University of Toronto

Term Test 2

Date - Nov 20, 2015 (1:10pm to 2:00pm)

Duration: 50 min

ECE331 — Analog Electronics
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS USING BACKS IF NECESSARY

1. Equation sheet is on last page of test.

2. Unless otherwise stated, use transistor parameters on equation sheet and assume \( g_m r_o \gg 1 \).

3. Non-programmable calculator allowed; No other aids allowed

4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

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Last Name: **SOLUTIONS**

First Name: 

Student #: 

(max grade = 24)
[6] Question 1: Consider the circuit below where \( v_{icm} = 3 \text{V} \).

\[ V_{DD} = 5 \text{V} \]

all \( L = 0.2 \mu\text{m} \)
\( W_1 = W_2 = 5 \mu\text{m} \)
\( W_3 = W_4 = W_5 = 5 \mu\text{m} \)

\( I_2 = 50 \mu\text{A} \)
\( I_1 = 100 \mu\text{A} \)

\[ \frac{(\omega)}{(L)} = 2.5 \]
\[ \frac{(\omega)}{(C)} = 2.5 \]

\[ I_{O1} = 50 \mu\text{A} = I_{O3} \]
\( MN_{Cox} = 240 \text{e}^{-6} \)
\( MP_{Cox} = 60 \text{e}^{-6} \)

\[ v_{op \_ min} = 2.6 \]
\[ v_{op \_ max} = 4.74 \]

a) What is the max and min for \( v_{op} \)? (assume \( v_{id} = 0 \))

\[ V_{OV,1,2} = \sqrt{\frac{2 I_{D1}}{MN_{Cox}(W)}} = 0.129 \text{V} \]

\[ V_{OV,3,4} = \sqrt{\frac{2 I_{D3}}{MP_{Cox}(W)}} = 0.258 \text{V} \]

\[ V_{OP \_ max} = V_{DD} - V_{OV3} = 4.74 \text{V} \]

\[ V_{OP \_ min} = V_{icm} - V_{TN} = 2.6 \text{V} \]

\[ V_{TN} = 0.4 \text{V} \]

b) Find the gain, \( v_o/v_{id} \). (\( v_o = v_{op} - v_{on} \))

\[ g_{m1} = \frac{2 I_{D1}}{V_{OV1}} = 0.775 \text{mA/V} \]

\[ R_{o1} = \frac{L}{\lambda I_{D1}} = 80 \text{K} \]

\[ R_{o3} = \frac{L}{\lambda I_{D3}} = 80 \text{K} \]

\( \frac{V_o}{V_{ID}} = g_{m1} \left( \frac{R_{o1}}{R_{o3}} \right) = 31 \)
[6] Question 2:

a) Write the transfer-function for an amplifier having an approximate gain of -100 at 100 rad/s and zeros at 1 and 10 rad/s (on the negative real axis) and poles at 5 and 1000 rad/s.

\[ H(s) = \frac{k \times (s+1)(s+10)}{(s+5)(s+1000)} \]

\[ |H(j\omega)| \approx -100 = \frac{k \times (100)(101)}{(100)(1000)} \Rightarrow k = -1000 \]

\[ H(s) = \frac{-1000 \times (s+1)(s+10)}{(s+5)(s+1000)} \]

b) What is the dc gain of the amplifier in a)?

\[ H(0) = \frac{-1000 \times (1)(10)}{(5)(1000)} \]

\[ H(0) = -2 \]

dc gain = -2

50 M rad/s

c) An amplifier has 2 poles at 50 and 100 Mrad/s. Using the open circuit time-constant approach, estimate the 3dB frequency for this amplifier in MHz.

\[ f_{3dB} = \left( \frac{2\pi}{50\text{MHz}} + \frac{2\pi}{100\text{MHz}} \right)^{-1} \]

\[ = 5.3 \text{ MHz} \]
[6] Question 3: Consider the amplifier circuit shown below and only the shown capacitors.

\[ g_m = 5 \text{mA/V} \]

ignore \( r_0 \)

\[ \frac{1}{g_m} = r_s = 200 \Omega \]

Given that a zero for this amplifier occurs at \( \omega_z = 1/(R_sC_1) = 1\text{k rad/s} \) and a zero occurs dc, draw the Bode plot for this amplifier showing pole locations (in rad/s) and approximate gain values at 100 rad/s and 25k rad/s.

\[ C_1 \Rightarrow \omega_{p1} = \frac{1}{C_1(R_s g_m)} = 6k \text{ rad/s} \]

\[ C_2 \Rightarrow \omega_{p2} = \frac{1}{C_2(R_0 + R_L)} = 12.5 \text{ rad/s} \]

\[ C_L \Rightarrow \omega_{pl} = \frac{1}{C_L(R_s g_m)} = 50k \text{ rad/s} \]

\[ \left| \frac{V_o}{V_i} \right|_{w_0} \approx \frac{-(R_0 / R_L)}{R_s + R_s} = -16.7 \frac{V}{V} \]

\[ \left| \frac{V_o}{V_i} \right|_{25k} \approx -\frac{R_0 / R_L}{R_s} = -100 \frac{V}{V} \]
[6] Question 4: Consider the differential amp shown below and only consider the capacitances shown.

\[ V_{DD} \]

\[ V_{in1} \]

\[ M_3 \]

\[ C_1 \]

80 nF

\[ V_{in2} \]

\[ M_5 \]

\[ M_4 \]

\[ M_1 \]

\[ M_2 \]

\[ V_{bias} \]

\[ V_{out} \]

\[ C_2 \]

150 nF

\[ g_{m1} = 1.5 \text{mA/V} \]

\[ r_{o1} = 26.7 \text{k\Ohm} \]

\[ g_{m2} = 1.5 \text{mA/V} \]

\[ r_{o2} = 26.7 \text{k\Ohm} \]

\[ g_{m3} = 1.5 \text{mA/V} \]

\[ r_{o3} = 26.7 \text{k\Ohm} \]

\[ g_{m4} = 1.5 \text{mA/V} \]

\[ r_{o4} = 26.7 \text{k\Ohm} \]

\[ g_{m5} = 3 \text{mA/V} \]

\[ r_{o5} = 13.3 \text{k\Ohm} \]

\[ r_{s3} = \frac{1}{g_{m3}} = 667 \text{k\Ohm} \]

a) Find the locations of the pole frequencies in this circuit (for the diff gain of the circuit).

\[ \omega_{P1} = \frac{1}{C_1 (r_{s3}||r_{o3})} \approx 18.7 \text{ GHz} \Rightarrow f_{P1} = 3 \text{ GHz} \]

\[ \omega_{P2} = \frac{1}{C_2 (r_{o2}||r_{o4})} \approx 499 \text{ MHz} \Rightarrow f_{P2} = 79 \text{ MHz} \]

b) Find the common-mode gain \( v_o/v_{icm} \) (in V/V). (Hint, \( v_o \) follows \( v_3 \) when a common-mode input signal is applied)

\[ \frac{v_o}{v_{icm}} = \frac{v_3}{v_{icm}} = \frac{-r_{s3}}{2r_{o5} + r_{s1}} = \frac{-667}{26.7k + 667} \]

\[ = 0.024 \frac{v}{v} \]
Analog Electronics

**Equation Sheet**

Constants: \( k = 1.38 \times 10^{-23} \text{JK}^{-1} \); \( q = 1.602 \times 10^{-19} \text{C} \); \( V_T = kT/q = 26 \text{mV at 300}^\circ \text{K} \);
\( \varepsilon_0 = 8.854 \times 10^{-12} \text{F/m} \); \( k_{ox} = 3.9 \); \( C_{ox} = (k_{ox}\varepsilon_0)/\ell_{ox} \)

**NMOS:** \( k_n = \mu_n C_{ox}(W/L) \); \( V_{in} > 0 \); \( V_{DS} \geq 0 \); \( V_{gs} = V_{DS} - V_{th} \)

(triode) \( V_{gs} \leq \gamma_{ov} \)

\[ (\text{active}) \quad V_{gs} \geq \gamma_{ov}; \quad i_D = 0.5k_n V_{gs}^2 (1 + \lambda V_{DS}); \quad g_m = k_n V_{gs} = 2I_D/V_{gs} = \sqrt{2k_n I_D}; \quad r_p = 1/g_m \]

**PMOS:** \( k_p = \mu_p C_{ox}(W/L) \); \( V_{ip} < 0 \); \( V_{SD} \geq 0 \); \( V_{gs} = V_{SD} - V_{th} \)

(triode) \( V_{gs} \leq \gamma_{ov} \)

\[ (\text{active}) \quad V_{gs} \leq \gamma_{ov}; \quad i_D = 0.5k_p V_{gs}^2 (1 + \lambda V_{DS}); \quad g_m = k_p V_{gs} = 2I_D/V_{gs} = \sqrt{2k_p I_D}; \quad r_p = 1/g_m \]

**BJT:** (active) \( i_C = I_C(e^{r_{eb}/V_T} - 1) \); \( g_m = \alpha_r = \frac{I_C}{V_T} \); \( r_e = V_T/I_E; \quad r_B = \beta/g_m; \quad r_O = V_T/I_C \)

\[ i_C = \beta i_b; \quad i_b = (\beta + 1)/V_T; \quad i_e = \frac{I_C}{V_T} \]

**Cascade:** \( V_{in} \)

\[ R_s = (1 + g_m R_s) g_m \]

\[ R_d = R_s \]

**Diff Pair:**

\[ A_d = \frac{g_m R_d}{A_{CM} = -R_d/(2R_{SS})(\Delta R_{SS})/R_d}; \quad V_{os} = V_s = (V_{gs} - V_{th})/2 \]

**Freq:**

for real axis poles/zeros \( T(s) = \frac{k_o(1 + s/\omega_{zz})}{(1 + s/\omega_{zz})(1 + s/\omega_{zz})} \)

OTC estimate \( f_H = 1/(2\pi \Sigma \omega) \); dominant pole estimate \( f_H = 1/(2\pi \omega_{max}) \)

**Miller:** \( Z_l = Z/(1 - K) \)

**MOS caps:** \( C_g = (2/3)W/L C_{ox} \); \( W/L C_{ox} \); \( C_{db} = C_{db}/(1 + V_{db}/V_0) \)

\[ f_c = g_m/2(\pi/C_{gs} + C_{gb}) \]

Feedback:

\[ A_f = A/(1 + A\beta); \quad s_c = (1/(1 + A\beta))s; \quad dA/dA = (1/(1 + A\beta))dA/A; \quad \omega_{Hz} = \omega_f(1 + A\beta) \]

**PM:** \( L = (s_c/\omega_b + 180); \quad GM = -[s_c/\omega_b(\omega_b > 1)] \)

**Pole Pair:** \( s = (0, \omega_0)/Q \)

**Power Amps:**

\[ A = (1/4)(V_p/\lambda R_c); \quad B = (1/4)(V_p/\lambda C); \quad P_{D_MAX} = V_p^2/(\pi^2 R_c) \]

\[ A_{AB} = i_{p1} = I_0 \]

2-stage cmos amp: \( \omega_{p1} = (1/(1 + R_c C_{in} C_{in}) \); \( \omega_p = (C_{in}^2/R_c); \omega_0 = (1/(C_{in}^2(1/C_{in}^2 - R_c)) \)

\[ SR = 1/C_c = \omega_0 V_{os}; \quad \text{will not SR limit if } \omega_0 V_{os} < SR \]

**MOS Transistor:** CMOS basic parameters. Channel length = 0.18μm

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<td>( V_f ) (V)</td>
<td>( \mu C_{ox} ) (μA/V^2)</td>
<td>( \lambda' ) (μm/V)</td>
<td>( C_{ox} ) (fC/μm^2)</td>
<td>( t_{ox} ) (nm)</td>
<td>( L_{ov} ) (μm)</td>
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<td>NMOS</td>
<td>0.4</td>
<td>240</td>
<td>0.05</td>
<td>8.5</td>
<td>4</td>
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<td>PMOS</td>
<td>-0.4</td>
<td>60</td>
<td>-0.05</td>
<td>8.5</td>
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