A Q–Enhanced Active–RLC Bandpass Filter

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Abstract—The passive elements R, L, and C are combined with an amplifier to realise a high-Q bandpass filter. Positive feedback is used to enhance the finite Q of the lossy integrated inductors, and design equations are given that take into account the finite gain and finite bandwidth of the amplifier. It is shown that a quality factor of 20 at 1 GHz is possible with 10 GHz transistors. In addition, preliminary noise analysis indicates promising results.

I Introduction

Monolithic, high-frequency RLC filters have recently been realised [1][2][3], but exhibit limited selectivity due to the small Q-factors of the integrated inductors; these inductors typically exhibit Q-factors of 3-8 in the 1-2 GHz range [4]. An active-RLC bandpass filter, which places the inductor in the feedback path of an amplifier to effect Q enhancement, is discussed below. It is expected that active-RLC will exhibit lower sensitivity to active components, and possibly lower noise, than corresponding active-RC filters. This is timely, since the growing need for, and popularisation of, personal communication systems will demand fully integrated solutions to almost all electronic functions [5][6]. This paper is about an active-RLC design in a 0.8μm BiCMOS technology.

II Background

Consider the resonance circuit of Fig. 1(a), in which r models the inductor losses. This circuit is important for it is the simplest prototype from which we can obtain an active-RLC filter. The characteristic equation describing this circuit is

\[ s^2 + \frac{s}{L} + \frac{1}{LC} = 0 \]

wherefrom the nominal centre-frequency and quality factor are defined as \( \Omega_0 = \frac{1}{(LC)^{1/2}} \) and \( Q_0 = \frac{\Omega_0}{L/r} \), respectively. That is, as expected, the quality factor of the filter is, at most, equal to that of the inductor. This is precisely the case for the circuit in [2]. The signal flow graph of this circuit when capacitively driven (Fig. 1(b)), suggests an approach for controlling the filter’s quality factor, bandwidth, and centre-frequency. The parameter \( b_1 \) is proportional to the filter’s bandwidth and can be controlled independently of centre-frequency; it represents a trans-impedance gain between the inductor’s voltage and current. Similarly, \( b_0 \) is proportional to the centre-frequency and can be controlled independently of bandwidth. The circuit of Fig. 2, which effects Q-enhancement, results from using this design approach. It is a positive feedback circuit and enhances the quality factor by, in effect, inserting a negative resistor in series with the inductor so as to cancel the ohmic loss. The characteristic equation describing this circuit is
\[
\frac{V_o(s)}{V_i(s)} = -\frac{s C (1 - \alpha)}{s^2 \left[ \frac{LC}{Z} + s C \left[ \frac{r}{Z} - \alpha \right] + \frac{1}{Z} \right]}
\]  
(3)

Assuming a single-pole amplifier response, \(Z(s) = \frac{Z_0}{1 + s/\omega_p}\), and substituting for \(Z(s)\) in (3), results in

\[
\frac{V_o(s)}{V_i(s)} = \alpha C Z_0 \omega_0^2 \frac{s}{s^2 + \omega_0^2 + \omega_p^2 + 1 + s/\omega_i}
\]  
(4)

where

\[
\omega_0 = \frac{\Omega_0}{\sqrt{1 + \frac{r}{L \omega_p}}}
\]  
(5)

\[
\frac{1}{\bar{Q}} = \frac{1}{Q_i + \omega_p \left( \frac{1}{1 + \frac{1}{L \omega_p}} \right)} \left[ \frac{1}{\sqrt{1 + \frac{r}{L \omega_p}}} \right]
\]  
(6)

\[
\omega_1 = \omega_p + \frac{r}{L}
\]  
(7)

and \(Q_i = \frac{Q_0}{1 - \alpha Z_0/r}\) is the ideal quality factor. The simplifying assumption \(Q > 1\) was used for ease of analysis, and is reasonable since we are designing for a \(Q\) greater than 10.

**IV Amplifier Frequency Response**

The requisite trans-impedance amplifier must exhibit a wide bandwidth and well-controlled variable gain. A resistively-loaded Gilbert quad [8], shown in Fig. 4, was chosen since it satisfies those requirements. The input current is sampled by the
Fig. 4. Amplifier Schematic.

Fig. 5. Realisation of Q-enhanced Filter.

low-impedance input diodes, \( Q_1 \) and \( Q_2 \), amplified, and converted to a voltage by output resistors \( R \). The frequency response of the circuit is approximated by

\[
\frac{V_o}{i_i} = R \frac{I_E}{2I_A} \frac{1 + sC_1 f_0}{1 + s(C_{s2} + g_m R C_r) r_d} (1 + s2C_{s2/2}g_m) \tag{8}
\]

where \( Z_0 = R \frac{I_E}{2I_A} \) and \( \omega_p = \frac{1}{(C_{s2} + g_m R C_r) r_d} \). As seen, there is also a non-dominant pole at half the unity-gain frequency of the devices, and a real zero. Thus, the assumed single-pole response of the amplifier in the previous section poorly approximates the response of the amplifier, but still provides useful circuit insight. The dc gain is easily controlled by the ratio of \( I_E \) to \( I_A \).

V Simulation Results

Fig. 5 shows the realisation of the Q-enhanced active-RLC filter. Not shown are the Q- and frequency-control circuitry [9]. This design uses 5 nH inductors with nominal Q of 5 at 2 GHz. The tuning capacitor is 1 pF, with \( \alpha = 0.7 \); the circuit is driven from 50 \( \Omega \) source and a large \( \alpha \) reduces input capacitance. The load resistors are 100 \( \Omega \) each. The simulation results of Fig. 6 show both the magnitude response, in dB, and phase response of the circuit as the current in \( Q_8 \) is varied. As seen, the filter's quality factor increases, from 15 - 70, with increasing bias current, with a relatively fixed center-frequency of 1.55 GHz.

VI First-Order Noise Analysis

Consider the passive circuit of Fig. 1(a), in which the resistor, \( r \) is the only source of noise. It is easily shown that the total current noise is given by

\[
\overline{i_n^2} = \frac{kT}{L} \tag{9}
\]

or alternatively, the voltage noise is \( \overline{v_n^2} = \frac{kT}{L} r^2 \), and is independent of the quality factor. Suppose, for ease of analysis, that the trans-impedance amplifier in Fig. 3 is represented as an output-buffered

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Fig. 6. Filter Frequency response.

common-base stage, with a resistive load $R_0$. The bandwidth of the filter is given by

$$B = \frac{r}{L} \left(1 - \frac{R_0}{r}\right) = \frac{rQ_0}{LQ}$$

(10)

Neglecting base resistances, it can be shown that the loss in the inductor dominates the output noise voltage. That is

$$\overline{v_{no}^2} = \frac{kTQ}{LQ_0} \left[\left(\frac{r}{\alpha}\right)^2 \left(1 - \frac{Q_0}{Q}\right)^2\right]$$

(11)

Thus, like active-RC, the dynamic range of this active-RLC design is inversely proportional to $Q$ [10].

VII Conclusion

Positive feedback was applied to a passive RLC circuit, and simulations show that a $Q$ of 20 is possible in the presence of lossy integrated inductors. The gain and bandwidth requirements on the amplifier were shown not to be excessive, lending credence to the viability of active-RLC for designing high-frequency monolithic filters.

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VIII References


