Electromigration Simulation and Design Considerations for Integrated Circuit Power Grids

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Due to continued technology scaling, electromigration has become a serious reliability concern in modern integrated circuits. This is further aggravated by the pervasive use of inaccurate models for electromigration based on traditional empirical black-box models. We will review the modern approach to electromigration verification, with emphasis on recent physical models, then summarize our work on a finite-difference based approach for power grid electromigration checking using these models. The method simulates the electromigration damage across the power grid, much like simulating for voltage or current. The lifetimes found using this physics-based approach are on average about twice, or more, those based on the traditional empirical approaches. Because this approach is computationally efficient, one is able to handle large grids with millions of branches. We then present detailed analysis of the steady state stress and its relation to voltages and currents in the grid, along with a number of design considerations that follow from this analysis.

I. INTRODUCTION

In a metal line carrying significant current density, the free electrons push and move the metal atoms in the direction of the electron wind, i.e., towards the anode end of the line; hence the name *electromigration* (EM) for this effect. The resulting atomic flow increases compressive stress at the anode and tensile stress at the cathode, which creates a stress gradient that presents an opposing driving force that retards EM[1]. If the levels of stress become high enough, a void may be created due to high tensile stress near the cathode, or a hillock (extrusion of metal through cracks in the dielectric) may form due to high compressive stress near the anode, which can either way result in circuit failure. With the confinement of metal lines in today's metal technology, voids are much more likely than hillocks and so one is often more concerned with the buildup of tensile stress. We will follow the common convention that tensile stress is positive and compressive stress is negative. A void is created once the stress exceeds a certain level of stress, called the *critical stress*[2], denoted σ_{crit}, which is an effective parameter that depends on a number of factors.

Integrated circuits (ICs, or simply chips) contain a variety of metal lines (interconnect) which fall under two major categories: 1) signal lines, including lines that provide intra- and inter-cell connectivity, signal buses, control signals, clock signals, etc. and 2) power supply and ground lines, which are supposed to deliver a well-regulated supply voltage to the whole chip. The term *power grid* refers to either the power supply network or the ground network. These networks carry mostly unidirectional currents, in contrast to other lines like signal lines. As a result, signal lines typically have longer EM lifetimes and so the focus of research is often on the power and ground networks[3]. Thus, this work is focused on EM failures in the power grid.

Modern power grids are fabricated using a dual-damascene process, in which the metal line and via are formed simultaneously using Copper (Cu). Since Cu diffuses easily in most dielectric materials, including SiO_2 and carbon-doped silica, thin refractory metal layers consisting of Ta/TaN or their combination are deposited on the sidewalls and bottom of the Cu trenches and vias, as interface barriers or metal liners to prevent Cu diffusion. Then, after CMP (chemical-mechanical polishing), Cu lines are capped with a dielectric diffusion barrier that is usually made of Si_3N_4 or SiC_xNi_y. In the dual-damascene structure, Cu-filled vias are used to connect multiple layers of metallization which, together with the use of sidewall metal liners, helps to minimize the electrical resistance of the Cu interconnects.

The power grid is a large and extensive metal structure, occupying parts of all metal layers. In the bottom two layers, the grid is connected to the underlying transistor circuitry. At the top layer, it connects through the C4 bumps to the external power supply and ground. In any given layer, the grid typically consists of many parallel stripes of metal that connect through vias to the stripes in the layers right above and right below; these stripes are alternating (power, ground) within each layer. As a result of the dual-damascene process, Cu atoms cannot flow from one metal layer to another, but they can easily flow within the same connected section of metal in their own layer. Effectively, Cu atoms are captive within their own metal islands on each level. Due to the structure of parallel stripes, these metal islands are mostly acyclic and so are referred to as *interconnect trees*[4] in the field.

In the remainder of this section, we will review a modern one-dimensional physical model for EM, called Korhonen's model, that is the basis for our work. Section II will summarize our work on an extension of this model for handling full chip power grids. Section III presents a theoretical analysis that captures the mass conservation constraint for whole interconnect trees. This analysis is then applied in Section IV, which develops efficient methods for computing the steady state stress in the full grid, deriving various design insights in the process. Finally, a brief conclusion is given in Section V.

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A. Korhonen's model

Highly accurate analysis of EM in complex on-die metal structures requires the use of expensive and time-consuming three-dimensional (3D) numerical solvers, and so is not suitable for large chip power grids in practice. Fortunately, in the early 90's three separate research groups, under Nix[2], Kirchheim[5,6] and Korhonen[7,8] contributed to the development of a one-dimensional (1D) model that provides a reasonable trade-off between accuracy and complexity, which has come to be called Korhonen's model. The model, which we will review briefly, describes the evolution of the *hydrostatic stress* σ over time. Additional details about the following analysis may be found in[2,5,7,9].

Atomic migration along a metal line that is subjected to electric current is caused mostly by a momentum exchange between the conducting electrons and lattice atoms. Because mass is conserved inside an interconnect tree due to the presence of diffusion barriers and the encapsulation of all lines and vias in modern metal systems, the migration of atoms results in a redistribution of mass along the line. Mass depletion occurs in the cathode region of the line and mass accumulation at the opposite anode region. This would normally result in volumetric deformation of the line, were it not for the rigid confinement by diffusion barriers and the surrounding dielectric, as mentioned earlier. The interaction between the inelastic volumetric strain and the rigid confinement generates an elastic stress that is distributed along the line, with the tensile stress at the cathode and compressive stress at the anode. We can express the relationship between the hydrostatic stress σ and the *volumetric strain* θ as

$$\sigma = -B\theta, \tag{1}$$

where B is the *effective bulk modulus*[10], which depends on the shape of the line cross-section, the elastic constants of the surrounding dielectric and barriers, and the Silicon substrate, as well as on the bulk modulus of the metal.

The mass transport along the line, which leads to stress generation, depends on the applicable diffusion mechanism. For Cu interconnects, the *vacancy mechanism of diffusion* is generally accepted as the underlying mechanism for atomic diffusion: atoms propagate along the line by jumping into vacant nearby lattice sites, i.e. the sites occupied by vacancies. Thus, the mass transport depends on *vacancy concentration*, which is extremely small at typical chip operating temperatures, and can be described by vacancy diffusion. Furthermore, the nonuniform distribution of stress along the line leads to an additional driving force that drives vacancy migration, which is due to the inhomogeneous elastic energy associated with the hydrostatic stress. Assuming that mass transport along the line is determined by grain boundary diffusion alone, with the vacancy diffusion coefficient D_v^{GB}, the effective vacancy transport corresponding to the entire cross-section of the line is characterized by the effective diffusion coefficient $D_v = D_v^{GB}\delta/d$. Here, δ is the grain boundary thickness and d the average grain size. Thus, as a 1D approximation, the total vacancy flux due to electric current and stress is given by

$$J_v = \frac{C_v D_v}{k_b T}\left(q^* \rho j - \Omega \frac{\partial \sigma}{\partial x}\right), \tag{2}$$

where C_v is the vacancy concentration corresponding to the entire cross-section of the line, D_v is the coefficient of vacancy diffusion, ρ is the metal resistivity, j is the current density (whose positive reference direction is in the direction of positive x), Ω is the atomic volume, $q^* = qZ$ is the *effective charge*, where q is the absolute value of the electron charge and Z is the *effective valence*, which is an effective charge value that can justify the measured driving forces exhibited in metal lines[11] under the applied electric field, including the electrostatic and the electron-wind contributions. The spatial and temporal nonuniformity of the vacancy flux is responsible for its divergence, which results in a time-dependent variation in the vacancy concentration along the line. The corresponding *atomic flux*, denoted $J_a = -J_v$, is

$$J_a = \frac{C D_a}{k_b T}\left(\Omega \frac{\partial \sigma}{\partial x} - q^* \rho j\right), \tag{3}$$

where C is the atomic concentration and D_a is the coefficient of atomic diffusion, both corresponding to the entire cross-section of the line. As before, if D_a^{GB} is the grain boundary atomic diffusivity, then $D_a = D_a^{GB}\delta/d$. It should be noted that a similar relationship exists between the local grain boundary concentration of vacancies c_v and their concentration normalized to the entire cross section of the line, $C_v = c_v\delta/d$.

A thermodynamic analysis shows that the vacancy concentration, in the presence of given stress σ, evolves towards the *equilibrium vacancy concentration*

$$c_{Eq} = \Omega^{-1}\exp\left(-\frac{E_v - f\Omega\sigma}{k_b T}\right) = c_{v0}\exp\left(\frac{f\Omega\sigma}{k_b T}\right), \tag{4}$$

where E_v is the energy of vacancy formation and f is the ratio of the lattice volume occupied by a vacancy to that occupied by an atom, while c_{v0} is the equilibrium vacancy concentration in the stress-free state ($\sigma = 0$), i.e., $c_{v0} = \Omega^{-1}\exp(-E_v/k_b T)$. A vacancy is generated in a lattice site when a lattice atom moves to another site that constitutes a lower energy state, which in the case of a confined line can be an interface, a grain boundary, or a dislocation edge. In this process, a vacancy is formed in pair with a displaced atom, which is added, or *plated*, to a free or a confined surface, such as a grain boundary, depending on the line structure. There are associated localized atomic relaxations around the vacancy and the plated atom that induce stress, leading to a new stress state. For a damascene line, the displaced atom is plated on the edge of the interface or at a grain boundary that requires the minimum energy. This yields a total volumetric strain

$$\theta \approx -\left(\frac{\delta}{d}\right)\left[(1-f)\Omega(c_{Eq} - c_{v0}) - \Omega c_{PL}\right], \tag{5}$$

where c_{PL} is the concentration of plated atoms. The ratio δ/d is used to get the volumetric strain of the line unit volume

when excessive or deficient vacancies and plated atoms are generated at the grain boundaries.

Hence, the evolution of the vacancy concentration in a volume dV of the line segment with coordinates x and $x+dx$ with a line thickness h and a width w, subjected to an electric current is described by the following *continuity equation* (mass balance)

$$\frac{\partial c_v(x,t)}{\partial t} = -\nabla J_v + R(x,t), \tag{6}$$

where $R(x,t)$ is the rate of generation/annihilation of vacancy-plated atom pairs. It has been shown[2,7] that there is a simple relation between $R(x,t)$ and the rate of hydrostatic stress evolution, given by

$$R \approx \frac{1}{B\Omega}\left(\frac{d}{\delta}\right)\frac{\partial \sigma}{\partial t}. \tag{7}$$

Assuming that EM-induced transport occurs through the grain boundaries and interfaces, which results in fast evolution of the vacancy concentration towards its equilibrium level, the continuity equation (6) after replacing the vacancy flux with the atomic flux, takes the form

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[\frac{\delta}{d}D_a^{GB}\frac{B\Omega}{k_bT}\left(\frac{\partial \sigma}{\partial x} - \frac{q^*\rho}{\Omega}j\right)\right] = B\Omega\frac{\partial J_a}{\partial x}. \tag{8}$$

where J_a is the atomic flux (3) with $C \approx 1/\Omega$. Then, following Korhonen[7], we denote $D = D_aB\Omega/k_bT$ as the diffusivity, so that equation (8) takes the form of the well-known 1D Korhonen equation describing the stress evolution in a metal line embedded in a rigid confinement under a current density j, as

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x}\left[D\left(\frac{\partial \sigma}{\partial x} - \frac{q^*\rho}{\Omega}j\right)\right]. \tag{9}$$

The most important feature of this result is that the rate of stress evolution is now characterized by the quasi-diffusion coefficient $D = D_aB\Omega/k_bT$, which is much smaller than the vacancy grain boundary diffusivity.

The major achievement of the effort leading to this Nix/Korhonen/Kirchheim model is the explanation of why the stress accumulation happens very slowly in comparison with vacancy diffusion, and why the resulting vacancy concentration at the cathode end of the line is not very large. The explanation is simple: a major portion of the vacancies moving towards the cathode are consumed by annihilation with the plated atoms in order to reach a stress-vacancy concentration equilibrium. Thus, the evolution of the plated atom concentration is a major cause of the stress evolution.

Because the diffusivity D_a depends on the micro-structure of the specific instance of a metal line, it is subject to manufacturing variations and is known to be lognormally distributed[12]. This leads to a statistical distribution for the time-to-failure (TTF). Note that D_a also has a dependence on stress, but this dependence is weak[7] and is often ignored[13–16].

II. EXTENDED KORHONEN'S MODEL (EKM)

In[17–19], we proposed a new EM checking approach for power grids that augments Korhonen's model[7] by enforcing *boundary laws* at all the junctions in order to be able to track material transport and the evolution of stress in interconnect trees. In contrast with previous work, the system equations are generated automatically and for arbitrary tree geometries. The method also extends Korhonen's model to full grid analysis by accounting for redundancy of the grid structure and using a voltage-drop based failure criterion, while being fast enough to be practically useful. We refer to this as the *extended Korhonen's model* (EKM). The system of equations is solved numerically, not analytically, and so essentially performs a simulation of the metal structure to find the stress evolution over time. We use a finite-difference based approach in which metal lines are discretized and the stress found only at the discrete points. In order to account for the redundancy of the many available current paths in modern power grids, we consider a grid to have failed only once a voltage-drop violation has been observed, rather than simply when the first void is formed.

The overall flow of the method is as follows. A random sample of the diffusivities (one per line) is used to create a specific instance of the grid. All trees are solved to track the stress over time, until a void is formed, then we check to see if this leads to a voltage drop violation. If not, and depending on where the void is exactly, the tree with the void may be broken up into multiple disconnected trees, and the solver for the time evolution of all stresses is restarted for all trees. This is repeated until a void is found that causes a voltage drop violation, and this gives the TTF for this instance of the grid. Another random sample is taken, a new fresh (undamaged) instance of the grid is created, and the process is repeated until the average of all the TTFs has converged to the mean time-to-failure (MTF) of the grid.

There are three technical capabilities inherent in this flow: 1) solving for the stress in an interconnect tree by a novel method of *stress simulation*, 2) assessing the impact of a new void on the grid voltage drop using *full grid analysis*, and 3) computing the MTF for the whole grid by an iterative process of *statistical sampling*. Each of these capabilities will be briefly described below, after some preliminaries.

A. Preliminaries

The time evolution of the EM-induced degradation in an interconnect tree consists of two phases, a *void nucleation* phase and a *void growth* phase. For a fresh undamaged tree, the initial time period during which the stress is evolving over time but no voids have yet developed, is called the void nucleation phase. During this phase, with the absence of voids, there is no impact on the line resistance, and so no impact on the tree's electrical behaviour. A void will form (nucleate) somewhere in the tree if/when the stress at that point exceeds the critical stress. When this happens, we have entered the void growth phase. Depending on the physical location where the void has nucleated, a failure may happen immediately upon voiding because it may have the effect of disconnecting a via and creating an open-circuit[20] (such failures are called *early failures*). But this is not always the case. In many cases, the

location of the void will be such that lines continue to conduct current even after a void has formed. As such a void grows in size, the resistance of the line increases and eventually reaches a steady-state value. Then, we are again in a void nucleation phase, on the way to the creation (possibly) of another void somewhere else. In this way, the analysis alternates between nucleation and growth phases, for the whole power grid.

Focus on junctions. It is typical in recent work in this area[7,13,21] to assume that line diffusivities D_a are fixed all along a metal branch. We too will adopt this approach, so that while different lines can have different diffusivities, all points in a given line have the same diffusivity. A consequence of this is that the highest value of the divergence of atomic flux is always at one of the line's endpoints, which we refer to as *junctions*. Thus, in our work, voids nucleate only at junctions. Furthermore, with D_a being independent of x, and j being independent of x anyway inside a uniform line, then (9) takes the simple form of the heat equation,

$$\frac{\partial \sigma}{\partial t} = D\frac{\partial^2 \sigma}{\partial x^2}. \tag{10}$$

Effective current model. Because EM takes a long time to manifest as a circuit failure, the short-term transients that one observes in typical chip operation don't play a significant role in EM failures. Therefore, it is common in the field to adopt an effective current model[22] for EM work. The effective current is a fixed (DC) value that is expected to give the same lifetime as the original current waveform. Thus, in our work we assume that the circuit loading currents are fixed over time, so that the current in every nucleation phase is a constant. The effective currents in the branches in any given nucleation phase can therefore be found by doing a simple DC analysis of the power grid.

Piecewise-constant voltage. We have assumed[23] that the void growth phase is short relative to the nucleation phase, so we ignore its duration and simply keep track of the duration of the nucleation phases. Therefore, both the branch currents and the voltage drops in the grid change abruptly once a void has formed, so that we are dealing with piecewise-constant (PWC) branch resistances, PWC branch currents and PWC voltage drops. With this, the grid electrical behaviour is captured by the system $G(t)v(t) = i_s$, where $G(t)$ is a time-varying conductance matrix whose elements are PWC over time, $v(t)$ is the PWC node voltage drop vector and i_s is the vector of effective DC sources that load the grid.

Reference directions. In our work, every metal line is given a reference direction, which is needed in order to consistently track the signs of all currents and atomic fluxes. Reference directions can be chosen arbitrarily, but remain fixed throughout the analysis. We create them using a simple breadth-first-search[24] graph algorithm. We adopt the convention that, for every branch k, both current density j_k and atomic flux J_{ak} are positive if they flow in the reference direction for that branch, which is also the positive direction for x in the branch. Finally, we assume that an initial stress value is given for every branch, typically based on up-front analysis of the residual thermal stress[25].

B. Capability 1: Stress simulation

Korhonen's model (9) provides a partial differential equation (PDE) that applies to every metal branch (line) of an interconnect tree, at every point x within the line. Our numerical solution approach starts with a discretization of every metal branch in every interconnect tree, using the standard *finite-difference* technique for solving PDEs[26], which effectively turns the collection of PDEs for every interconnect tree into a system of ODEs (ordinary differential equations), which are then solved using well-established numerical methods.

We use the *central difference* approximation[26] to estimate the derivative, based on a spatial discretization of the PDE (10) in every metal line, as

$$\frac{d\sigma}{dt}(x_i,t) \approx \frac{D}{\Delta^2}\left[\sigma(x_{i+1},t) - 2\sigma(x_i,t) + \sigma(x_{i-1},t)\right], \tag{11}$$

where Δ is the distance between consecutive discretized points in the line. This may be viewed as a *line model*, which we then augment with a *junction model*, which is the key part that allows us to "stitch" together all the line models into a full model for the tree, as follows.

A junction enforces an interdependence among the atomic fluxes in the branches connected to it, based on mass-conservation considerations. An exact model for a junction would require a detailed representation of the stress and flux in the spatial volume, in which current and flux are not necessarily uniform across the cross-section as they are in a metal branch. As such, it does not seem to be amenable for use in a 1D model. However, because the volume and mass of a typical junction is very small in comparison with the volume and mass of typical metal lines, we have found that the following approximation works very well. We assume that the mass of a junction is negligibly small, and so effectively assume that the junction is compressed into a single point where the 1D lines are connected to each other. With an infinitesimal *point junction* of this sort, it follows that the sum of all incoming atomic mass through some branches must be equal to the sum of all outgoing mass through the other branches, i.e.,

$$\sum_{k\in\mathscr{B}_{\text{out}}} w_k J_{ak} - \sum_{k\in\mathscr{B}_{\text{in}}} w_k J_{ak} = 0, \tag{12}$$

where w_k is the width of branch k (all branches on the same metal layer have the same height), J_{ak} is the flux in branch k and $\mathscr{B}_{\text{out}}$ (respectively, $\mathscr{B}_{\text{in}}$) is the set of branches connected to the junction whose reference directions are outgoing (respectively, incoming). This equation is a sort of *mass-conservation boundary law* that is enforced for every junction. Using (3), this gives a relationship among the stress derivatives in every connected line, right at the junction. These derivatives are then approximated using a central difference approximation that involves "ghost points," as is often done in finite-difference numerical methods; the details are available in the cited papers.

As a result of the line model and junction model, the overall system of equations for the tree becomes a linear time-invariant (LTI) state-space system,

$$\dot{\sigma}(t) = A\sigma(t) + Bi(t) \tag{13}$$

where σ is a vector of the stresses at all discretized points, including junctions, i is a vector of all the branch currents, while A and B are coefficient matrices that are automatically generated for every tree. This system is solved numerically in order to track the time-evolution of the stress. This is based on well-known methods for solving large systems of ODEs, such as one encounters in circuit simulation[27]. The time derivative is discretized at successive points in time, which converts the differential system into an algebraic one, which turns out to be a linear system in this case; this is then solved at every point in time to get the stress. In our experience[19], this system turns out to be a *stiff system*, just like in circuit simulation. The best numerical formulas for such situations are the backward difference formulas, like the 2nd order Gear (BDF2). We have tested this approach against analytical solutions, and compared to published experimental results, with very good agreement. Furthermore, it has been compared[28] to a detailed 3D solver, again with very good agreement.

C. Capability 2: Full grid analysis

During the void nucleation phase, the stress is compared to a user-specified critical stress threshold σ_{crit}. Once the stress at any point exceeds σ_{crit}, we stop the simulation of all trees and enter the *void growth phase*. An additional *void boundary law* is applied to express the stress derivative at the void surface and a simulation is applied to the relevant tree to determine the final size of that void. The void could end up being an open circuit, if it's right under a via for example, or it can be resistive. The tree is restructured (broken up) accordingly, and the voltage drop in the whole grid is checked, based on solving the full grid DC system $Gv = i_s$ using updated conductance values. If no voltage drop violation is found, all tree currents are updated to reflect the structural change and a new nucleation phase is initiated for all trees. We repeat this process until, after sufficiently many voids have formed, one observes a voltage-drop violation, at which time we declare this to be the TTF, and we terminate the simulation of this specific grid instance.

D. Capability 3: Statistical sampling

Because the TTF is random, one is often interested in its mean (the MTF), which we compute using the Monte Carlo method. We use random sampling (from the distribution of line diffusivities) to generate, within each Monte Carlo iteration, a fresh (undamaged) instance of the same power grid. We then simulate each grid instance until it develops a voltage-drop failure, which gives us its TTF. The iterations continue until the Monte Carlo estimate of the MTF has converged to within a given error tolerance. We improve the computational efficiency by means of a filtering scheme, whereby up-front analysis is done to identify the set of trees that are deemed to have minimal impact on the MTF and these are excluded from the simulation. This has been found to significantly reduce the runtime with very little error[23]. We have also found ways to extrapolate the solution (the curve of stress over time), based on a few early time points, which further speeds up the estimation of the MTF.

III. TREE-WIDE MASS CONSERVATION

It will be useful to express the fact that mass is conserved across the whole interconnect tree, in terms of both stress and flux. The total number of atoms in the interconnect tree is a reasonable proxy for the total mass in the tree. The atomic concentration at any point in the tree can be written in terms of stress, as follows. Combining (1) and (5), and using the fact that $c_{PL} \gg (c_{Eq} - c_{v0})$ under typical conditions, we have

$$\sigma \approx -(\delta/d)B\Omega c_{PL}. \tag{14}$$

However, because $(\delta/d)c_{PL}$ is the EM-induced change in concentration of plated atoms per unit volume of the line, i.e., $(\delta/d)c_{PL} \approx C - C_0$, where $C_0 = 1/\Omega$, we arrive at

$$C \approx C_0 - C_0\sigma/B, \tag{15}$$

so we can write the total number of atoms in a branch of length l, width w and height h, at time t, as

$$\phi(t) = C_0 whl - C_0\frac{wh}{B}\int_0^l \sigma(x,t)dx. \tag{16}$$

We will now see how this can be expressed as a simple linear combination for the whole interconnect tree, first in terms of stress, then in terms of flux.

A. Stress analysis

For an interconnect tree with m branches, and ignoring any additional mass in the junctions' 3D structures, consistent with the EKM framework, the total number of atoms in the tree can be expressed as

$$\Phi(t) = \sum_{k=1}^{m}\phi_k(t) = C_0 h\sum_{k=1}^{m} w_k l_k - \frac{C_0 h}{B}\sum_{k=1}^{m} w_k l_k \overline{\sigma}_k(t), \tag{17}$$

where $\overline{\sigma}_k(t)$ is the *average stress* in branch k at time t, defined as

$$\overline{\sigma}_k(t) = \frac{1}{l_k}\int_0^{l_k}\sigma_k(x,t)dx. \tag{18}$$

Let $a_k = w_k l_k$ be the *layout area* of branch k and $\gamma = \sum_{k=1}^{m} a_k$ be the total layout area of the metal lines in the interconnect tree. We now define what will be a very useful metric for the *average distributed stress across the tree*, as

$$\mathscr{E}_t(\sigma) \triangleq \sum_{k=1}^{m}\frac{a_k}{\gamma}\frac{1}{l_k}\int_0^{l_k}\sigma_k(x,t)dx = \sum_{k=1}^{m}\frac{a_k}{\gamma}\overline{\sigma}_k(t), \tag{19}$$

which is a *weighted average* based on the relative areas of the branches, as fractions of the total area. Note that the 'σ'

when we write $\mathscr{E}_t(\sigma)$ in this equation is merely a symbol that stands for the set of all stress values in the tree, rather than a stress variable for any specific point. With this slight abuse of notation, we can write

$$\Phi(t) = C_0 h\gamma - \frac{C_0 h\gamma}{B}\mathscr{E}_t(\sigma). \tag{20}$$

Due to mass conservation, both $\Phi(t)$ and therefore $\mathscr{E}_t(\sigma)$, are fixed and time-invariant over all time, even as the stresses change over time. So, we will drop the time dependence and simply use $\mathscr{E}(\sigma)$ to represent the time-invariant tree-wide average stress (19), so that

$$\mathscr{E}_t(\sigma) = \mathscr{E}(\sigma) = \text{ constant}, \quad \forall t. \tag{21}$$

The value of $\mathscr{E}(\sigma)$ can be computed from the (assumed given) initial stresses at time zero. As a result of high temperature process steps in manufacturing, it is reasonable to consider that the stress in the final packaged chip is uniform within every interconnect tree, but possibly different in different trees. For a given tree, we will denote this initial uniform stress value as σ_0, assumed given, and we therefore have

$$\mathscr{E}(\sigma) = \sum_{k=1}^{m} \frac{a_k}{\gamma}\overline{\sigma}_k(0) = \sigma_0, \tag{22}$$

so that the mass-conservation requirement for the whole tree, at any time t, can be expressed as

$$\sum_{k=1}^{m} \frac{a_k}{\gamma}\overline{\sigma}_k(t) = \mathscr{E}(\sigma) = \sigma_0, \quad \forall t. \tag{23}$$

B. Flux analysis

Recall from (8) that

$$\frac{\partial \sigma}{\partial t} = B\Omega\frac{\partial J_a}{\partial x}. \tag{24}$$

Let $J_{ak}(x,t)$ be the atomic flux in branch k at any given point $x \in [0, l_k]$, at time t, and take the integral of both sides of (24) over $[0, l_k]$, to get

$$J_{ak}(l_k,t) - J_{ak}(0,t) = \frac{1}{B\Omega}\int_0^{l_k} \frac{\partial}{\partial t}\sigma(x,t)dx. \tag{25}$$

The Leibniz integration rule provides

$$\frac{\partial}{\partial t}\int_0^{l_k} \sigma(x,t)dx = \int_0^{l_k} \frac{\partial}{\partial t}\sigma(x,t)dx, \tag{26}$$

so that

$$J_{ak}(l_k,t) - J_{ak}(0,t) = \frac{l_k}{B\Omega}\frac{\partial}{\partial t}\overline{\sigma}_k(t). \tag{27}$$

Therefore, we can write

$$\sum_{k=1}^{m} w_k\left[J_{ak}(l_k,t) - J_{ak}(0,t)\right] = \frac{1}{B\Omega}\sum_{k=1}^{m} a_k\frac{\partial}{\partial t}\overline{\sigma}_k(t). \tag{28}$$

However, by simply taking the time-derivative of (23) we get $\sum_{k=1}^{m} a_k \partial\overline{\sigma}_k(t)/\partial t = 0, \forall t$, so that

$$\sum_{k=1}^{m} w_k\left[J_{ak}(l_k,t) - J_{ak}(0,t)\right] = 0, \quad \forall t. \tag{29}$$

This constitutes a time-invariant condition for the incident fluxes at the junctions in the whole interconnect tree, which is an expression of the mass conservation requirement in terms of flux. Indeed, this is an extension to the whole tree of the conservation of mass condition at every junction (12).

IV. GRID DESIGN AND THE STEADY STATE STRESS

Assuming the initial stress σ_0 is less than σ_{crit} and that DC currents are applied to the grid, then every junction with rising stress will show a stress evolution that eventually *saturates* at a value that we will call its *steady state stress*, if it doesn't fail before that time. Under these conditions, an interconnect tree would achieve its highest stress levels in the steady state. Thus, a tree is *immortal* if and only if all its steady state stresses are below σ_{crit}. This is a key motivation for studying the steady state stress in the grid. Of course, it would be overly pessimistic to actually require every junction to have a *safe* (i.e., less than σ_{crit}) steady state stress. A design of that sort would be hard to fit within the available chip area and would likely have poor performance anyway. Instead, we are interested in what the steady state stress has to "tell us" about the dynamic stress in the grid and about the tree layout. Trees with the highest steady state stress are probably also going to have the highest stresses under dynamic stress analysis (e.g., with EKM). The distribution of the steady state stress across the tree may indicate certain tree layout optimizations that can be performed in order to avoid obvious hot-spots, before more detailed dynamic analysis is performed. In other words, coarse-grain optimizations may be indicated from analysis of the steady state stress. Indeed, and as we will see in the following, we can glean many design hints and considerations from looking at the steady state stress.

For a straight metal line carrying constant current I, with uniform height h and width w, and assuming a steady state has been reached (i.e., stress is no longer changing over time) with no voids created, this steady state is characterized by a balance between the atomic flux due to the current in the line and the opposing flux due to the stress gradient[1,29], as

$$\Omega\frac{d\sigma}{dx} = \frac{q^*\rho}{wh}I, \tag{30}$$

where we have followed our convention that tensile stress is positive and that x (the distance variable along the line) has the same reference direction as the current I and the flux J_a. Note from (30) that the stress becomes *linear* in x in the steady state due to the constant current, and that we have not made any assumptions about the line terminations at its endpoints, such as whether they are blocking boundaries to atomic flow, or not. Because $dv = -(\rho\, dx/wh)I$, where $v(x)$ is the voltage

at the point x, then $dv/dx = -(\rho/wh)I$ and (30) becomes

$$\frac{d\sigma}{dx} = -\frac{q^*}{\Omega}\frac{dv}{dx} \tag{31}$$

from which, as in[29], $\sigma(x) - \sigma(0) = -(q^*/\Omega)\,(v(x) - v(0))$. In general, in fact, we have $\Delta\sigma = -(q^*/\Omega)\Delta v$ between any two points within the line, including notably between its two endpoints i and j, so that

$$\sigma_i - \sigma_j = -\frac{q^*}{\Omega}\,(v_i - v_j)\,. \tag{32}$$

It's clear from this that the steady state stress at any interior point of a metal line is always less than the stress at its cathode and greater than that at its anode. Thus, in a multi-branch metal structure like an interconnect tree, we mostly care about the steady state stresses at the junctions.

It is also clear from (32) that all stress differences across branches are easily computable once the voltage differences are known. Voltage/current analysis is normally required before any EM risk assessment can be made, so that one can assume that the voltages are already known, and it should be possible to easily compute the steady state stresses. For an interconnect tree with n junctions and m branches, consider a corresponding *graph* with n vertices and m edges, where every edge in the graph is assigned a direction consistent with the reference direction of the corresponding branch. Let M be the $n \times m$ matrix defined as follows. The (i, j)-entry of M is 0 if vertex i is not an endpoint of edge j, and otherwise it is $+1$ (or -1) if the edge direction is from node i (or towards it). As a result, every column k of M contains a single $+1$ and a single -1, corresponding to the two endpoints of edge k, and is otherwise zero. This matrix is called the *incidence matrix* of the graph[30], and we can use it to write the full set of m branch equations based on (32) as the matrix equation

$$M^T\sigma = -\frac{q^*}{\Omega}M^T v, \tag{33}$$

where $\sigma = \{\sigma_i\}_{i=1}^n$ is a vector of all the n junction steady state stresses and $v = \{v_i\}_{i=1}^n$ is a vector of all the n junction voltages. Note that, if the vector σ^* is a solution to (33), then *any* other $\sigma = \sigma^* + \theta\mathbb{1}$ is also a solution, for any scalar $\theta \in \mathbb{R}$, where $\mathbb{1}$ is a vector of all 1s, due to the fact that $M^T\mathbb{1} = 0$. The system of linear equations (33) is not sufficient to find the steady state stresses, because it provides only the stress differences. More information is needed to have a solvable system which, as we will see, can be obtained from the tree-wide mass conservation requirement.

This problem has been considered previously and the deficiency recognized by others, starting with[29] where, in addition to the stress system based on (32), they formulate another system of equations for the atomic *concentrations* C_i at the junctions. In their approach, they identify a linearized relationship between the ratio of the concentrations at the branch endpoints and the voltage difference across the branch. For the same reasons given above, they too need additional information in order to solve the concentration system. For this, they make use of the conservation of mass constraint, applied to the whole interconnect tree. They only point to this as a viable approach but don't actually show the algorithmic details or demonstrate numerical results.

In other work, the authors in[31,32] and more recently[33,34], have introduced additional equations for the *displacements* at the line endpoints. Displacement is a measure of the deformation of the atomic lattice under stress. In their case, the additional information used to augment the stress system based on (32) is a full set of displacement equations, one for every junction, based again on conservation of mass considerations. The work was fully implemented and tested. In the latter work[34], they simplify the system back to the same size as the original stress system, by doing further work on the displacement equations, although it turns out that they assume zero initial stress everywhere.

We will show how the stresses can be found without the intermediate steps of using concentrations or displacements, and we will provide a solution in the general case, without the assumption of zero initial stress. The method will build on, improve and extend the work in[29] and[33,34]. We will employ a mass conservation requirement for the whole tree, and the method will have linear time complexity, so that it is optimal in terms of computational cost.

A. Finding the steady state stress

The mass conservation requirement (23) holds at all times, including of course at steady state. In the steady state, we will drop the time dependence and simply use $\overline{\sigma}_k$ to denote the steady state average stress in branch k, instead of $\overline{\sigma}_k(t)$. Because the steady state stress is linear in x inside every branch, the average stress $\overline{\sigma}_k$ is the simple average of the two stresses at the endpoints of branch k, which we will denote as the two junctions k_0 and k_1, i.e.,

$$\overline{\sigma}_k = \frac{\sigma_{k_0} + \sigma_{k_1}}{2}, \tag{34}$$

so that,

$$\mathscr{E}(\sigma) = \sum_{k=1}^{m}\frac{a_k}{2\gamma}(\sigma_{k_0} + \sigma_{k_1}). \tag{35}$$

We can rearrange this summation to enumerate first by junction instead of by branch, and then by the branches connected to each junction, so that

$$\mathscr{E}(\sigma) = \sum_{j=1}^{n}\sum_{k\in\mathscr{B}_j}\frac{a_k}{2\gamma}\sigma_j, \tag{36}$$

where $\mathscr{B}_j$ is the set of branches connected to junction j. With the value of $\mathscr{E}(\sigma) = \sigma_0$ available, this constitutes what we will call the *steady state mass conservation constraint*, as an additional linear relation among all the junction steady state stresses. To simplify this somewhat, let

$$\alpha_j = \sum_{k\in\mathscr{B}_j}\frac{a_k}{2\gamma} \tag{37}$$

and notice that

$$\sum_{j=1}^{n}\sum_{k\in\mathscr{B}_j} a_k = 2\sum_{k=1}^{m} a_k = 2\gamma, \tag{38}$$

so that $\sum_{j=1}^{n}\alpha_j = 1$ or $\alpha^T\mathbb{1} = 1$, where $\alpha = \{\alpha_j\}_{j=1}^{n}$ is a vector of all the α_j coefficients. With this, we can express the mass conservation constraint as simply

$$\mathscr{E}(\sigma) = \sum_{j=1}^{n}\alpha_j\sigma_j = \sigma_0 \qquad \text{or} \qquad \alpha^T\sigma = \sigma_0. \tag{39}$$

With the values of α_j and $\mathscr{E}(\sigma) = \sigma_0$ in hand, we can now combine (39) with (33) to get a complete system of equations that allows us to find the values of all the steady state stresses, as will be given below. This will provide a simple closed-form expression for the stress at every node, as was found in[33], except that in this case we will allow for non-zero initial stress, and we will not need to use a displacement-based analysis. The result will have a computational cost of $\mathscr{O}(n+m)$.

Note that $\sigma = -(q^*/\Omega)v + \theta\mathbb{1}$ is a general solution to the stress system (33), where $\theta \in \mathbb{R}$ is an arbitrary scalar, as may be seen by pre-multiplying this σ with M^T: $M^T\sigma = -(q^*/\Omega)M^Tv + M^T\mathbb{1}\theta = -(q^*/\Omega)M^Tv$, due to $M^T\mathbb{1} = 0$. We then enforce mass conservation to find a unique value of θ, by plugging $\sigma = -(q^*/\Omega)v + \theta\mathbb{1}$ into (39), to get

$$-\frac{q^*}{\Omega}\alpha^Tv + \theta\alpha^T\mathbb{1} = \mathscr{E}(\sigma) = \sigma_0, \tag{40}$$

so that

$$\theta = \sigma_0 + \frac{q^*}{\Omega}\alpha^Tv = \mathscr{E}(\sigma) + \frac{q^*}{\Omega}\alpha^Tv. \tag{41}$$

With θ in hand, the steady state stress at any junction can be computed using $\sigma = -(q^*/\Omega)v + \theta\mathbb{1}$, i.e.,

$$\sigma_j = -\frac{q^*}{\Omega}v_j + \theta. \tag{42}$$

This result is identical to that in[33,34], except that their θ did not include the σ_0 initial condition.

B. Stress-voltage relationships

The intimate relationship seen above between the steady state stress and voltage merits some more investigation. We start by defining a similar metric for voltage as was defined earlier for stress,

$$\mathscr{E}_t(v) \triangleq \sum_{k=1}^{m}\frac{a_k}{\gamma}\frac{1}{l_k}\int_0^{l_k} v_k(x,t)dx. \tag{43}$$

Because the voltage is linear in x inside every branch, and is invariant over time, the average voltage in any branch k can be written, using the earlier notation of k_0 and k_1 for the terminals of edge k, as

$$\overline{v}_k \triangleq \frac{1}{l_k}\int_0^{l_k} v_k(x)dx = \frac{v_{k_0}+v_{k_1}}{2}, \tag{44}$$

and, dropping the time dependence,

$$\begin{aligned}\mathscr{E}(v) = \sum_{k=1}^{m}\frac{a_k}{\gamma}\overline{v}_k &= \sum_{k=1}^{m}\frac{a_k}{2\gamma}(v_{k_0}+v_{k_1}) \\ &= \sum_{j=1}^{n}\sum_{k\in\mathscr{B}_j}\frac{a_k}{2\gamma}v_j = \sum_{j=1}^{n}\alpha_jv_j = \alpha^Tv.\end{aligned} \tag{45}$$

Therefore, we can write (41) as

$$\theta = \mathscr{E}(\sigma) + \frac{q^*}{\Omega}\mathscr{E}(v) \tag{46}$$

and, from (42),

$$\sigma_j - \mathscr{E}(\sigma) = -\frac{q^*}{\Omega}(v_j - \mathscr{E}(v)), \tag{47}$$

or, in vector form,

$$\sigma - \mathbb{1}\mathscr{E}(\sigma) = -\frac{q^*}{\Omega}(v - \mathbb{1}\mathscr{E}(v)). \tag{48}$$

So, for any junction, the deviation of its steady state stress from the tree-wide average stress $\mathscr{E}(\sigma) = \sigma_0$, is proportional to the deviation of its voltage from the tree-wide average voltage $\mathscr{E}(v)$. As an example, suppose the critical stress is $\sigma_{\text{crit}} = 600$ MPa. The proportionality constant $(-q^*/\Omega)$ is about -48.26 MPa/mV, so it takes a deviation in voltage of about -12.43 mV to produce a deviation of 600 MPa in stress. In other words, if $\sigma_0 = 0$ and all voltage deviations are within 12.43 mV of $\mathscr{E}(v)$, then $\sigma_j < \sigma_{\text{crit}}, \forall j$, and the tree is immortal.

These results can help us better understand the relationship between stress and the *voltage drop* in the grid. For example, we can give an answer to the following question: if one achieves voltage drop safety by design for the power grid, is that enough to ensure safety from EM? For a chip with a 1 V supply, if the voltage drop is guaranteed to never exceed 0.1 V (10% drop), then the junction voltages in any interconnect tree may be spread over a 100 mV band that contains $\mathscr{E}(v)$ and so, using the above results, voltage drop safety is *not* enough to conclude that the grid is immortal.

We will find it useful to introduce the vector $\beta = \{\beta_j\}_{j=1}^{n}$, where $\beta_j = \gamma\alpha_j$ so that

$$\beta_j = \sum_{k\in\mathscr{B}_j}\frac{a_k}{2} \quad \text{and} \quad \beta = \gamma\alpha, \tag{49}$$

from which, due to (39) and (45),

$$\gamma\mathscr{E}(\sigma) = \beta^T\sigma \quad \text{and} \quad \gamma\mathscr{E}(v) = \beta^Tv. \tag{50}$$

We can now rewrite (47) as $(\gamma\sigma_j - \beta^T\sigma) = -(q^*/\Omega)(\gamma v_j - \beta^Tv)$, and in vector form for the whole tree as

$$\left(\gamma I_n - \mathbb{1}\beta^T\right)\sigma = -\frac{q^*}{\Omega}\left(\gamma I_n - \mathbb{1}\beta^T\right)v, \tag{51}$$

where I_n is the $n\times n$ identity matrix. Using the $n\times n$ matrix $\mathscr{Z} = (\gamma I_n - \mathbb{1}\beta^T)$, which depends only on the layout geometry of the tree, this can be written compactly as

$$\mathscr{Z}\sigma = -\frac{q^*}{\Omega}\mathscr{Z}v. \tag{52}$$

Note that this $\mathscr{Z}$ is not the same as M^T of (33); this equation *can* be used to find the stress values.

Finally, let κ be any real number, then because $\alpha^T \mathbb{1} = 1$, we have $\mathscr{E}(v+\kappa\mathbb{1}) = \mathscr{E}(v) + \kappa$, so that

$$(v+\kappa\mathbb{1}) - \mathbb{1}\mathscr{E}(v+\kappa\mathbb{1}) = v - \mathbb{1}\mathscr{E}(v). \tag{53}$$

Therefore, and using (48), we see that $(\sigma - \mathbb{1}\sigma_0)$ is invariant under a *voltage translation* (a shift in voltage that is applied equally to all voltages in the tree). Thus, the junction stresses in the tree remain the same if a uniform shift is applied to all the voltages. This suggests that the reliability of an interconnect tree is not necessarily strongly dependent on the variations in current demand in the underlying transistor circuitry. If these current variations create roughly the same voltage shift across the whole tree (which would probably be more likely for smaller trees), then the steady state EM reliability in the tree would not be affected. The steady state EM reliability depends primarily on the spread (in values) of the junction voltages in the tree, not on their actual values. The stresses in the tree are more intimately related to its terminal currents and branch currents than to its nodal voltages. We will explore this further in the next section.

C. Stress-current relationships

With the tree stress distribution being invariant under a translation of its voltages, as we saw above, it would be interesting to express the tree stresses in terms of its currents only, i.e., with no regard to its voltages. This will open up various interesting questions that can be explored for a tree in isolation, without reference to the whole power grid.

To start, we will consider the fairly straightforward relationship between the node stresses and the branch currents, which we represent with the vector $i = \{i_k\}_{k=1}^m$. Consider the diagonal $m \times m$ matrix R whose every (kth) diagonal entry is equal to the resistance of the corresponding edge k, to be denoted r_k. With this, we simply have $M^T v = Ri$, so that (33) leads to

$$M^T \sigma = -\frac{q^*}{\Omega} Ri. \tag{54}$$

If the branch currents are known, this can be easily solved for the stresses, after enforcing mass conservation.

Next, we will focus on the tree's external loading currents. Consider a tree driven by (only) *ideal current sources*, carrying what we will refer to as its *terminal currents*. Because the current sources are ideal, the tree voltages are therefore *indeterminate*, in the sense that if vector v^* is a valid assignment of node voltages in the tree, then $v^* + \kappa\mathbb{1}$ is also a valid assignment, for any $\kappa \in \mathbb{R}$. We will refer to such a tree as a *floating tree*. Let $c = \{c_i\}_{i=1}^n$ be the vector of all the terminal currents, where c_i is set to 0 if no current source is actually connected to node i. We set the positive reference direction for every current source c_i to be away from the node i. If node i is connected by a resistive branch to node j, then we say that these two nodes are *neighbours*, and we denote by r_{ij} the resistance of that branch. If that branch has been designated as edge k, then r_{ij} is just another symbol for the r_k introduced earlier. Let $\mathscr{N}_i$ be the set of all the neighbours of node i, then applying Kirchhoff's Current Law (KCL) at node i, gives

$$\sum_{j \in \mathscr{N}_i} \frac{1}{r_{ij}}(v_i - v_j) + c_i = 0. \tag{55}$$

Notice that there are no resistors to ground in this equation, as a result of the assumption of ideal current sources and the floating tree. The set of all such equations for the nodes in the tree can be assembled as the matrix equation $Gv = -c$, where G is the familiar *conductance matrix*[27] of the tree. If one starts with a blank (all zeros) matrix G, then considers all the edges in sequence to see how each of them contributes to G according to (55), one finds that every edge k adds $\pm 1/r_k$ to only four locations in the matrix, as

$$g_{k_0k_0} := g_{k_0k_0} + \frac{1}{r_k} \qquad g_{k_0k_1} := g_{k_0k_1} - \frac{1}{r_k} \tag{56}$$

$$g_{k_1k_0} := g_{k_1k_0} - \frac{1}{r_k} \qquad g_{k_1k_1} := g_{k_1k_1} + \frac{1}{r_k}, \tag{57}$$

where k_0 and k_1 are the two endpoints (nodes) of edge k. As a result of the contributions of all the tree edges, the matrix G is completely defined and has a known structure. We can express this compactly as

$$g_{ij} = \begin{cases} \sum_{k \in \mathscr{N}_i} (1/r_{ik}), & \text{for } i = j; \\ -1/r_{ij}, & \text{for } i \neq j \text{ and } j \in \mathscr{N}_i; \\ 0, & \text{otherwise;} \end{cases} \tag{58}$$

so that G is a symmetric $n \times n$ diagonally-dominant matrix with positive diagonal entries and non-positive off-diagonal entries. In addition, the entries in every column add up to zero, and the entries in every row add up to zero, so that in particular $G\mathbb{1} = 0$ and the matrix is singular. Using (32), we can rewrite (55) at every node i in terms of stress, as

$$-\frac{\Omega}{q^*} \sum_{j \in \mathscr{N}_i} \frac{1}{r_{ij}}(\sigma_i - \sigma_j) + c_i = 0, \tag{59}$$

which leads to the matrix equation

$$G\sigma = \frac{q^*}{\Omega} c. \tag{60}$$

This is a very interesting relationship between stress and the terminal currents involving a matrix with well-known and useful structure, which can be very useful for considering various optimizations of the tree for steady state EM reliability.

D. Supply and ground network distinctions

As mentioned earlier, the term *power grid* refers to both the on-chip *power supply distribution network*, which is tied at one or more points to a single external voltage source V_{dd}, and the on-chip *ground distribution network*, tied at one or

more points to the external ground. For the supply network, we're interested in the *voltage drop* below V_{dd} at every node; if $v_j \le V_{dd}$ is the actual voltage at node j, then $u_j = V_{dd} - v_j \ge 0$ is the voltage drop at that node. Proper design requires all voltage drops to remain below some $V_{th} > 0$. For the ground network, we're interested in the *voltage rise* above ground at every node, i.e., the actual voltages $v_j \ge 0$ at the nodes, which must also be maintained below some $V_{th} > 0$. Typical values of V_{th} may be around 10% of V_{dd}. At the bottom metal layers, many grid nodes are attached to circuit blocks that draw supply current from the supply network and channel these currents into the ground network. We will represent these circuit blocks as ideal current sources, i.e., with no shunt resistors, but we allow the external V_{dd} voltage source to have non-zero series source resistance.

Considering first the *supply network*, we assume it is loaded by a vector of constant circuit currents $c = \{c_i\}$ that represents all the underlying circuit block currents (with $c_i = 0$ if there is no circuit block attached to node i), with a positive reference direction *away from* the network and into the circuit blocks. By superposition, the voltage vector v may be found in three steps: 1) open-circuit all the current sources and find the voltage response, which would obviously be $v^{(1)} = V_{dd}\mathbb{1}$ in this case, 2) short-circuit the voltage source and find the voltage response, in this case some $v^{(2)} \le 0$, and 3) compute $v = v^{(1)} + v^{(2)}$. To find $v^{(2)}$, KCL at every node provides, as in nodal analysis[27],

$$Gv^{(2)}(t) = -c. \tag{61}$$

This G matrix has almost exactly the same structure as for the case of a floating tree (58); the only difference is due to the voltage source resistance connected to ground. The effect of that is to increase some of the diagonal values of G, relative to the structure given for the floating tree, so that this G is known to be non-singular. We are interested in the voltage drop vector $u = V_{dd}\mathbb{1} - v = V_{dd}\mathbb{1} - v^{(1)} - v^{(2)} = -v^{(2)}$, so that $v^{(2)} = -u$ and (61) becomes

$$Gu = c. \tag{62}$$

As for the *ground network*, if it is loaded by a vector of constant circuit currents c, whose positive reference direction is *into* the ground network, then KCL provides, using a different G matrix that's specific for the network under consideration,

$$Gv = c. \tag{63}$$

Thus, the governing equations for voltage drop analysis in the supply network have exactly the same form as for the voltage rise in the ground network. Turning next to the stress equations, as we saw earlier, (47) applies to every node j in an interconnect tree in either the *ground or supply network*. For an interconnect tree in the *supply network* specifically, where we're interested in the voltage drop u_j, we have from (45)

$$\mathscr{E}(v) = \sum_{j=1}^{n} \alpha_j (V_{dd} - u_j) = V_{dd} - \mathscr{E}(u), \tag{64}$$

so that, for every node j, we have $v_j - \mathscr{E}(v) = (V_{dd} - u_j) - (V_{dd} - \mathscr{E}(u)) = -(u_j - \mathscr{E}(u))$, so that we can write (47) as

$$\sigma_j - \mathscr{E}(\sigma) = \frac{q^*}{\Omega}(u_j - \mathscr{E}(u)). \tag{65}$$

The above relations will be used below to study the immortality of nodes and trees in the power and ground networks.

1. Immortal grid

For an immortal grid, we need to ensure that $\sigma_j \le \sigma_{\text{crit}}$ for every grid node j. Note that at time zero, before the chip has been deployed, we would fully expect that all the initial stresses in the grid are less than σ_{crit}, otherwise the grid would be "dead on arrival." Therefore, we assume that the time-invariant $\mathscr{E}(\sigma) = \sigma_0$ is always less than σ_{crit}. We now define a new voltage constant for each tree,

$$\delta_0 = \frac{\Omega}{q^*}(\sigma_{\text{crit}} - \sigma_0) > 0. \tag{66}$$

For example, if $\sigma_0 = 0$ and $\sigma_{\text{crit}} = 600\,\text{MPa}$, then δ_0 is about $12.4\,\text{mV}$. Considering the actual voltages v_j in the grid, with $\sigma_j = \mathscr{E}(\sigma) - (q^*/\Omega)(v_j - \mathscr{E}(v))$, the requirement $\sigma_j \le \sigma_{\text{crit}}$ translates to

$$\mathscr{E}(v) - v_j \le \delta_0 \quad \text{or} \quad v_j \ge \mathscr{E}(v) - \delta_0. \tag{67}$$

So, all nodes with $v_j \ge \mathscr{E}(v)$ are automatically immortal, while for those nodes with v_j below $\mathscr{E}(v)$, they must not be too far below it – they must be within δ_0 below it. It's the nodes with very low voltages (well below $\mathscr{E}(v)$) that are the most vulnerable. In order for the whole tree to be immortal, the whole distribution of v_j in the tree must be narrow enough so that $\mathscr{E}(v) - v_j \le \delta_0$ for all its nodes. Fig. 1 shows an illustration of the ranges of voltages that correspond to mortal nodes and immortal nodes, in both the ground and supply networks. It is notable that the most vulnerable nodes are those with voltages close to zero in the ground network (so in the top metal layers) and those with voltage drops close to V_{th} in the supply network (so in the bottom metal layers). This suggests that *it may be easier to secure the ground network against EM than to secure the supply network*, because it may be easier to widen metal lines in the top metal layers, compared to those in the bottom layers.

If a tree has a minimum node voltage of $v_{\text{min}} \ge 0$, then the immortality requirement for that tree becomes

$$\mathscr{E}(v) \le v_{\text{min}} + \delta_0. \tag{68}$$

For some trees in the *ground network*, v_{min} may well be very close to zero, so that $\mathscr{E}(v) \le \delta_0$ is the requirement for these trees. For trees in the *supply network*, v_{min} may well be very close to V_{th}, so that their requirement is $\mathscr{E}(v) \le V_{th} + \delta_0$. We can also express these supply network results in terms of the voltage drop u. As a replacement for (67), starting with $\sigma_j = \mathscr{E}(\sigma) + (q^*/\Omega)(u_j - \mathscr{E}(u))$, the relation $\sigma_j \le \sigma_{\text{crit}}$ becomes

$$u_j - \mathscr{E}(u) \le \delta_0 \quad \text{or} \quad u_j \le \mathscr{E}(u) + \delta_0. \tag{69}$$

Then, let $u_{\max} = (V_{dd} - v_{\min})$ be the highest voltage drop in the tree, so that (68) becomes

$$u_{\max} \leq \mathscr{E}(u) + \delta_0. \tag{70}$$

V. CONCLUSION

We have reviewed our recent work on a simulation approach to EM checking, based on a 1D physical model. We also gave detailed analysis of the steady state stress, including ways of computing it and design insights and considerations that result from this analysis, for both the power and ground on-die networks.

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FIGURE CAPTIONS

Figure 1. Ranges of node voltages for mortal and immortal nodes.

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