Oversampling Converters

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Motivation

• Popular approach for medium-to-low speed A/D and D/A applications requiring high resolution

Easier Analog

• reduced matching tolerances
• relaxed anti-aliasing specs
• relaxed smoothing filters

More Digital Signal Processing

• Needs to perform strict anti-aliasing or smoothing filtering
• Also removes shaped quantization noise and decimation (or interpolation)
Quantization Noise

- Above model is exact
  - approx made when assumptions made about $e(n)$
- Often assume $e(n)$ is white, uniformly distributed number between $\pm \Delta/2$
- $\Delta$ is difference between two quantization levels

Quantization Noise

- White noise assumption reasonable when:
  - fine quantization levels
  - signal crosses through many levels between samples
  - sampling rate not synchronized to signal frequency
- Sample lands somewhere in quantization interval leading to random error of $\pm \Delta/2$
**Quantization Noise**

- Quantization noise power shown to be $\Delta^2/12$ and is *independent of sampling frequency*
- If white, then spectral density of noise, $S_e(f)$, is constant.

\[
S_e(f) = \frac{\Delta^2}{12} \left( \frac{1}{f_s} \right)
\]

\[
\text{Height } k_x = \left( \frac{\Delta}{\sqrt{12}} \right) \frac{1}{f_s}
\]

**Oversampling Advantage**

- Oversampling occurs when signal of interest is bandlimited to $f_0$ but we sample higher than $2f_0$
- Define oversampling-rate
  \[
  \text{OSR} = \frac{f_s}{(2f_0)}
  \]
- After quantizing input signal, pass it through a brickwall digital filter with passband up to $f_0$
Oversampling Advantage

• Output quantization noise after filtering is:

\[ P_e = \int_{-f_s/2}^{f_s/2} S_e^2(f)|H(f)|^2 df = \int_{-f_0}^{f_0} k_x^2 df = \frac{\Delta^2}{12} \left( \frac{1}{OSR} \right) \]  

(2)

• Doubling OSR reduces quantization noise power by 3dB (i.e. 0.5 bits/octave)

• Assuming peak input is a sinusoidal wave with a peak value of \( 2^N (\Delta/2) \) leading to

\[ P_s = \left( \left( \frac{2^N}{\sqrt{2}} \right)^2 \right)^2 \]

• Can also find peak SNR as:

\[ SNR_{max} = 10 \log \left( \frac{P_s}{P_e} \right) = 10 \log \left( \frac{3}{2} 2^{2N} \right) + 10 \log(OSR) \]  

(3)

Example

• A dc signal with 1V is combined with a noise signal uniformly distributed between \( \pm \sqrt{3} \) giving 0 dB SNR. — \{0.94, –0.52, –0.73, 2.15, 1.91, 1.33, –0.31, 2.33\}.

• Average of 8 samples results in 0.8875

• Signal adds linearly while noise values add in a square-root fashion — noise filtered out.

Example

• 1-bit A/D gives 6dB SNR.

• To obtain 96dB SNR requires 30 octaves of oversampling ( (96-6)/3 dB/octave )

• If \( f_0 = 25 \text{ kHz} \), \( f_s = 2^{30} \times f_0 = 54,000 \text{ GHz} \)! 


**Advantage of 1-bit D/A Converters**

- Oversampling improves SNR but not linearity
- To achieve 16-bit linear converter using a 12-bit converter, 12-bit converter must be linear to 16 bits
  — i.e. integral nonlinearity better than $1/2^4$ LSB
- A 1-bit D/A is *inherently linear*
  — 1-bit D/A has only 2 output points
  — 2 points always lie on a straight line
- Can achieve better than 20 bits linearity without trimming (will likely have gain and offset error)
- Second-order effects (such as D/A memory or signal-dependent reference voltages) will limit linearity.

**Oversampling with Noise Shaping**

- Place the quantizer in a feedback loop

![Diagram](attachment:image.png)

- Linear model
Oversampling with Noise Shaping

- Shapes quantization noise away from signal band of interest

**Signal and Noise Transfer-Functions**

\[ S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)} \quad (4) \]

\[ N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)} \quad (5) \]

\[ Y(z) = S_{TF}(z)U(z) + N_{TF}(z)E(z) \quad (6) \]

- Choose \( H(z) \) to be large over 0 to \( f_0 \)
- Resulting quantization noise near 0 where \( H(z) \) large
- Signal transfer-function near 1 where \( H(z) \) large

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Oversampling with Noise Shaping

- Input signal is limited to range of quantizer output when \( H(z) \) large
- For 1-bit quantizers, input often limited to 1/4 quantizer outputs
- Out-of-band signals can be larger when \( H(z) \) small
- Stability of modulator can be an issue (particularly for higher-orders of \( H(z) \))
- Stability defined as when input to quantizer becomes so large that quantization error greater than \( \pm \Delta/2 \) — said to “overload the quantizer”
**First-Order Noise Shaping**

- Choose $H(z)$ to be a discrete-time integrator
  \[
  H(z) = \frac{1}{z - 1} \quad (7)
  \]

- If stable, average input of integrator must be zero
- Average value of $u(n)$ must equal average of $y(n)$

**Example**

- The output sequence and state values when a dc input, $u(n)$, of $1/3$ is applied to a 1st order modulator with a two-level quantizer of $\pm 1.0$. Initial state for $x(n)$ is 0.1.

<table>
<thead>
<tr>
<th>n</th>
<th>$x(n)$</th>
<th>$x(n + 1)$</th>
<th>$y(n)$</th>
<th>$e(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.5667</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>1</td>
<td>-0.5667</td>
<td>0.7667</td>
<td>-1.0</td>
<td>-0.4333</td>
</tr>
<tr>
<td>2</td>
<td>0.7667</td>
<td>0.1</td>
<td>1.0</td>
<td>0.2333</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>-0.5667</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>-0.5667</td>
<td>0.7667</td>
<td>-1.0</td>
<td>-0.4333</td>
</tr>
<tr>
<td>5</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

- Average of $y(n)$ is $1/3$ as expected
- Periodic quantization noise in this case
Transfer-Functions

Signal and Noise Transfer-Functions

\[ S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1 + 1/(z-1)} = z^{-1} \quad (8) \]

\[ N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + 1/(z-1)} = (1 - z^{-1}) \quad (9) \]

- Noise transfer-function is a discrete-time differentiator (i.e. a highpass filter)

\[ N_{TF}(f) = 1 - e^{-j2\pi f/f_s} = \frac{e^{j\pi f/f_s} - e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s} \]

\[ = \sin\left(\frac{\pi f}{f_s}\right) \times 2j \times e^{-j\pi f/f_s} \quad (10) \]

Signal to Noise Ratio

Magnitude of noise transfer-function

\[ |N_{TF}(f)| = 2 \sin\left(\frac{\pi f}{f_s}\right) \quad (11) \]

Quantization noise power

\[ P_e = \int_{-f_0}^{f_0} S_e^2(f)|N_{TF}(f)|^2 \, df = \int_{-f_0}^{f_0} \left(\frac{\Delta^2}{12}\right) \frac{1}{f_s} \left[2 \sin\left(\frac{\pi f}{f_s}\right)\right]^2 \, df \quad (12) \]

- Assuming \( f_0 \ll f_s \) (i.e., \( OSR >> 1 \))

\[ P_e \approx \left(\frac{\Delta^2}{12}\right) \left(\frac{\pi^2}{3}\right) \left(\frac{2f_0}{f_s}\right)^3 = \frac{\Delta^2 \pi^2}{36} \left(\frac{1}{OSR}\right)^3 \quad (13) \]
Max SNR

- Assuming peak input is a sinusoidal wave with a peak value of $2^N(\Delta/2)$ leading to $P_s = ((\Delta 2^N)/(2\sqrt{2}))^2$

- Can find peak SNR as:

$$\text{SNR}_{\text{max}} = 10 \log \left( \frac{P_s}{P_e} \right)$$

$$= 10 \log \left( \frac{3}{2} 2^N \right) + 10 \log \left[ \frac{3}{\pi^2} (\text{OSR})^3 \right]$$  \hspace{1cm} (14)

or, equivalently,

$$\text{SNR}_{\text{max}} = 6.02N + 1.76 - 5.17 - 30 \log (\text{OSR})$$  \hspace{1cm} (15)

- Doubling OSR gives an SNR improvement 9 dB or,

equivalently, a benefit of 1.5 bits/octave
**SC Implementation**

![Diagram of SC Implementation](image)

- **1-bit D/A**
- **Quantizer**
- **H(z)**
- **1-bit D/A**
- **Analog**
- **Digital**

**Second-Order Noise Shaping**

![Diagram of Second-Order Noise Shaping](image)

- **Equation 16:**
  \[ S_{TF}(f) = z^{-1} \]

- **Equation 17:**
  \[ N_{TF}(f) = (1 - z^{-1})^2 \]

- **Equation 18:**
  \[ SNR_{max} = 6.02N + 1.76 - 12.9 + 50 \log(OSR) \]

- **Doubling OSR improves SNR by 15 dB**
  (i.e., a benefit of 2.5 bits/octave)
Noise Transfer-Function Curves

- Out-of-band noise increases for high-order modulators
- Out-of-band noise peak controlled by poles of noise transfer-function
- Can also spread zeros over band-of-interest

Example

- 90 dB SNR improvement from A/D with $f_0 = 25$ kHz

Oversampling with no noise shaping
- From before, straight oversampling requires a sampling rate of 54,000 GHz.

First-Order Noise Shaping
- Lose 5 dB (see (15)), require 95 dB divided by 9 dB/octave, or 10.56 octaves — $f_s = 2^{10.56} \times 2f_0 \approx 75$ MHz

Second-Order Noise Shaping
- Lose 13 dB, required 103 dB divided by 15 dB/octave, $f_s = 5.8$ MHz (does not account for reduced input range needed for stability).
Quantization Noise Power of 1-bit Modulators

- If output of 1-bit mod is \( \pm 1 \), total power of output signal, \( y(n) \), is normalized power of 1 watt.
- Signal level often limited to well below \( \pm 1 \) level in higher-order modulators to maintain stability.
- For example, if maximum peak level is \( \pm 0.25 \), max signal power is 62.5 mW.
- Max signal is approx 12 dB below quantization noise (but most noise in different frequency region).
- Quantization filter must have dynamic range capable of handling full power of \( y(n) \) at input.
- Easy for A/D — digital filter
- More difficult for D/A — analog filter

Zeros of NTF are poles of \( H(z) \)

- Write \( H(z) \) as

\[
H(z) = \frac{N(z)}{D(z)}
\]  
(19)

- NTF is given by:

\[
\text{NTF}(z) = \frac{1}{1 + H(z)} = \frac{D(z)}{D(z) + N(z)}
\]
(20)

- If poles of \( H(z) \) are well-defined then so are zeros of NTF.
Error-Feedback Structure

- Alternate structure to interpolative

\[ u(n) \rightarrow \sum x(n) \rightarrow y(n) \]

- Signal transfer-function equals unity while noise transfer-function equals \( G(z) \)
- First element of \( G(z) \) equals 1 for no delay free loops
- First-order system — \( G(z) - 1 = -z^{-1} \)
- More sensitive to coefficient mismatches

Architecture of Delta-Sigma A/D Converters

- Anti-aliasing filter
- Sample-and-hold
- \( \Delta \Sigma \) Modulator
- Digital low-pass filter
- Decimation filter
- OSR

Time

Frequency
Architecture of Delta-Sigma A/D Converters

- Relaxes analog anti-aliasing filter
- Strict anti-aliasing done in digital domain
- Must also remove quantization noise before downsampling (or aliasing occurs)
- Commonly done with a multi-stage system
- Linearity of D/A in modulator important — results in overall nonlinearity
- Linearity of A/D in modulator unimportant (effects reduced by high gain in feedback of modulator)
**Architecture of Delta-Sigma D/A Converters**

- Relaxes analog smoothing filter (many multibit D/A converters are oversampled without noise shaping)
- Smoothing filter of first few images done in digital (then often below quantization noise)
- Order of lowpass filter should be at least one order higher than that of modulator
- Results in noise dropping off (rather than flat)
- Analog filter must attenuate quantization noise and should not modulate noise back to low freq — strong motivation to use multibit quantizers

**Multi-Stage Digital Decimation**

- Sinc filter removes much of quantization noise
- Following filter(s) — anti-aliasing filter and noise
Sinc Filter

- \( \text{sinc}^{L+1} \) is a cascade of \( L+1 \) averaging filters

**Averaging filter**

\[
T_{\text{avg}}(z) = \frac{Y(z)}{U(z)} = \frac{1}{M} \sum_{i=0}^{M-1} z^{-i}
\]  \( (21) \)

- \( M \) is integer ratio of \( f_s/(8f_0) \)
- It is a linear-phase filter (symmetric coefficients)
- If \( M \) is power of 2, easy division (shift left)
- Can not do all decimation filtering here since not sharp enough cutoff

**Sinc Filter**

- Consider \( x_{\text{in}}(n) = \{ 1, 1, -1, 1, 1, -1, \ldots \} \) applied to \( M = 4 \) averaging filters in cascade

\[
T_{\text{avg}}(z) \xrightarrow{x_{\text{in}}(n)} x_1(n) \xrightarrow{T_{\text{avg}}(z)} x_2(n) \xrightarrow{T_{\text{avg}}(z)} x_3(n)
\]

- \( x_1(n) = \{ 0.5, 0.5, 0.0, 0.5, 0.5, 0.0, \ldots \} \)
- \( x_2(n) = \{ 0.38, 0.38, 0.25, 0.38, 0.38, 0.25, \ldots \} \)
- \( x_3(n) = \{ 0.34, 0.34, 0.31, 0.34, 0.34, 0.31, \ldots \} \)
- Converging to sequence of all 1/3 as expected
Sinc Filter Response

- Can rewrite averaging filter in recursive form as

\[ T_{avg}(z) = \frac{Y(z)}{U(z)} = \frac{1}{M} \left( 1 - \frac{z^{-M}}{1 - z^{-1}} \right) \]  

and a cascade of \( L + 1 \) averaging filters results in

\[ T_{sinc}(z) = \frac{1}{M^{L+1}} \left( 1 - \frac{z^{-M}}{1 - z^{-1}} \right)^{L+1} \]  

- Use \( L + 1 \) cascade to roll off quantization noise faster than it rises in \( L \)’th order modulator

Sinc Filter Frequency Response

- Let \( z = e^{j\omega} \)

\[ T_{avg}(e^{j\omega}) = \frac{sinc\left( \frac{\omega M}{2} \right)}{sinc\left( \frac{\omega}{2} \right)} \]  

where \( sinc(x) \equiv \frac{\sin(x)}{x} \)

\[ \left| T_{avg}(e^{j\omega}) \right| \]

\[ \begin{array}{c|c|c}
0 & \pi & 2\pi \\
\hline
1 & & \\
\end{array} \]
**Sinc Implementation**

\[ T_{sinc}(z) = \left( \frac{1}{1-z^{-1}} \right)^{L+1} \left( 1-z^{-M} \right)^{L+1} \frac{1}{M^{L+1}} \]  

(25)

- If 2's complement arithmetic used, wrap-around okay since followed by differentiators

\[
\begin{align*}
T_{sinc}(z) &= \left( \frac{1}{1-z^{-1}} \right)^{L+1} \left( 1-z^{-M} \right)^{L+1} \frac{1}{M^{L+1}} \\
&= \left( 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \cdots + \frac{z^{-M}}{M!} \right)^{L+1} \left( 1 + Mz^{-1} + \frac{(Mz)^{-2}}{2!} + \frac{(Mz)^{-3}}{3!} + \cdots \right)^{L+1} \frac{1}{M^{L+1}} \\
&= \left( 1 + \frac{z^{-1}}{L+1} + \frac{z^{-2}}{(L+1)(L+2)} + \cdots + \frac{z^{-M}}{(L+1)(L+2)\cdots(L+M)} \right)^{L+1} \left( 1 + \frac{Mz^{-1}}{L+1} + \cdots \right)^{L+1} \frac{1}{M^{L+1}} \\
&= \left( 1 + \frac{z^{-1}}{L+1} + \cdots + \frac{z^{-M}}{(L+1)(L+2)\cdots(L+M)} \right)^{L+1} \left( 1 + \frac{Mz^{-1}}{L+1} + \cdots \right)^{L+1} \frac{1}{M^{L+1}} \\
&= \left( 1 + \frac{z^{-1}}{L+1} + \cdots + \frac{z^{-M}}{(L+1)(L+2)\cdots(L+M)} \right)^{L+1} \left( 1 + \frac{Mz^{-1}}{L+1} + \cdots \right)^{L+1} \frac{1}{M^{L+1}} \\
&= \left( 1 + \frac{z^{-1}}{L+1} + \cdots + \frac{z^{-M}}{(L+1)(L+2)\cdots(L+M)} \right)^{L+1} \left( 1 + \frac{Mz^{-1}}{L+1} + \cdots \right)^{L+1} \frac{1}{M^{L+1}}
\end{align*}
\]

- If 2’s complement arithmetic used, wrap-around okay since followed by differentiators

**Higher-Order Modulators**

- An L’th order modulator improves SNR by 6L+3 dB/octave

**Interpolative Architecture**

- Can spread zeros over freq of interest using resonators with \( f_1 \) and \( f_2 \)
- Need to worry about stability (more later)
MASH Architecture

• Multi-stAge noise SHaping - MASH
• Use multiple lower order modulators and combine outputs to cancel noise of first stages

\[
u(n) + e_1(n) + e_2(n) + z^{-1}u(n) + (1 - z^{-1})e_1(n) + z^{-1}e_1(n) + (1 - z^{-1})e_2(n)
\]

Output found to be:

\[Y(z) = z^{-2}U(z) - (1 - z^{-1})^2 E_2(z)\]  \hspace{1cm} (26)

Multibit Output

• Output is a 4-level signal though only single-bit D/A’s
  — if D/A application, then linear 4-level D/A needed
  — if A/D, slightly more complex decimation

A/D Application

• Mismatch between analog and digital can cause first-order noise, \(e_1\), to leak through to output
• Choose first stage as higher-order (say 2’nd order)
Bandpass Oversampling Converters

- Choose $H(z)$ to have high gain near freq $f_c$
- NTF shapes quantization noise to be small near $f_c$
- OSR is ratio of sampling-rate to twice bandwidth — not related to center frequency

$\frac{f_s}{4} = 1 \text{ MHz}$
$\frac{f_s}{2} = 4 \text{ MHz}$
$f_s = 4 \text{ MHz}$
$f_s/2 = 2f_0 = 200$
$f_s/4 = 1 \text{ MHz}$
$f_s = 4 \text{ MHz}$
$f_s/2 = 2f_\Delta = 200$

Lowpass OSR = $\frac{f_s}{2f_0}$ = 200

Bandpass OSR = $\frac{f_s}{2f_\Delta}$ = 200

- Above $H(z)$ has poles at $\pm j$ (which are zeros of NTF)
  — $H(z)$ is a resonator with infinite gain at $f_s/4$
  — $H(z) = \frac{z}{(z^2 + 1)}$
- Note one zero at $+j$ and one zero at $-j$
  — similar to lowpass first-order modulator
  — only 9 dB/octave
- For 15 dB/octave, need 4'th order BP modulator
Modulator Stability

• Since feedback involved, stability is an issue
• Considered stable if quantizer input does not overload quantizer
• Non-trivial to analyze due to quantizer
• There are rigorous tests to guarantee stability but they are too conservative
• For a 1-bit quantizer, heuristic test is:
  \[ |N_{TF}(e^{j\omega})| \leq 1.5 \quad \text{for } 0 \leq \omega \leq \pi \]  
  (27)
• Peak of NTF should be less than 1.5
• Can be made more stable by placing poles of NTF closer to its zeros
• Dynamic range suffers since less noise power pushed out-of-band

Stability Detection

• Might look at input to quantizer
• Might look for long strings of 1s or 0s at comp output

When instability detected ...

• reset integrators
• Damp some integrators to force more stable
Linearity of Two-Level Converters

- For high-linearity, levels should NOT be a function of input signal
  — power supply variation might cause symptom
- Also need to be memoryless
  — switched-capacitor circuits are inherently memoryless if enough settling-time allowed
- Above linearity issues also applicable to multi-level
- A nonreturn-to-zero is NOT memoryless
- Return-to-zero is memoryless if enough settling time
- Important for continuous-time D/A
Idle Tones

- 1/3 into 1’st order modulator results in output
  \[ y(n) = \{1, 1, -1, 1, 1, -1, 1, 1, \ldots \} \]  
  (28)
- Fortunately, tone is out-of-band at \( f_s/3 \)
- \( (1/3 + 1/24) = 3/8 \) into modulator has tone at \( f_s/16 \)
- Similar examples can cause tones in band-of-interest and are not filtered out — say \( f_s/256 \)
- Also true for higher-order modulators
- Human hearing can detect tones below noise floor
- Tones might not lie at single frequency but be short term periodic patterns.
  — could be a tone varying between 900 and 1100 Hz varying in a random-like pattern

Dithering

Dither signal

- Add pseudo-random signal into modulator to break up idle tones (not just mask them)
- If added before quantizer, it is noise shaped and large dither can be added.
  — A/D: few bit D/A converter needed
  — D/A: a few bit adder needed
- Might affect modulator stability
Opamp Gain

- Finite opamp gain, $A$, moves pole at $z = 1$ left by $1/A$

- Flattens out noise at low frequency — only 3 dB/octave for high OSR

- Typically, require

\[ A > \frac{OSR}{\pi} \]   (29)

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Multi-bit Oversampled Converters

- A multi-bit DAC has many advantages
  — more stable - higher peak $|\text{NTF}|$
  — higher input range
  — less quantization noise introduced
  — less idle tones (perhaps no dithering needed)

- Need highly linear multi-bit D/A converters

Example

- A 4-bit DAC has 18 dB less quantization noise, up to 12 dB higher input range — perhaps 30 dB improved SNR over 1-bit

Large Advantage in DAC Application

- Less quantization noise — easier analog lowpass filter
Multi-bit Oversampled Converters

- Randomize thermometer code
- Can also “shape” nonlinearities

Third-Order A/D Design Example

- All NTF zeros at $z = 1$

$$NTF(z) = \frac{(z - 1)^3}{D(z)}$$  \hspace{1cm} (30)

- Find $D(z)$ such that $|NTF(e^{j\omega})| < 1.4$
- Use Matlab to find a Butterworth highpass filter with peak gain near 1.4
- If passband edge at $f_s/20$ then peak gain = 1.37

$$NTF(z) = \frac{(z - 1)^3}{z^3 - 2.3741z^2 + 1.9294z - 0.5321}$$  \hspace{1cm} (31)
Third-Order A/D Design Example

• Find $H(z)$ as

$$H(z) = \frac{1 - NTF(z)}{NTF(z)} \quad (32)$$

$$H(z) = \frac{0.6259z^2 - 1.0706z + 0.4679}{(z - 1)^3} \quad (33)$$

Third-Order A/D Design Example

• Choosing a cascade of integrator structure

• $\alpha_i$ coefficients included for dynamic-range scaling
  — initially $\alpha_2 = \alpha_3 = 1$
  — last term, $\alpha_1$, initially set to $\beta_1$ so input is stable for a reasonable input range

• Initial $\beta_i$ found by deriving transfer function from 1-bit D/A output to $V_3$ and equating to $-H(z)$
Third-Order A/D Design Example

$H(z) = \frac{z^2(\beta_1 + \beta_2 + \beta_3) - z(2\beta_3) + \beta_3}{(z-1)^3}$ (34)

• Equating (33) and (34) results in

$\alpha_1 = 0.0232, \quad \alpha_2 = 1.0, \quad \alpha_3 = 1.0$
$\beta_1 = 0.0232, \quad \beta_2 = 0.1348, \quad \beta_3 = 0.4679$ (35)

Dynamic Range Scaling

• Apply sinusoidal input signal with peak value of 0.7 and frequency $\pi/256$ rad/sample
• Simulation shows max values at nodes $V_1, V_2, V_3$ of 0.1256, 0.5108, and 1.004
• Can scale node $V_1$ by $k_1$ by multiplying $\alpha_1$ and $\beta_1$ by $k_1$ and dividing $\alpha_2$ by $k_1$
• Can scale node $V_2$ by $k_2$ by multiplying $\alpha_2/k_1$ and $\beta_2$ by $k_2$ and dividing $\alpha_3$ by $k_2$

$\alpha'_1 = 0.1847, \quad \alpha'_2 = 0.2459, \quad \alpha'_3 = 0.5108$
$\beta'_1 = 0.1847, \quad \beta'_2 = 0.2639, \quad \beta'_3 = 0.4679$ (36)
Third-Order A/D Design Example