use different models for the nonlinearities, and make approximations. In addition, the SFG can also illustrate the coupling between nonlinear elements which is difficult to see when considering a single response at a time.

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