MOS DIFFERENTIAL PAIR

\[ V_{DD} \]

\[ R_D \]

\[ V_{DD} \]

\[ M_1, M_2 \text{ IDENTICAL} \]

\[ V_{G1} \]

\[ V_{G2} \]

\[ I \]

\[ V_{SS} \]

\[ i_{D1} + i_{D2} = I \text{ IF } M_1, M_2 \text{ ACTIVE} \]

\[ \text{IF } V_{G1} = V_{G2} \Rightarrow i_{D1} = i_{D2} = \frac{I}{2} \]

\[ \text{IF } V_{G1} > V_{G2} \Rightarrow i_{D1} > i_{D2} \]

\[ \text{IF } V_{G1} < V_{G2} \Rightarrow i_{D1} < i_{D2} \]

CAN "STEER" CURRENT \( I \)
COMMON-MODE INPUT OPERATION

\[ V_{o1} = V_{o2} = V_{dd} - \frac{I}{2} R_0 \quad \text{WHILE M1, M2 ACTIVE} \]

\[ C_{o1} = \frac{I}{2} = \frac{\mu W C_{ox}}{2} \left( \frac{W}{L} \right) \left( V_0 \right)^2 \]

\[ V_{ov} (W) = \sqrt{I / \left( \mu W C_{ox} \left( \frac{W}{L} \right) \right)} \]

\[ V_{cm-max} = V_t + V_{dd} - \frac{I}{2} R_0 \quad ( ONE \ V_T \ ABOVE \ V_{o1} ) \]

\[ V_{cm-min} = -V_{ss} + V_{cs} + V_t + V_{ov} (W) \]

( \text{VCS IS VOLTAGE NEEDED ACROSS I CURRENT SOURCE} )
LARGE SIGNAL WITH DIFFERENTIAL VOLTAGE

IF $V_{id} >> 0$ ALL $I$ FLOWS THROUGH $M_1$

$V_{id} << 0$ "" "" "" $M_2$

WHEN $I_{D2} = 0$ ($M_2$ JUST TURNS OFF)

$V_{GS2} = V_{thn} \Rightarrow V_S = -V_{thn}$

$V_{GS1} = V_{th} + V_{OUT} \text{ WHERE}$

$V_{OUT} = \sqrt{2I/(M \cdot C_{ox} W)}$

$= \sqrt{2} \cdot V_{OUT}(I_2)$
Since \( n id = \sqrt{5} + V_{651} \)
\[= -V_{6n} + V_{6n} + \sqrt{2} V_{0v1} \]

\[V_{id \text{ max}} = \sqrt{2} V_{0v} \left( \frac{1}{2} \right) \]

WHERE CURRENT CAN BE STEERED TO \( M_1 \) OR \( M_2 \)

\[\frac{i_{d1}}{I} = \frac{i_{d2}}{I} = \frac{1}{4} \]

\[V_{id \text{ max}} = n \sqrt{2} V_{0v} \left( \frac{1}{2} \right) \]

CAN SHOW

\[i_{d1} = \frac{I}{2} + \left( \frac{I}{V_{0v} \left( \frac{1}{2} \right)} \right) \left( \frac{V_{id}}{2} \right) \left( \sqrt{1 - \left( \frac{V_{id} / 2}{V_{0v} \left( \frac{1}{2} \right)} \right)^2} \right) \]

\[i_{d2} \text{ similar eqn.} \]
- $i_D$ is approximately linear relationship to $V_{id}$ near $V_{id} = 0$
- More non-linear as $V_{id}$ increases.

**Small-Signal Operation**

$V_{G1} = V_{cm} + \frac{V_{id}}{2}$

$V_{G2} = V_{cm} - \frac{V_{id}}{2}$
2 WAYS TO LOOK AT CIRCUIT.
FOR SMALL-SIGNAL ANALYSIS

\[ i_{S1} = \frac{v_{id}}{r_{S1} + r_{S2}} = \frac{v_{id}}{2r_S} \]

Assume \( r_S = r_{S1} = r_{S2} \)

\[ v_{o1} = -i_{S1}R_D \quad \text{and} \quad i_{S2} = -i_{S1} \]

\[ v_{o2} = -i_{S2}R_D = i_{S1}R_D \]

\[ v_{od} = v_{o2} - v_{o1} = 2i_{S1}R_D = \frac{R_D}{r_S}v_{id} \]

\[ \frac{v_{od}}{v_{id}} = \frac{R_D}{r_S} = gmR_D \]
\[ V_{01} = -\left( \frac{R_0}{R_s} \right) \left( \frac{V_{id}}{2} \right) \]

\[ V_{02} = -\left( \frac{R_0}{R_s} \right) \left( -\frac{V_{id}}{2} \right) \]

\[ V_{0d} = V_{02} - V_{01} = \left( \frac{R_0}{R_s} \right) V_{id} = g_m R_0 V_{id} \]
DIFFERENTIAL HALF CIRCUIT.

IF CIRCUIT IS BALANCED => DO ANALYSIS ON HALF CIRCUIT

\[ V_{CM} + \frac{V_{id}}{2} \]

\[ R_0 = 1k \quad R_L = 2k \quad R_0 = 1k \]

\[ V_{CM} + \frac{V_{id}}{2} \]

\[ R_S = 100 \]

\[ I = 1mA \]

\[ V_{CM} = \frac{V_{id}}{2} \]

ASSUME

\[ V_{0V} = V_{0V}^{(2)} = 0.2V \]

\[ I = 0 \]

\[ R_0 = 1k \]

\[ \frac{-V_{od}}{2} \]

\[ R_S = 100 \]

\[ R_L = 1k \]

\[ \frac{R_S}{2} = 1k \]

\[ q_m = \frac{1}{r_S} = \frac{2I_0}{V_{0V}} = \frac{2(I/2)}{V_{0V}} = \frac{2(0.5)}{0.2} = 5mA/V \]

\[ r_S = 200\Omega \]
\[
- \frac{V_{od}}{2} = - \frac{V_{od}}{V_{id}} = \frac{-[R_{01} + (R_{02})]}{R_S + R_S} \\
- \frac{V_{od}}{V_{id}} = - \frac{500}{300} = -1.67 \frac{V}{V} \\
\]

\[
\frac{V_{od}}{V_{id}} = 1.67 \frac{V}{V} \\
\]

DIFF PAIR WITH CURRENT SOURCE LOADS

\[
A_d = \frac{V_{od}}{V_{id}} = g_{ma} \left( \frac{R_{01} + R_{02}}{R_S + R_S} \right) \\
\]
\[ A_d = \frac{V_{od}}{V_{id}} = g_{m1} \left( R_{on} \parallel R_{op} \right) \]

\[ R_{on} \approx (g_{m2} R_{o2}) R_{in} \]

\[ R_{op} \approx (g_{m5} R_{o5}) R_{o7} \]
DIFFERENTIAL & COMMON-MODE SIGNALS

- MANY CIRCUITS RELY ON DIFFERENTIAL SIGNALS AS THEIR SIGNALS AND TRY TO IGNORE (OR REJECT) COMMON-MODE SIGNALS

- ETHERNET
  
  TWISTED WIRE PAIR

  $\text{V}_{\text{lid}} ^{+} - \text{V}_{\text{lad}} ^{-} = \text{V}_{\text{od}}$

  TRANSMIT & RECEIVE DIFF SIGNALS

  SO COMMON MODE INTERFERENCE CAN BE REJECTED

- TWISTED PAIR HELPS BUT COMMON-MODE SIGNALS STILL GETS THROUGH

  $\text{V}_{\text{lid}} ^{+} - \text{V}_{\text{lad}} ^{-} = \frac{\text{V}_{\text{ol}} ^{+}}{2} + \text{V}_{\text{NOISE}}$

  $\text{V}_{\text{ol}} ^{-} = -\frac{\text{V}_{\text{lid}} ^{-}}{2} + \text{V}_{\text{NOISE}}$
\[ V_{od} = V_{o1} - V_{o2} = V_{id} \]

**DIFF AMP**

\[
\begin{align*}
V_{oni} + \frac{V_{id}}{2} & \rightarrow V_{i1} \\
V_{oni} - \frac{V_{id}}{2} & \rightarrow V_{i2}
\end{align*}
\]

\[
\begin{align*}
V_{o1} & \rightarrow V_{CM0} + \frac{V_{od}}{2} \\
V_{o2} & \rightarrow V_{CM0} - \frac{V_{od}}{2}
\end{align*}
\]

\[ V_{id} = V_{i1} - V_{i2} \]

\[ V_{oni} = \frac{V_{i1} + V_{i2}}{2} \]

\[ V_{od} = V_{o1} - V_{o2} \]

\[ V_{CM0} = \frac{V_{o1} + V_{o2}}{2} \]

**DEFINE A GAINS THROUGH AMP**
\[
A_{d} = \frac{V_{od}}{V_{id}} \quad \text{DIFFERENTIAL GAIN}
\]

\[
A_{cm-cm} = \frac{V_{cm0}}{V_{cmi}} \quad \text{COMMON MODE TO COMMON MODE GAIN}
\]

\[
A_{cm} = \frac{V_{od}}{V_{imi}} \quad \text{COMMON MODE TO DIFFERENTIAL GAIN}
\]

\[
A_{d-cm} = \frac{V_{cm0}}{V_{id}} \quad \text{DIFF TO CM GAIN (USUALLY UNIMPORTANT)}
\]

\[A_{cm-cm} \text{ KEEP SMALL SO COMMON-MODE SIGNALS DO NOT AMPLIFY (< 1)}\]

\[\text{MUST IMPORTANT ARE } A_{d} \text{ & } A_{cm}, \text{ WANT } A_{d} \gg A_{cm} \text{ SO CM SIGNALS ARE REJECTED.}\]
EQUIV TO HALF CIRCUIT

\[ \frac{V_{01}}{V_{CM1}} = -\frac{R_0}{R_S + 2R_{SSS}} = \frac{V_{02}}{V_{CM2}} \]

So \( V_{od} = V_{01} - V_{02} = 0 \) \( \Rightarrow \) \( A_{CM} = 0 \)

IF PERFECTLY MATCHED.

IF RESISTORS MISMATCHED BY \( \Delta R_0 \)

\( R_{01} = R_0 \quad \text{and} \quad R_{02} = R_0 + \Delta R_0 \)

CAN SHOW \( A_{CM} = -\left(\frac{R_0}{2R_{SSS}}\right)\left(\frac{\Delta R_0}{R_0}\right) \)
If transistors $g_m$ mismatched by $\frac{\Delta g_m}{g_m}$

Can show

$$i ACm = \left( \frac{R_o}{2R_{ss}} \right) \left( \frac{\Delta g_m}{g_m} \right)$$

CMRR (common-mode rejection ratio) usually care about ratio $\frac{A_d}{A_{cm}}$

So define

$$CMRR = \frac{|A_d|}{|A_{cm}|}$$

Or in dB

$$CMRR (dB) = 20 \log \left( \frac{|A_d|}{|A_{cm}|} \right)$$

Since $A_d = g_m R_0$

Then for $\Delta R \Rightarrow CMRR = \frac{2g_m R_{ss}}{(\Delta R_0 / R_0)}$

For $\Delta g_m \Rightarrow CMRR = \frac{2g_m R_{ss}}{(\Delta g_m / g_m)}$
**Input Offset of Diff Pair (DC Value)**

**Define** $V_{os}$ as input offset voltage

**Needed to set output offset to zero**

$V_{os}$ input offset  
$V_o$ output offset

$$V_{os} = \frac{V_o}{Ad}$$

![Circuit Diagram](diagram.png)

If $V_{id} = 0$  
$V_o$ is output offset voltage
OFFSET DUE TO $R_D$, $(\frac{w}{L})$, $\Delta V_{th}$ MATCH

typically $V_{th}$ dominates

$V_{os} = \Delta V_{th}$

typically $\Delta V_{th} = 1 \rightarrow 10$ mV

for $\Delta R_D$

$V_{os} = \left(\frac{V_{os}}{2}\right) \left(\frac{\Delta R_D}{R_D}\right)$

for $\Delta (\frac{w}{L})$

$V_{os} = \left(\frac{V_{os}}{2}\right) \left(\frac{\Delta (\frac{w}{L})}{\frac{w}{L}}\right)$
ACTIVE LOADS

FULLY DIFF

DIFF - SINGLE-ENDED

\[ V_{id} = V_{61} - V_{62} \]
WANT TO MODEL ABOVE AS

\[ \text{To find } G_m \]

\[ G_m = \frac{i_o}{V_{id}} \text{ or } i_o = G_m V_{id} \]

Can show \( G_m = g_m \text{ TETX} \)

Where \( g_m = g_m' = g_m'' \)
To find $R_0$

\[ R_0 = \frac{N_X}{i_X} \]

Can show $R_0 \approx R_{02} \parallel R_{04}$

$R_{02} \approx 2R_{02} \implies i = \frac{N_X}{2R_{02}}$

\[ i_X = i + \frac{N_X}{R_{04}} = \frac{2V_X}{2R_{02}} + \frac{V_X}{R_{04}} \]

\[ i_X = \frac{N_X}{R_{02}} + \frac{V_X}{R_{04}} \implies R_0 = \frac{N_X}{i_X} = R_{02} \parallel R_{04} \]
\[ R_{03} = \frac{1}{g_{m3}} \parallel \frac{1}{R_{03}} = \frac{1}{g_{m7}} \]

\[ R_{01} = \frac{1}{g_{m1}} + \frac{R_{03}}{g_{m1} R_{01}} = \frac{1}{g_{m1}} + \frac{1}{g_{m1} g_{m3} R_{01}} \]

\[ R_{01} \approx \frac{1}{g_{m4}} \]

\[ R_{02} = \left[1 + g_{m2} (R_{01})\right] R_{02} \quad g_{m1} = g_{m2} \]

\[ = \left[1 + (g_{m2}) \left(\frac{1}{g_{m1}}\right)\right] R_{02} \]

\[ = 2 \frac{R_{02}}{R_{02}} \]
**DIFFERENTIAL GAIN**

$$A_d = \frac{V_o}{V_{id}} = G_m R_0 = g_m \left( \frac{r_2 || r_0}{r_0} \right)$$

If $$r_0 = r_{o2} = r_{o4}$$

$$A_d = \frac{1}{2} g_m r_0$$

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**2 STAGE CMOS OPAMP**

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**OVERALL GAIN**

$$\frac{V_o}{V_{id}} = A_1 A_2$$

(CC IS A COMPENSATION CAP FOR STABILITY)

$$A_1 = \frac{V_{o1}}{V_{id}} = -g_{m1} \left( \frac{r_2 || r_0}{r_0} \right)$$

$$A_2 = \frac{V_o}{V_{o1}} = -g_{m6} \left( \frac{r_{o6} || r_{o7}}{r_{o7}} \right)$$
SIZING TRANSISTORS FOR ZERO SYSTEMATIC OFFSET

(RANDOM OFFSET WILL STILL OCCUR & RESULT IN 1-20 mV INPUT OFFSET VOLTAGE)

LET V_ID = 0

SINCE M3 & M4 MATCHED

I_{D3} = I_{D4} = \frac{I}{2} \Rightarrow V_{O53} = V_{O54}

SO \quad V_{G65} = V_{O54} = V_{O53} = V_{G53}

\Rightarrow I_{D6} = \frac{(W/L)_6}{(W/L)_3} \left(\frac{I}{2}\right)

FOR NO SYSTEMATIC OFFSET I_{D7} = I_{D6}

\Rightarrow I_{D7} = \frac{(W/L)_2}{(W/L)_5} \frac{I}{I}

SO FOR I_{D6} = I_{D7}

\frac{(W/L)_6}{(W/L)_3} = 2 \frac{(W/L)_7}{(W/L)_5}
Example

\[ V_{B} - V_{0} \]

\[ V_{c} - I_{1} - I_{2} \]

\[ I = 50 \mu A \]

\[ V_{B} \text{ chosen so } I_{03} = I_{02} \]

Find \[ \frac{V_{0}}{V_{c}} \]

DC

\[ I_{01} = 2I_{02} + I_{01} + I_{02} = I = 50 \mu A \]

\[ I_{02} = 16.7 \mu A \quad I_{01} = 33.3 \mu A \]

\[ q_{m1} = \sqrt{2 \cdot \mu \cdot \alpha \cdot \left( \frac{W}{L} \right) \cdot I_{01}} = 406 \text{ mA/V} \]

\[ R_{S1} = 2.5 \text{ k} \Omega \]

\[ q_{m2} = 200 \text{ mA/V}^{2} \quad R_{S2} = 5 \text{ k} \Omega \]

\[ R_{01} = \frac{L}{\chi' \cdot I_{01}} = 60 \text{ k} \quad R_{02} = 120 \text{ k} \]

\[ V_{0V1} = V_{0V2} = 167 \text{ mV} \]

\[ R_{03} = \frac{L}{\chi' \cdot I_{03}} = 120 \text{ k} \]
Find $i_{sc}$ (ignore $r_o$)

\[ i_{sc} = -i_{s2} = i_{s1} = \frac{v_i}{r_{s1} + r_{s2}} = \frac{v_i}{7.5k} \]

\[ R_o \]

\[ R_{op} = r_{o3} \]
\[ R_{on} = R_{i1} + r_{o2} + g_m r_2 R_{i1} \]
\[ = 120k \]
\[ = 180k \]

\[ R_o = R_{op} \parallel R_{on} = 72k \]
\[ V_0 = \text{i}_{\text{sc}} R_0 = (72k) \left( \frac{V_i}{7.5k} \right) \]

\[ \frac{V_0}{V_i} = 9.6 \sqrt{V} \]