Distortion Analysis of a LNA

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0. Abstract

This paper presents an overview and comparison of the common solution in determining distortion in LNA. Brief background information regarding how distortion affects the performance of CMOS LNA is presented. Recent CMOS LNA distortion analysis methods based on SPICE simulation and analytical analysis from three research works are presented. Finally, suggestion for further research work on distortion analysis of LNA is presented.

I. Introduction

In the last decade, there had been an explosive growth in the consumer wireless telecommunication sector, along with the growth there had been also a growing demand from the consumers for portable wireless systems that are capable of operating with lower power consumption, better performances, and lower cost. The increase in demand from the consumers had led to numerous developments in radio frequency (RF) circuits by taking advantage of technology advances in CMOS processes.

In a present-day wireless receiver, low noise amplification had to be carried out prior to frequency translation. From the typical modern receiver setup as shown in Figure 1, a low-noise amplifier (LNA) stage is often included to provide sufficient gain to overcome the noise in subsequent stages. Apart from the need to provide gain while adding as little noise as possible, LNA must also possess good linearity and a wide useable dynamic range.

This paper provides some insight in the distortion analysis methods that are commonly found in the literature. To provide some background, Section II presents mathematical derivation showing how distortions and nonlinearities exist and present in a realistic device. Section II also provides short descriptions on the two categories of distortions. Recent distortion analysis works on amplifier stage made from simple single transistor stage intended for implementation as LNA is presented in Section III. These works will provide a reference solution that is commonly used in distortion analyses in LNA. Finally, Section IV provides possible future works as to how analysis of nonlinearities can be better carried out.

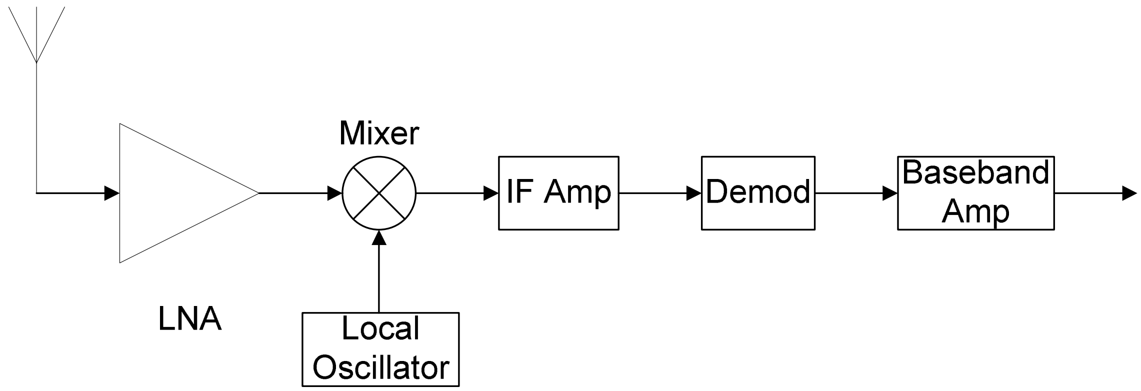

Figure 1: Typical Receiver Block Diagram.

II. Background Information

Distortions in most analog and RF circuit blocks arise from the nonlinearities inherent with the devices used. Looking at the a single input amplifier block in Figure 2, if a system does not contain any distortion then the output of the system would just be the input multiplied with a gain constant.

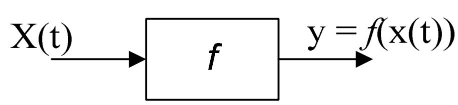

Figure 2: Amplifier Block with Single Input.

However, realistic systems always contain some inherent nonlinear properties and from [1] the output of the system such as that shown in Figure 2 can be described by its Volterra series

$$y(t) = \sum_{n=1}^{\infty} \mathbf{H}_n\left[x(t)\right]. \tag{1}$$

with Volterra operator, $\mathbf{H}_n$, in (1) being derived from the nth order Volterra kernel, $h_n(\tau_1,\tau_2,\cdots,\tau_n)$, using

$$\mathbf{H}_n[x(t)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \cdots, \tau_n) x(t-\tau_1) x(t-\tau_2) \cdots x(t-\tau_n) d\tau_1 d\tau_2 \cdots d\tau_n . \quad (2)$$

Although Volterra series is a powerful tools and it fully describe all the nonlinear properties of a system, for practical analysis it is not often used. Being a power series with memory requires all its volterra operators to be computed from convolutions and for high order of n^{th} kernel it can results in excessive computations. For most of the situations however, CMOS transistors can be assumed to be memoryless and time-variant [2]. Hence an amplifier block created from transistors can also assumed to be memoryless; the output of such system can instead be described by a power series of the form

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots , \quad (3)$$

where x(t) is the input to the system and a_n are the n^{th} order nonlinearities constants tabulated using

$$a_n = \frac{1}{n!} \frac{\partial^n f(x(t))}{\partial x^n(t)} . \quad (4)$$

From (1) and (3), it is evident that any systems will display some sort distortions at the output due to their inherent nonlinearities. These nonlinearities can be classified into two categories—nonlinear distortion effects and nonlinear interference effects [3]. Nonlinear distortions effects include distortions that occur in the absence of any other signals beside the desired input signals, whereas nonlinear interference effects are the results involving one or more undesired input signals.

Two nonlinear phenomena are often associated with LNA, they .are the gain compression (expansion) and third order intermodulation. Gain compressions (expansion) is a nonlinear distortion effect cause by the inherent nonlinearities of the device, assuming that the input to the system is a sinusoidal signal with amplitude A and frequency ω and substituting into (3), the power series for the output becomes

$$y(t)=\left(a_0+\frac{a_2}{2}A^2\right)+\left(a_1+\frac{3}{4}a_3A^2\right)A\cos(\omega\, t)+\frac{a_2}{2}A^2\cos(2\omega\, t)+\frac{a_3}{4}A^3\cos(3\omega\, t)+\cdots \quad (5)$$

From (5), the fundamental frequency component of the output is modified by the a_3 and causes distortion in the form of gain compressions or gain expansion depending on the sign of a_3.

Third order intermodulation on the other hand is a nonlinear interference effects caused by interferences between one or more undesired signals at the input. Assume that the input to the system is sum of the two sinusoidal signals with frequency components of ω_1 and ω_2, having amplitudes of A. After some algebraic manipulation the output of the system becomes

$$y(t)=\begin{array}{l}\left(a_0+a_2A^2\right)+\left(a_1+\frac{9}{4}a_3A^2\right)A\left[\cos(\omega_1\, t)+\cos(\omega_2\, t)\right]+\\ \left(\frac{a_3}{2}A^2\right)\left[\cos(2\omega_1\, t)+\cos(2\omega_2\, t)\right]+\left(\frac{a_3}{4}A^3\right)\left[\cos(3\omega_1\, t)+\cos(3\omega_2\, t)\right]+\\ \left(a_2A^2\right)\left[\cos(\omega_1+\omega_2)t+\cos(\omega_1-\omega_2)t\right]+\\ \left(\frac{3}{4}a_3A^3\right)\left[\cos(\omega_1\pm 2\omega_2)t+\cos(2\omega_1\pm\omega_2)t\right]+\cdots\end{array}. \quad (6)$$

From (6), ignoring the output terms produced by the 4th and higher order of nonlinearities, the cubic term of the system produces third-order intermodulation products with frequency components ($\omega_1 \pm 2\omega_2$) and ($2\omega_1 \pm \omega_2$). (5) and (6) demonstrate that for a system with inherent nonlinearities, it is possible to generate frequency components that are not present at the input and causes distortions.

The gain compression is often measured as the 1dB compression point which by definition is the power level when the gain deviates away from linearity by 1dB. The 3rd order intermodulation distortion is often measure as the 3rd intercept point (IP3).

III. Recent LNA Distortion Analysis Research

Linearity of LNA is determined by the gain compression and 3rd order intermodulation. The most common practice in determine the linearity of systems found in published research works, [4]-[6], made use of time-domain simulators such as SPICE or actual experimental to measure the first-order output and third-order intermodulation terms. The output power levels of both the fundamental and third-order intermodulation are recorded and plotted against input power levels to produce a graph similar to that shown in Figure 3, the 1dB compression and third-order intercept points are then determined via extrapolation.

The nonlinear distortions results obtained from simulations can vary significantly depending on the models used; the degree of accuracy in the distortion analysis obtained from SPICE depends on the degree of accuracy of the models used by the SPICE simulator. Often computed nonlinearity coefficients are acquired from the data of DC analyses, in situations in which low level BSIM models are used then inaccurate distortion analysis might occurred.

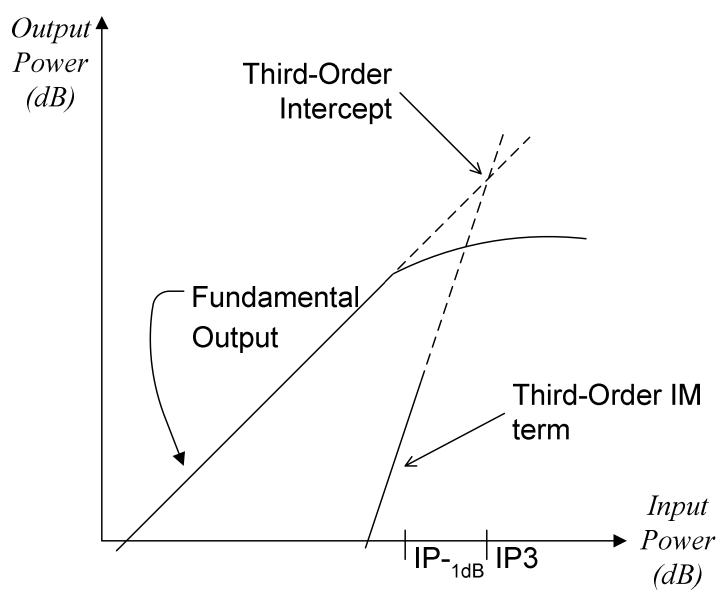

Figure 3: Graphical Representation of LNA Linearity Parameters

To analyze distortion in LNA, it is generally sufficient to analyze the distortion of single transistor as LNA typically a single transistor gain stage (i.e. common-source and common-gate) with tuned impedance and low noise figure. A review of the literatures indicated three papers that investigated distortion of LNA using different methods.

The first example in nonlinearities analysis of LNA is [2], it involves analytical analysis of nonlinear distortion when a system can be described by a power series similar to (3). A degenerated common-source stage was used in the analytical analysis, as it is one of the most simple and common LNA setup. Small signal model of the LNA as shown in Figure 4 was used in the research work, the models neglects several effects such as output conductance and the body effect.

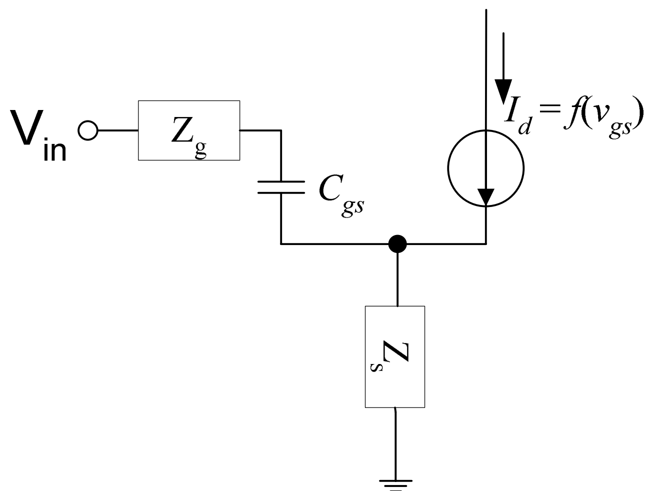

Figure 4: Small Signal of Common-Source Amplifier used to calculate Linearity.

From the small signal model the input voltage is written as

$$V_{in} = Z_s i_d + \left(j\omega\, C_{gs} Z_s + j\omega\, C_{gs} Z_g + 1\right) v_{gs}\,, \tag{7}$$

with some algebraic manipulation the long-channel drain current of the transistor is then described in [2] to be

$$i_d = \begin{array}{c} 2k\left(V_{GS} - V_T\right) v_{in} + \left(\dfrac{k}{F^2} - \dfrac{2Z_s k^2 \left(V_{GS} - V_T\right)}{F^3}\right) v_{in}^2 + \\ \left(\dfrac{4Z_s^2 k^3 \left(V_{GS} - V_T\right)}{F^5} - \dfrac{2Z_s k^2}{F^4}\right) v_{in}^3 + \cdots \end{array}, \tag{8}$$

where

$$F = j\omega\, C_{gs}\left(Z_s + Z_g\right) + 2kZ_s\left(V_{GS} - V_T\right) + 1\,, \tag{9}$$

and

$$k = \frac{\mu C_{OX}}{2}\frac{\mathrm{W}}{\mathrm{L}}. \tag{10}$$

3rd order intermodulation reported in the paper is

$$|IM3| = k\left|\frac{1}{F^3}\right|\left|\frac{2Z_s k}{F} - \frac{Z_s}{(V_{GS} - V_T)}\right|\left|v_{in}^2\right|. \tag{11}$$

Beside the common-source configuration amplifier, the nonlinear analysis of a common-gate amplifier stage had also been mentioned in [2]. Schematic setup of the common-gate amplifier stage mentioned in [2] is reproduced in Figure 5.

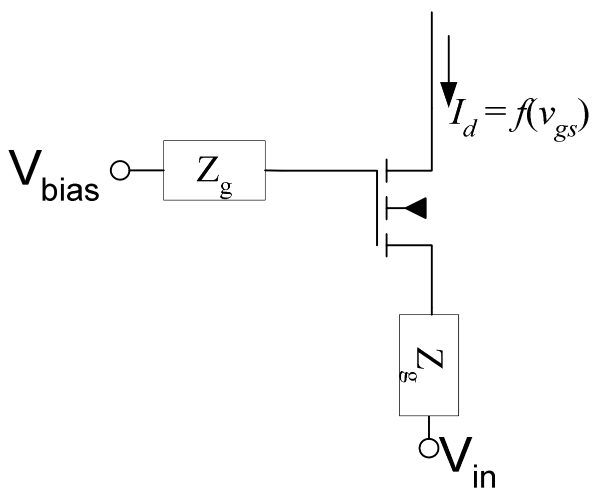

Figure 5: Common-Gate Transconductance Stage

From [2], the 3rd order intermodulation product for the common-gate amplifier stage is the same as that for the common-source amplifier stage and is given by (11).

The second work on distortion analysis found in the literature is [7]; the work involves the construction of a low-noise amplifier in 0.35μm CMOS process with the schematic setup shown in Figure 6. Transistors used in the schematic were assumed to be memoryless and time-invariant with the AC drain current expressed in a Taylor series with the small signal gate-source, drain-source and source-bulk voltages as inputs. SPICE simulation was used in this research work to determine the nonlinear distortion of the amplifier, coefficients of the Taylor series were determined in this case by numerical differentiation on data from DC analyses. These distortion coefficients were check against the third order Volterra kernel evaluated at all frequency combinations in the band of interest and was reported to have a maximum error on the magnitude of 13%.

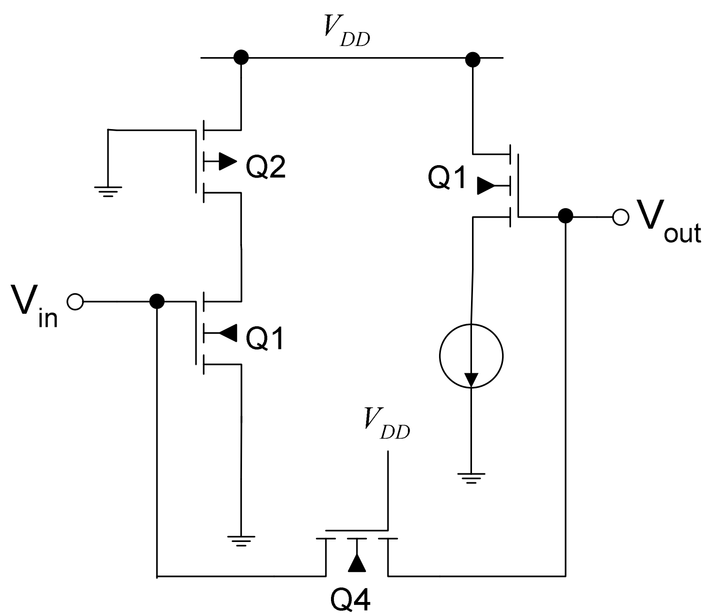

Figure 6: Simplified CMOS Amplifier.

The third research work dedicated to nonlinear analysis is [8], it discussed the nonlinearities in short-channel MOSFETs. With the minimum effective length of transistors decreasing with advancement in CMOS processes, in many applications it is possible for the transistors to operate in short-channel regime instead of the usual long-channel operation. When a transistor operates in short-channel regime, the general expression with the drain-source current being proportional to the square of the effective voltage becomes invalid. Instead, the drain current and effective voltage are related by the following expression

$$I_{D,SAT} = W v_{SAT} C_{OX} \frac{V_{eff}^2}{V_{eff} + E_{SAT} L}, \tag{12}$$

with E_{SAT} being the velocity saturation field strength and V_{SAT} is the saturation velocity. The saturation field strength and saturation velocity are defined as

$$E_{SAT} = \frac{2v_{SAT}}{\mu_{SAT}}, \tag{13}$$

$$\mu_{SAT} = \frac{\mu_o}{1+\theta V_{eff}}. \tag{14}$$

The drain-source current was then expressed in power series to determine linearity, using (12) the coefficients of power series was then determined and the input-referred 1dB compression and 3rd intercept points were reported in [8] to be

$$P_{1dB} \approx 0.29 \frac{v_{SAT}}{\mu_1 R_S} v_{eff} \left(1+\frac{\mu_1 v_{eff}}{4v_{SAT}L}\right)\left(1+\frac{\mu_1 v_{eff}}{2v_{SAT}L}\right)^2 , \tag{15}$$

$$P_{IP3} \approx \frac{8}{3} \frac{v_{SAT}}{\mu_1 R_S} v_{eff} \left(1+\frac{\mu_1 v_{eff}}{4v_{SAT}L}\right)\left(1+\frac{\mu_1 v_{eff}}{2v_{SAT}L}\right)^2 , \tag{16}$$

where

$$\mu_1 \equiv \mu_0 + 2\theta v_{SAT} L. \tag{15}$$

Values for 1dB compression and 3rd intercept points predicted by (15) and (16) were plotted against values obtained from BSIM and LEVEL3 simulations and the results obtained in [8] are reproduced in Figure 7.

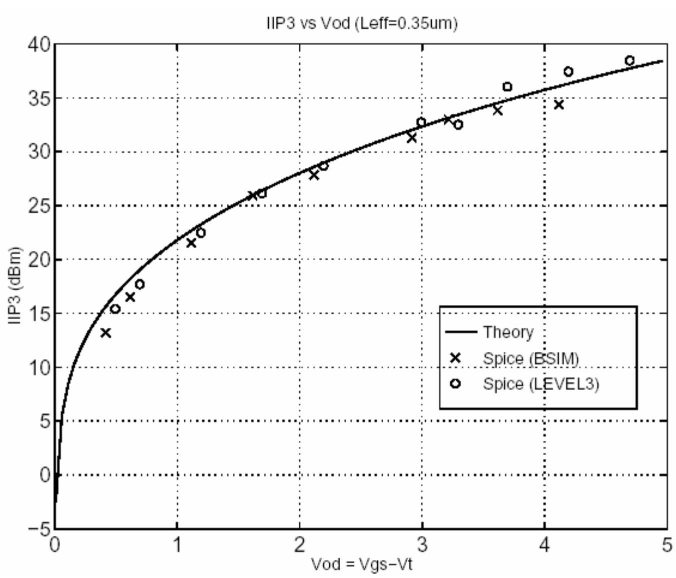

Figure 7: Comparison between Theory and Simulation of IPP3.

From the figure showing the 3rd intercept point against simulation results, the analytical expression matches within 10% of simulation values.

IV. Future Work

From the research works shown in Section III, due to the inherent nonlinearities of CMOS transistors, the dynamic range of LNA will always be limited by nonlinear distortions. Analyses of these distortions involve solving for the coefficients of power series by the coefficients either by computing numerical differentiation on data from DC analyses or analytical analysis. To obtain better accuracy in nonlinear distortion analysis results, transistors and passive devices must be accurately modeled by their I-V characteristic and its first n^{th} derivatives [9]. Higher order of accuracy of modeling transistors in the BSIM or LEVEL3 models used for SPICE simulations is necessarily in order to achieve better and more accurate distortion analysis. The challenge is to provide models that describe transistors' operation perfectly.

V. Conclusion

This paper discussed how distortion exists and affects the performances of CMOS LNA; analytical expressions for a memoryless and time-variant amplifier block were derived. Examples of linearity analyses in three different research works using different methods ranging from using SPICE simulator and analytical calculation were demonstrated. Analytical expressions for 1dB compression and 3rd order intercept points found in the literature were reproduced in the paper.

References

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