

# EQUIVALENCE CLASSES OF CLONE CIRCUITS FOR PHYSICAL-DESIGN BENCHMARKING

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## Abstract

To provide a better understanding of physical design algorithms and the underlying circuit architecture they are targeting, we need to exercise the algorithms and architectures with many benchmark circuits. The lack of existing benchmarks makes us consider the automatic generation of netlists.

In this paper, we formally define the equivalence class of circuit “clones” of a given seed circuits, based upon physical properties of the seed circuit’s netlist graph. Using these equivalence classes of circuits, a given seed circuit can be used to generate many similar circuits. A more finely grained statistical analysis of algorithm behaviour can then be obtained from using the multiple benchmarks than would be available from using the seed benchmark alone.

## 1. INTRODUCTION

There are two primary motivations for the automatic generation of benchmark netlists.

The first motivation is that there are simply not enough benchmark circuits available, or not enough at a given density range. With the exception of the recent efforts such as the ISPD’98 partitioning benchmarks from Charles Alpert at IBM [1], the major drawback of public-domain benchmarks is that they lag industry by multiple orders of magnitude in numbers of gates. ASIC designs can use over 5M gates, next-generation programmable logic designs up to 1M gates. Most public domain benchmarks for partitioning, placement, routing and programmable-logic architecture design (e.g. [2]) range from 100 to 2000 4-input LUTs, or less than 10K gates. Neither logic nor algorithms scale well enough for experimentation at small density to be directly applicable to larger density problems. Even within industry, there is a fundamental problem in that new algorithms and architectures must be defined and evaluated a full design-cycle before there are any true benchmarks at the next-generation density range.

The second motivation for automatic generation of benchmarks is to improve the statistical validity of tests that we can perform on a limited number of circuits. Since algorithms for NP-hard problems are always heuristic, they can exhibit varied performance under only slightly different circumstances. Thus, it is advantageous for us to have multiple circuits which are similar enough to group in the same class, yet different enough

to isolate the experiment from minor variations due to the particular heuristic bias of a given algorithm. This latter issue is the primary focus of the current ISCAS special session [3].

There have been several previous attempts at generating automatic benchmark circuits. Hutton *et. al.* [4,5,6,7] have described methods and a tool for generating circuits subject to a number of statistical properties about the physical nature of the circuit. This can be done either by extracting the parameterization from a seed circuit, or by random selection of the properties from defaults extrapolated from a collection of benchmarks. Darnauer and Dai [8] generated circuits by recursive decomposition, using Rent’s Rule [9,10] as a basic paradigm. In both these approaches, the user is able to generate circuits of virtually any size, without an existing seed circuit, but the correspondence of the circuits to reality diverges as the size grows. Iwama and Hino [11], Kapur *et. al.* [12], Ghosh *et. al.* [13,14] and Harlow and Brglez [15] have addressed the “mutation” of initial circuits using various transformation rules in order to effect a circuit with similar overall structure but differing local connectivity.

One of the reasons that heuristic algorithms work well in practice is that “real netlist graphs” exhibit fundamental restrictions on simple graph-theoretic properties such as fanout, distance on paths and the location of sequential elements (flip-flops) in the netlist. In order to do valid experimentation it is thus important to capture these properties in the circuits we use for benchmarks.

In this paper, we formally define equivalence classes of circuits based upon purely physical netlist characteristics: fanout of nodes, path-length, the relative location of nodes on paths through the design, and the length of edges induced by these locations. When an equivalence class is based on the extracted characteristics of an existing seed benchmark circuit, we call the artificially generated members of the same class “clone-circuits” of the original. The work reported on in this paper formalizes and builds upon previous work [4,5,6,7] in which we created two tools: CIRC, a tool to analyze netlists and extract the properties of interest, and GEN, a tool to generate a new circuit given a set of properties. In the previous work we used pairwise comparison of clone circuits to validate the generation tool, but did not make formal definitions of clones or analyze the behaviour of equivalence classes induced by clones.

In a companion paper [16], we will outline a methodology for the evaluation of competing heuristic algorithms for physical design using clone circuits. We will apply the methodology to several different problems and comment on practical experience doing so for these and other issues requiring benchmarks, such as the design of programmable logic architectures. In doing so we will make conclusions as to where clone circuits can and cannot be used effectively.

## 2. EQUIVALENCE CLASSES

The purpose of the benchmarks we are generating is to analyze the performance of physical design algorithms and the interconnection networks of programmable-logic architectures. Thus, our circuit parameterization is based on purely physical graph-theoretic properties of the netlist: node types, connectivity and path-lengths, as opposed to functional properties such as “this is a multiplier.”

We will assume that all netlists under discussion are synchronous designs with no bidirectional pins. Nodes in the netlist  $N$  consist of primary inputs (PI), primary outputs (PO), 4-input lookup tables (LUT) and D-type flip-flops (DFF) gated by a single global clock called ‘clock’ and no other secondary signals. The fanin and fanout of a node have their standard meaning, and  $\max\_fanout(N)$  is the maximum fanout value for any node  $x$  in  $N$ . There are no combinational cycles allowed.

To describe a netlist,  $N$ , we need to formalize a number of terms.

**Definition 2.1:** Define the unit *delay* of a node  $x$ : If  $x$  is a PI or DFF, then  $\text{delay}(x)$  is 0. Otherwise  $\text{delay}(x)$  is 1 + the maximum unit delay of any fanin of  $x$ . Define  $\text{delay}(N)$  as the maximum, over all nodes  $x$  in  $N$ , of  $\text{delay}(x)$ .

**Definition 2.2:** Define the sequential *level* of a node  $x$ . If  $x$  is a PI, then  $\text{level}(x)$  is 0. If  $x$  is a DFF, then  $\text{level}(x)$  is 1+ the level of the D-input of  $x$ . Otherwise  $\text{level}(x)$  is the minimum level over all fanins of  $x$ . Define  $\text{levels}(N)$  as 1 + the maximum, over all nodes  $x$  in  $N$ , of  $\text{level}(x)$ . Define *level  $i$*  of  $N$  as the subgraph  $N_i$  induced by the set of nodes in  $N$  which are at level  $i$ .

**Definition 2.3:** A netlist  $N$  is *combinational* if it contains no DFF nodes, and *sequential* otherwise. If  $N$  is combinational it must have exactly 1 level, and all nodes  $x$  satisfy  $\text{level}(x) = 0$ . Otherwise  $N$  has at least 2 levels, and at least one node at each level.

Under the restrictions mentioned previously (no combinational cycles or bidirectional I/Os and a single global clock), both  $\text{level}(x)$  and  $\text{delay}(x)$  are well-defined.

**Definition 2.4:** The *shape* function of a combinational netlist  $N$  is defined as an integer vector  $\text{shape}[d]$ ,  $d = 0..\text{delay}(N)$ , where  $\text{shape}[d]$  is the number of nodes in  $N$  which have delay  $d$ .

**Definition 2.5:** Given a directed edge  $e=(x,y)$  in a netlist  $N$ , define  $\text{length}(e) = \text{delay}(y) - \text{delay}(x)$ . If  $\text{level}(y) < \text{level}(x)$  then  $e$  is a *back-edge*. If  $\text{level}(y) = \text{level}(x)$  then  $\text{delay}(y) > \text{delay}(x)$  and  $e$  is a *forward-edge*. Otherwise  $e$  is a

*FF-edge*, and we must have  $\text{delay}(y) = 0$ ,  $\text{level}(y) = \text{level}(x) + 1$ , and  $x$  is a DFF node. There are no other cases possible under the definitions of delay and level.

**Definition 2.6:** The *edges* function of a netlist  $N$  is defined as an integer vector  $\text{edges}[d]$ ,  $d = 0..\text{delay}(N)$ , where  $\text{edges}[d]$  is the number of edges in  $N$  of length  $d$ .

**Definition 2.7:** The fanout function of a netlist  $N$  is defined as an integer vector  $\text{fanout}[f]$ ,  $f = 0..\max\_fanout(N)$ , where  $\text{fanout}[f]$  is the number of nodes  $x$  of  $N$  with fanout  $f$ .

We can now outline a mechanism to decompose or partition a netlist into two or more parts. Given  $N$  and a bipartition  $X$  and  $Y$  of the nodes of  $N$ , create two graphs  $N_x$  and  $N_y$  induced by the partition. For every edge  $e=(x,y)$  where  $x$  is in  $N_x$  and  $y$  is in  $N_y$ , create a new primary input  $x'$  in  $N_y$  for  $x$ , and a new primary output  $y'$  in  $N_x$  for  $y$  (and similarly for edges from  $Y$  to  $X$ ). The netlist graphs  $X$  and  $Y$  are now disjoint, yet by identifying or gluing the appropriate nodes  $\{x,x'\}$  and  $\{y,y'\}$  together we can re-create  $N$ .

**Definition 2.8:** Under the process just described for an edge  $e=(x,y)$ , define additional nodes  $x'$  to be a *ghost input* (GI) in  $N_y$  and  $y'$  to be a *ghost output* (GO) in  $N_x$ . Define  $\text{delay}(x')$  to be that of  $\text{delay}(x)$  and  $\text{delay}(y')$  to be that of  $\text{delay}(y)$ , supplementing the previous definitions with that of the new node-types GI and GO. Along with primary output nodes (PO), we can infer new shape functions  $\text{POshape}[d]$ ,  $\text{GOshape}[d]$  and  $\text{GIshape}[d]$  as we did for the delay-based shape function on the appropriate subset of nodes.

One obvious decomposition of a graph is into levels, whereby each flip-flop or back-edge induces a ghost input in one level and a ghost output in another. It is this usage that we will use to define the complete signature of a sequential netlist. We will call this process the *sequential decomposition* of a netlist  $N$  into its level-netlists  $N_0, N_1$ , etc.

For a netlist, we use the scalar parameters  $nPI, nPO$ , etc. to denote the number of nodes of the corresponding node-type.

**Definition 2.9:** The *signature* of a level-netlist  $N_i$  is composed of  $\{i, n, nPI, nPO, nLOG, nDFF, nPO, nGI, nGO, \text{delay}(N_i), \max\_fanout(N_i), \text{shape}[], \text{edges}[], \text{fanouts}[], \text{POshape}[], \text{GOshape}[], \text{and GIshape}[]\}$ . The signature of a sequential netlist  $N$  is defined by the collective signatures of its sequential decomposition. For an exact specification the scalar parameters are redundant given the vector parameters in the signature but are part of the signature for clarity.

Given the concept of a signature of a netlist, we can now formally define equivalence classes of netlists.

**Definition 2.10.** Two netlists are *equivalent* if they have the same signature. Given the set of all netlists of any size, we can then induce a mathematical equivalence class to properly partition all netlists into equivalence classes under signatures.

**Definition 2.11.** Given a set of circuits generated to have the same signature as a given input circuit, we refer to the original circuit as the *seed* circuit, and the other members of the equivalence class as *clone* circuits.



problem for simple specifications such as the one in Figure 2, but as the specification is further and further constrained there is more onus on the user to choose parameters which are compatible. It is possible to generate glued circuits with multiple different components manually, but the user has to match the GI and GO distributions carefully to ensure that the circuit is “glueable”. In future versions of GEN we hope to have a better solution to the issue of conflicting parameters so that we can do multi-level and hierarchical gluing without help from the user.

## 4. CONCLUSIONS

In this paper we have formally defined equivalence classes of clone circuits, and shown how the tools CIRC and GEN can be used to generate circuits which are equivalent to a given seed circuit.

Using the cloning mechanism we can generate a large number of benchmark circuits from a given seed circuit and effect better and more statistically significant tests than would be possible with just one benchmark. Because GEN can also create a circuit from scratch, this mechanism is not limited to the density range of current benchmark circuits. However, we point out that the validity of circuits diminishes as the number of nodes increases – we will address this further in the companion paper when we discuss methodology and applications.

CIRC and GEN are available under public-domain license via the website <http://www.eecg.toronto.edu/~jayar>.

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