High-frequency cutoff approximation \( f_H \)

For accurate \( f_H \) value
    Can do exact analysis or simulation

However, open-circuit time-constant method together with Miller's theorem (for coupled capacitors) gives \( f_H \) estimate

Procedure

1) Use Miller's theorem to separate coupled capacitors

2) For each high freq. (HF) cap
    Find \( C_i \) associated with that cap while other HF caps opened.

Each pole is estimated to be

\[ f_{pi} = \frac{1}{2\pi C_i} \]
3) 2 methods to estimate $f_H$

- Open-Circuit Time Constant (OTC) estimate

\[
\frac{1}{2\pi i \sum c_i} = \left( \frac{1}{f_{p_1}} + \frac{1}{f_{p_2}} + \ldots \right)^{-1}
\]

- Dominant Pole estimate

\[
f_H \approx \frac{1}{2\pi i c_{\text{max}}}
\]

where $c_{\text{max}}$ is largest $c_i$

or equivalently

\[
f_H \approx f_{p_{\text{min}}}
\]

where $f_{p_{\text{min}}}$ is smallest $f_{p_i}$
SHORT/OPEN CIRCUIT TIME CONSTANT

EXAMPLE
(WIDELY SPACE POLES SO USE DOMINANT ESTIMATE)

\[ \begin{align*}
   & C_1 \\ & C_2 \\ & R_3 \\ & R_4 \\
\end{align*} \]

\[ \begin{align*}
   & 10\mu F \\ & 10\mu F \\ & 1k \\ & 100k \\
\end{align*} \]

\[ \begin{align*}
   & R_1 \\ & R_2 \\ & C_3 \\ & C_4 \\
\end{align*} \]

\[ \begin{align*}
   & 1k \\ & 100k \\ & 1 \mu F \\ & 10F \\
\end{align*} \]

First determine which caps are low freq cutoff and which high freq cutoff:

- LF \( \Rightarrow \) \( C_1, C_2 \)
- HF \( \Rightarrow \) \( C_3, C_4 \)

For midband gain short LF caps, open HF caps:

\[ \begin{align*}
   \frac{V_o}{V_i} = 1
\end{align*} \]
\[ \frac{f_L}{A_L} \]

**For LF cutoff =** OPEN HF CAPS  
**ZERO INDEPENDENT SOURCE**

**Find poles due to each LF cap (while other LF cap shorted)**

\[ C_1 \Rightarrow f_{p1} = \frac{1}{2\pi C_1 \left( R_1 || R_2 \right)} \approx \frac{1}{2\pi C_1 R_1} \]

\[ f_{p1} = 15.9 \text{ Hz} \]

\[ C_2 \Rightarrow f_{p2} = \frac{1}{2\pi C_2 R_2} = 0.159 \text{ Hz} \]

**Low Freq cutoff is higher of \( f_{p1} \) and \( f_{p2} \)**

\[ f_L = 15.9 \text{ Hz} \]
For HF cutoff => Short LF caps zero vs

Find poles due to each HF cap (while other HF caps open)

\[ C_3 = \frac{1}{f_{p3}} = 159 \text{ MHz} \]

\[ C_4 = \frac{1}{f_{p4}} = \frac{1}{2 \pi C_4 (R_3 + R_4)} \approx \frac{1}{2 \pi C_4 R_4} \]

= 1.59 MHz

High freq cutoff is lower of \( f_{p3} + f_{p4} \)

\[ f_H = 1.59 \text{ MHz} \]
HOW ACCURATE ARE POLE ESTIMATES FOR OPEN-CIRCUIT TIME CONSTANT (OTC)?

Consider:

\[ \frac{V_o}{V_i} = \frac{1}{s^2(C_1 R_1 R_2) + s(C_2 (R_1 + R_2) + C_1 R_1) + 1} \]

 CAN SHOW

Using open-circuit time constant method

\[ \tau_1 = C_1 R_1 \quad \tau_2 = C_2 (R_1 + R_2) \]

NOTE

\[ \frac{V_o}{V_i} = \frac{1}{s^2(C_1 R_1 R_2) + s(C_1 + C_2) + 1} \]

SUM OF OPEN-CIRCUIT TIME-CONSTANTS
If \( T(s) \) expressed as \((\text{real poles \& zeros})\):

\[
T(s) = \frac{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2}) \cdots (1 + \frac{s}{\omega_m})}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}}) \cdots (1 + \frac{s}{\omega_{pn}})}
\]

\[
= \frac{1 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n}
\]

Then \( b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn}} \)

And it can be shown that

\[
b_1 = \sum_{i=1}^{n} C_i R_i
\]

Sum of open circuit-time constants

If zeros not dominant + one-pole dominant then

\[
b_1 \approx \frac{1}{\omega_{p1}}
\]
**Examples**

\[ C_1 = C_2 = 1.59 \, \text{pF} \]

**Ex 1**  \( R_1 = 1 \, \text{k} \quad R_2 = 100 \, \text{k} \)

**Actual** => \( f_p_2 = 990 \, \text{kHz} \quad f_p_1 = 101 \, \text{MHz} \)

**OTC** => \( f_{p_2} = 990 \, \text{kHz} \quad f_{p_1} = 100 \, \text{MHz} \)

**Ex 2**  \( R_1 = 10 \, \text{k} \quad R_2 = 100 \, \text{k} \)

**Actual** => \( f_{p_2} = 901 \, \text{kHz} \quad f_{p_1} = 11.1 \, \text{MHz} \)

**OTC** => \( f_{p_2} = 909 \, \text{kHz} \quad f_{p_1} = 10 \, \text{MHz} \)

**Ex 3**  \( R_1 = 30 \, \text{k} \quad R_2 = 100 \, \text{k} \)

**Actual** => \( f_{p_2} = 723 \, \text{kHz} \quad f_{p_1} = 4.61 \, \text{MHz} \)

**OTC** => \( f_{p_2} = 769 \, \text{kHz} \quad f_{p_1} = 3.33 \, \text{MHz} \)
COMMON SOURCE AMP

ALL $C_1$, $C_2$, $C_3$ ARE LOW FREQ COUPLING & BYPASS
SO SHORT THEM FOR HIGH FREQ ANALYSIS

Midband Gain
$$\frac{V_o}{V_{ip}} = -g_m \left( \frac{1}{R_L || R_0} \right)$$
$$\approx -g_m R_L'$$

LET $R_L' = \frac{V_{il} R_L}{R_0}$
Using Miller Theorem

\[ C_{m1} = C_{gd} \left( 1 + \frac{q_m R_L'}{q_m R_L'} \right) \]

\[ C_{m2} = C_{gd} \left( 1 + \frac{1}{q_m R_L'} \right) \approx C_{gd} \text{ if } q_m R_L' \gg 1 \]

So

\[ W_{p1} = \left[ (C_{gd} + C_{m1})(R_{516} || R_6) \right]^{-1} = 2 \pi f_{p1} \]

\[ W_{p2} = \left[ (C_{gd} + C_{L} + C_{m1}) R_L' \right]^{-1} = 2 \pi f_{p2} \]

So

\[ V_{S16} \ll \frac{-q_m R_L'}{(1 + \frac{1}{W_{p1}})(1 + \frac{1}{W_{p2}})} \]

Dominant \( f_H \) estimate

\[ f_H \approx f_{p1} \text{ or } f_{p2} \text{ whichever is lower } \]
OTC \( f_H \) ESTIMATE

\[
f_H = \left( \frac{1}{f_{P1}} + \frac{1}{f_{P2}} \right)^{-1} = \frac{f_{P1}}{f_{P2}}
\]

A mathematical parallel

Often \( W_{P1} \) is low due to Miller effect on \( C_{gd} \)

Can keep \( W_{P1} \) high if \( R_{S16} \) is small

EX: let \( R_{S16} = 0 \)

Find \( f_H = f_T \) (unity gain freq)

Assuming \( g_m R_L \gg 1 \)

\[
f_H = \frac{W_{P2}}{2\pi} = \frac{1}{2\pi (C_{d6} + C_L + C_{gd}) R_L}
\]

And low freq gain \( A_m = -g_m R_L \)

\[
f_T = |A_m| f_H \quad \text{[See single time constant pg 13]}
\]

\[
f_T = \frac{g_m}{2\pi (C_{d6} + C_L + C_{gd})}
\]
Gain (dB)

\[ \leq 20 \log |\lambda M| \]

\[ R_L'' \leq \text{CHANGING } R_L' \]

\[ f_H \text{ but } f_L \text{ unchanged} \]

\[ \frac{1}{(C_{db} + C_L + C_{gd}) R_L'} \]

\[ = \frac{\frac{Q_{max}}{(C_{db} + C_L + C_{gd})}}{\text{log } w} \]
CASCODE AMP

CONSIDER

Common Source (CS)

\[ V_{in} \quad \text{V}_{o} \quad \text{IMPEDANCE} \quad \text{V}_{o} \]

\[ \text{CAP} \quad \text{FREE} \quad \text{TRANSISTOR} \]

\[ \text{DC GAIN} \Rightarrow -g_{m} \left( \frac{r_{o}}{2} \right) \]

\[ f_{3dB} \Rightarrow \frac{1}{2 \pi \text{C} \left( \frac{r_{o}}{2} \right)} \]

\[ f_{T} \Rightarrow \frac{g_{m}}{2 \pi \text{C} \text{L}} \]
CASCODE AMP

IMPEDEANCE \(= qm r_0^2\)

\[ gm = qm = q_2 \]

\[ V_i \]

\[ V_{B2} \]

\[ M_2 \]

\[ R_0 \]

\[ V_o \]

\[ CL \]

\[ R_0 \approx qm r_0^2 \]

DC GAIN \(=\) \(-\frac{gm^2 r_0^2}{2}\)

\[ f_{3dB} \Rightarrow \frac{1}{2\pi CL \left( \frac{r_0}{2} \right) (qm r_0)} \]

\[ f_t \Rightarrow \frac{gm}{2\pi CL} \]

GAIN dB

CASCODE

CS

\[ W_{3dB} (CASCODE) \]

\[ W_{3dB} (CS) \]

\[ \text{LOG } W \]
CASE CODE AMP

IF OUTPUT CAPACITANCE LOAD DETERMINES \( f_H \) THEN
CASE CODE AMP HAS SAME \( f_T \)
AS COMMON-SOURCE AMP
BUT HIGHER DC GAIN
(AND LOWER \( f_H \))
Other caps in cascode amp

- Typically gain $\frac{V_i}{V_{gs1}}$ not large

So no large Miller effect on $C_{gd1}$.

- There is Miller effect on $C_{gd2}$ but since $V_{gs2}$ not in signal path does not affect poles of system.
So approx circuit is

\[ V_{S16} \]

\[ C_{g1} + (1 - \frac{1}{k}) C_{gd1} = C_{g1} \]

\[ C_{L'} = C_{gd2} + C_{db2} + C_L \]

\[ Req \]

\[ C_{g1} \Rightarrow R_{S16} \]

\[ C_{D1} \Rightarrow \frac{1}{gm^2} \quad \text{depends on } R_L \]

\[ C_{L'} \Rightarrow R_L || (gm^2 R_0^2) \quad \text{(usually dominates)} \]
**Source Follower**

\[ V_{0d} \]

\[ R_{S16} \]

\[ V_{S16} \]

\[ C_{gd} \]

\[ R_{L} \]

\[ C_{L} \]

\[ V_{o} \]

\[ R_{L'} \]

\[ V_{q} \]

\[ C_{qp} \]

\[ R_{o} \]

\[ V_{0} \]

Let \( R_{L'} = R_{L} || R_{o} \)

\[ \frac{V_{o}}{V_{S16}} = A_{m} = \frac{R_{L'}}{R_{L'} + r_{s}} \]

\[ r_{s} = \frac{1}{q_{m}} \]

\[ R_{o} = r_{s} || R_{o} \]
3 POLES \( C_{gd}, C_{g2}, C_L \)

\[ f_{p1} = \frac{1}{2\pi \cdot C_{gd} \cdot R_{516}} \]

\[ f_{p2} = \frac{1}{2\pi \cdot (C_L)(R_L || R_0 || R_S)} \approx \frac{1}{2\pi (C_L)(R_S)} \]

\[ f_{p3} = \frac{1}{2\pi \cdot C_{g2} \cdot (R_{pg})} \]

WHERE

\[ R_{pg} = \frac{R_{516} + R_{L}'}{1 + q_0 \cdot R_{L}'} \]

(REQUIRES ANALYSIS)

ALL 3 POLES USUALLY HIGHER THAN IN COMMON-SOURCE CASE.