University of Toronto

Term Test 1

Date - Oct 21, 2013 (11:10pm to 12:00pm)

Duration: 50 min

ECE331 — Analog Electronics
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS USING BACKS IF NECESSARY

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
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Last Name: SOLUTIONS

First Name: ____________________________

Student #: _____________________________

(max grade = 24)
[6] Question 1: Assume an nmos transistor (based on the 0.18um parameters given on the equation sheet) with \( W = 4 \mu m, \ L = 0.5 \mu m \) is biased with \( V_{ov} = 0.4 \, V \) and \( V_{DS} = 1 \, V \).

a) Find the \( r_o \) for this device.

\[
I_D = \frac{\mu N C_ox (\frac{W}{L})(V_{ov})^2}{2} = \left( \frac{2.4 \times 10^{-6}}{2} \right) \left( \frac{4}{0.5} \right) (0.4)^2 = 1.54 \, mA
\]

\[
\lambda = \frac{V_{ov}}{L} = \frac{0.05}{0.5} = 0.1 \, V^{-1}
\]

\[
r_o = \frac{1}{\lambda I_D} = 65.1 \, k
\]

b) If \( V_{DS} \) is increased by 0.5V, what is the corresponding change in \( I_D \)?

\[
\Delta I_D = \frac{\Delta V}{r_o} = \frac{0.5}{65.1 \, k} = 7.68 \, mA
\]
[6] Question 2: Given 2 current sources from $V_{DD}$ where each are $10 \mu A$, design a wide-swing cascode current-mirror circuit (including bias voltage generation) that gives an nmos output of $40 \mu A$. Assume all transistor lengths are $0.18 \mu m$ and the nmos current mirror output transistors have $W = 4 \mu m$. Show the widths of all transistors on your schematic. (Hint: there should be 5 nmos transistors and 2 current sources on your schematic).

\[ V_B \hspace{1cm} i_c \]
\[ v_i \]
\[ M_1 \quad M_2 \]
\[ r_0 \]
\[ I \]
\[ g_{m1} = 1 \text{mA/V} \]
\[ g_{m2} = 0.5 \text{mA/V} \]
\[ g_{m3} = 1 \text{mA/V} \]

a) Find the small-signal short-circuit current gain, \( i_{sc} / v_i \) assuming all \( r_o \to \infty \).

\[ r_{s1} = 1k \quad r_{s2} = 2k \]
\[ i_{sc} = \frac{v_i}{r_{s1} + r_{s2}} = \frac{v_i}{3k} \]
\[ \hat{i}_{sc} = \frac{1}{3k} = 0.333 \text{ mA/V} \]

b) Find the small-signal gain, \( i_{sc} / v_i \) assuming \( r_o = 20k \) for all transistors and making no approximations. Compare this result with that found in part a) in terms of percentage error.

\[ V_x \]
\[ i_{sc} \]
\[ r_0 \]
\[ r_{s1} \]
\[ v_i \]
\[ r_{s2} \]
\[ i_x \]
\[ \frac{v_x - v_i}{r_{s1}} + \frac{v_x}{r_0} + \frac{v_x}{r_{s2}} = 0 \Rightarrow v_x = \frac{r_{s1} r_{s2} r_0 v_i}{r_{s1} + r_{s2} + r_0} \]
\[ i_{sc} = \frac{v_x}{r_0} + \frac{v_x}{r_{s2}} = 0.625 v_i \left( \frac{1}{r_0} + \frac{1}{r_{s2}} \right) = 0.3438 \text{ mA/V} \]
\[ \text{ERROR} = \frac{0.333 - 0.3438}{0.3438} \times 100 = -3.00 \text{ Error} \]

Aside:

\[ q_{m1} r_{o1} = 2.0 \quad q_{m2} r_{o2} = 10 \]
\[ \text{IF} \quad q_{m1} r_{o1} = q_{m2} r_{o2} \]
\[ \text{THEN} \quad \text{ERROR} = 0.70 \]
[6] **Question 4:** Consider the circuit shown below. All current sources and the transistors have the same output impedance of \( r_o = 40k \). Also the transistors have \( g_m = 1\text{mA/V} \). Estimate the small-signal gains, \( v_1/v_i \), and \( v_2/v_1 \). Make the assumption that \( g_m r_o \gg 1 \).

\[
\begin{array}{c|c}
\hline
v_1/v_i & 0.4 \quad \text{v/V} \\
v_2/v_1 & 2.0 \quad \text{v/V} \\
\hline
\end{array}
\]

\[
R_x = \frac{1}{g_m} + \frac{r_o}{g_m r_o} = \frac{2}{g_m} = 2 \text{k}\Omega
\]

\[
\frac{v_1}{v_i} = \frac{(R_x \parallel r_o \parallel 1k)}{(R_x \parallel r_o \parallel 1k) + 1k} \approx \frac{(R_x \parallel 1k)}{(R_x \parallel 1k) + 1k}
\]

\[
= \frac{0.667}{0.667 + 1} = 0.4 \quad \text{v/V}
\]

\[
\frac{v_2}{v_1} \approx g_m (r_o \parallel r_o) = 2.0 \quad \text{v/V}
\]
Analog Electronics

Equation Sheet

Constants:
\[ k = 1.38 \times 10^{-23} \text{ JK}^{-1} \cdot \text{K} = 1.602 \times 10^{-19} \text{ C} ; \ V_T = kT/q = 26 \text{ mV at 300 } ^\circ \text{K} ; \]
\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} ; \ k_{\text{ox}} = 3.9 \cdot C_{\text{ox}} = (k_{\text{ox}}\varepsilon_0)/t_{\text{ox}} \]

NMOS:
\[ k_n = \mu_n C_{\text{ox}}(W/L) ; \ V_{\text{in}} > 0 ; \ \nu_{\text{DS}} > 0 ; \ \nu_{\text{ov}} = \nu_{\text{DS}} - \sqrt{2k_n V_{\text{ov}}} \]
(triode) \[ \nu_{\text{DS}} \leq \nu_{\text{ov}} \] (or \( \nu_{\text{DR}} < \nu_{\text{DR}} \)) \[ I_D = k_n(\nu_{\text{DS}} - \sqrt{2k_n V_{\text{ov}}}) \]

(active) \[ \nu_{\text{DS}} > \nu_{\text{ov}} \]
\[ I_D = 0.5k_n\nu_{\text{ov}}(1 + \nu_{\text{ov}}) ; \ \gamma_{\text{m}} = k_n V_{\text{ov}} = 2I_D/V_{\text{ov}} = \sqrt{2k_n I_D} ; \ r_s = 1/\gamma_{\text{m}} ; \ r_o = L/(|\nu_{\text{ID}}|) \]

PMOS:
\[ k_p = \mu_p C_{\text{ox}}(W/L) ; \ V_{\text{tp}} < 0 ; \ \nu_{\text{SD}} > 0 ; \ \nu_{\text{ov}} = \nu_{\text{SD}} - |\nu_{\text{tp}}| \]
(triode) \[ \nu_{\text{SD}} \leq \nu_{\text{ov}} \] (or \( \nu_{\text{DR}} > \nu_{\text{DR}} \)) \[ I_D = k_p(\nu_{\text{SD}} - \sqrt{2k_p V_{\text{ov}}}) \]

(active) \[ \nu_{\text{SD}} > \nu_{\text{ov}} \]
\[ I_D = 0.5k_p\nu_{\text{ov}}(1 + |\nu_{\text{ov}}|) ; \ \gamma_{\text{m}} = k_p V_{\text{ov}} = 2I_D/V_{\text{ov}} = \sqrt{2k_p I_D} ; \ r_s = 1/\gamma_{\text{m}} ; \ r_o = L/(|\nu_{\text{ID}}|) \]

BJT:
\[ I_C = I_{\text{S}}(1 + \beta)/V_T ; \ r_e = V_T/I_C ; \ r_e = \beta/\gamma_{\text{m}} ; \ r_o = |V_{\text{BE}}|/I_C \]
\[ I_C = \beta I_E ; \ r_e = (\beta + 1)/\beta ; \ r_o = \beta/\gamma_{\text{m}} ; \ r_o = |V_{\text{BE}}|/I_C \]

Cascode:
\[ V_{\text{S}} = (1 + \gamma_{\text{m}}/R_S) r_o ; \]
\[ V_{\text{S}} = (1 + \gamma_{\text{m}}/R_S) r_o \]

Diff Pair:
\[ A_D = \gamma_{\text{m}} R_D ; \ A_{\text{CM}} = -(R_D/(2R_S))((\Delta V_{\text{DS}})/R_D) ; \ A_{\text{CM}} = -((R_D/(2R_S))((\Delta V_{\text{DS}})/R_D) \]
\[ V_{\text{os}} = \Delta V_{\text{T}} ; \ V_{\text{os}} = (V_{\text{DD}}/2)((\Delta V_{\text{DL}})/R_D) ; \ V_{\text{os}} = (V_{\text{DD}}/2)((\Delta V_{\text{DL}})/(W/L)) \]

1st order:
step response \( y(t) = y_0(1 + s/\tau) (1 + s/\omega_{0\text{dB}}) \) unity gain freq for \( T(t) = \frac{A_{\text{M}}}{1 + s/\omega_{0\text{dB}}} \)

Freq:
for real axis poles/zeros \( T(s) = k_{\text{dc}}(1 + s/\omega_{0\text{dB}})(1 + s/\omega_{0\text{dB}}) \)

OTC estimate \( f_H = 1/(2\pi\tau_{\text{max}}) \); dominant pole estimate \( f_H = 1/(2\pi\tau_{\text{max}}) \)

Miller:
\[ Z_2 = \frac{k}{2(1 - k)} ; \ Z_2 = Z/(1 - 1/K) \]

MOS caps:
\[ \begin{align*}
\kappa_{\text{ox}} &= \frac{2}{3} WLC_{\text{ox}} + WL_{\text{ox}} C_{\text{ox}} ; \\
C_{\text{gd}} &= WL_{\text{ox}} C_{\text{ox}} ; \\
C_{\text{db}} &= C_{\text{db0}}/(\sqrt{1 + \nu_{\text{DB}}/V_{\text{DD}}})
\end{align*} \]

Feedback:
\[ A_f = A/(1 + \beta) ; \ x_1 = (1/(1 + \beta))/x_2 ; \ dA_f/A_f = (1/(1 + \beta))/dA/A ; \]
\[ \omega_{0r} = \omega_{0r}(1 + A\beta) ; \ \omega_{0f} = \omega_{0f}(1 + A\beta) \]

Loop Gain
\[ L = -s_f/s_f ; \ A_f = A_f(L/(1 + l) + d/(1 + l) + Z_{\text{port}} = Z_{\text{port}}((1 + l)/(1 + l)) \]

PM
\[ \angle(\omega_{0f} + 180) ; \ GM = \angle(\omega_{0f} + 180) \]

Pole Splitting
\[ \omega_{0f} = \frac{1}{(R_{\text{Gm}} R_{\text{C}})} ; \ \omega_{0f} = \frac{(G_{\text{m}} C_{\text{C}})}{(C_{\text{C}} + C_{\text{C}} + C_{\text{C}})} \]

Pole Pair
\[ \frac{s^2 + \omega_0^2}{Q^2} + \frac{s^2 + \omega_0^2}{Q^2} \Rightarrow \text{real poles} ; \ Q > 1; \ \text{freq resp peaking} \]

Power Amps:
Class A: \( \eta = (1/4)(\theta_{\text{V}}'/\theta_{\text{C}}) \); Class B: \( \eta = (1/4)(\theta_{\text{V}}'/\theta_{\text{C}}) \); \( P_{\text{DN max}} = V_{\text{C}}^2/(\pi^2 R_L) \)

Class AB: \( I_{\text{DP}} = I_{\text{DP}} \)

2-stage cmos opamp: \( \omega_0 = 1/(R_1 G_{\text{m}2} R_2 C_{\text{C}}) ; \ \omega_0 = (G_{\text{m}2} C_{\text{C}}) ; \ \omega_0 = 1/(C_{\text{C}}((1/G_{\text{m}2}) - R)) \)

\[ SR = 1/C_{\text{C}} = \omega_0 V_{\text{OV}} ; \text{will not SR limit if } \omega_0 V_{\text{OV}} < \text{SR} \]

MOS Transistor: CMOS basic parameters. Channel length = 0.18 \mu m