8.1
(a) Referring to Fig. 8.2,
note that \( I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.2 \text{ mA}}{2} \)
\[ = 0.1 \text{ mA} \]
\[ I_D = \frac{1}{2} k_i \left( \frac{W}{L} \right) V_O^2 \]
\[ V_{ov} = \sqrt{\frac{2 I_D}{\eta k_i (W/L)}} = \sqrt{\frac{2(0.1 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} \]
\[ = 0.2 \text{ V} \]
\[ V_{G1} = V_{m1} + V_{ov} = 0.5 + 0.2 = 0.7 \text{ V} \]
(b) If \( V_{cm} = 0 \),
\[ V_{s1} = V_{s2} = V_G - V_{GS} = 0 - 0.7 = -0.7 \text{ V} \]
\[ I_{D1} = I_{D2} = \frac{0.2 \text{ mA}}{2} = 0.1 \text{ mA} \]
\[ V_{D1} = V_{D2} = V_{DD} - I_{D1} R_D \]
\[ = 1 \text{ V} - (0.1 \text{ mA})(10 \text{ K}) = 0 \text{ V} \]
(c) Now, if \( V_{cm} = 0.1 \text{ V} \),
\[ V_{S1} = V_{S2} = V_G - V_{GS} = 0.1 \text{ V} - 0.7 \text{ V} \]
\[ = -0.6 \text{ V} \]
Since \( I \) is a constant current source, 
\( I_{D1} \) and \( I_{D2} \) remain at 0.1 mA
This means that 
\( V_{D1} \) and \( V_{D2} \) are still 0 V
(d) \( V_{cm} = -0.1 \text{ V} \),
\[ V_{S1} = V_{S2} = V_G - V_{GS} = -0.1 \text{ V} - 0.7 \text{ V} \]
\[ = -0.8 \text{ V} \]
Still, \( I_{D1} = I_{D2} = 0.1 \text{ mA} \)
\[ V_{D1} = V_{D2} = 0 \text{ V} \]
(e) \( V_{cm} \) (max) = \( V_{DD} - I_{D1} R_D - V_{ov} + V_{GS} \)
\[ = 1 - (0.1 \text{ mA})(10 \text{ K}) - 0.2 = 0.7 = +0.5 \text{ V} \]
(f) \( V_{S} \) (min) = \( -0.1 + 0.7 = -0.8 \text{ V} \)
\[ V_{cm} \) (min) = \( V_{S} \) (min) + \( V_{GS} \)
\[ = -0.8 + 0.7 = -0.1 \text{ V} \]

8.2
\[ V_{ip} = -0.8 \text{ V} \ k_i \left( \frac{W}{L} \right) = 4 \text{ mA/} \sqrt{V^2} \]
(a) \( V_{G1} = V_{G2} = 0 \text{ V} \)
\[ |V_{ov}| = \sqrt{0.5 \text{ mA/} \sqrt{V^2}} = 0.354 \text{ V} \]
\[ |V_{GS}| = |V_{ip}| + |V_{ov}| = 1.154 \text{ V} \]
\[ V_S = V_{G1} + |V_{GS}| = 1.154 \text{ V} \]
\[ V_{D1} = V_{D2} = -0.5 \text{ V} + \left( \frac{0.5 \text{ mA}}{2} \right)(4 \text{ k} \Omega) \]
\[ = -1.5 \text{ V} \]
(b) Current source requires 0.5 V
\[ V_{cm} \) (max) = \( 2.5 \text{ V} - 0.5 \text{ V} - 0.8 \text{ V} - 0.354 \text{ V} \]
\[ = 0.846 \text{ V} \]
\[ V_{cm} \) (min) = \( -2.5 \text{ V} + \left( \frac{0.5 \text{ mA}}{2} \right)(4 \text{ k} \Omega) - 0.8 \text{ V} \]
\[ = -2.3 \text{ V} \]

8.3
Refer to Fig. 8.2
\[ V_{OV} = \sqrt{\frac{2 I_D}{\eta k_i (W/L)}} = \sqrt{\frac{2(0.1 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} \]
\[ = 0.2 \text{ V} \]
(a) \( V_{GS} = V_{OV} + V_t = 0.2 \text{ V} + 0.5 \text{ V} = 0.7 \text{ V} \)
\[ V_S = V_G - V_{GS} = 0 - 0.7 \text{ V} = -0.7 \text{ V} \]
\[ V_{D1} = V_{D2} = V_{DD} - i_{D1} R_D = 1.0 \text{ V} - 0.1 \text{ mA} \]
\[ (10 \text{ k} \Omega) = 0 \text{ V} \]
\[ V_{D2} - V_{D1} = 0 \text{ V} \]
(b) For \( i_{D1} = 0.15 \text{ mA} \), \( i_{D2} = 0.05 \text{ mA} \),
\[ i_{D1} = \frac{I}{2} \frac{V_{OV}}{V_{OV}} \quad v_{id} = \left[ \frac{2i_{D1}}{I} - 1 \right] V_{OV} \]
\[ v_{id} = \left[ \frac{2(0.15 \text{ mA})}{0.2 \text{ V}} - 1 \right] 0.2 \text{ V} = 0.1 \text{ V} \]
\[ V_{GS1} = \sqrt{\frac{2(0.15 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} + 0.5 \text{ V} = 0.745 \text{ V} \]
\[ V_S = V_G - V_{GS1} = 0.1 \text{ V} - 0.745 \text{ V} = -0.645 \text{ V} \]
\[ V_{D1} = V_{DD} - i_{D1} R_D = 1.0 \text{ V} - 0.15 \text{ mA} (10 \text{ k} \Omega) \]
\[ = -0.5 \text{ V} \]
\[ V_{D2} = 1.0 \text{ V} - (0.05 \text{ mA})(10 \text{ k} \Omega) = 0.5 \text{ V} \]
\[ V_{D2} - V_{D1} = 1.0 \text{ V} \]
(c) \( i_{D1} = 0.2 \text{ mA} \), \( i_{D2} = 0 \):
\[ V_{G1} = v_{id} = \sqrt{2} \cdot V_{OV} = 1.414(0.2 \text{ V}) = 0.283 \text{ V} \]
\[ V_{GS} = \sqrt{\frac{2(0.2 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} + 0.5 \text{ V} = 0.783 \text{ V} \]
\[ V_S = V_G - V_{GS} = 0.283 \text{ V} - 0.783 \text{ V} = -0.5 \text{ V} \]
\[ V_{D1} = 1.0 \text{ V} - (0.2 \text{ mA})(10 \text{ k} \Omega) = -1.0 \text{ V} \]
\[ V_{D2} = +1.0 \text{ V} \]
\[ V_{D2} - V_{D1} = 2.0 \text{ V} \]
\[ i_{D1} = 0.05 \text{ mA} \], \( i_{D2} = 0.05 \text{ mA} \)
\[ i_{D1} = 0.05 \text{ mA} \]
\[ i_{D2} = 0.05 \text{ mA} \]
\[ V_{GS} = \sqrt{\frac{2(0.05 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} + 0.5 \text{ V} = 0.641 \text{ V} \]
\[ V_S = V_G - V_{GS} = -0.1 \text{ V} - 0.641 \text{ V} = -0.741 \text{ V} \]
For example,
\[ v_{id} = \left[ \frac{2(0.05 \text{ mA})}{0.2 \text{ V}} - 1 \right] (0.2 \text{ V}) = -0.1 \text{ V} \]
\[ V_{GS} = \sqrt{\frac{2(0.05 \text{ mA})}{0.4 \text{ mA/} \sqrt{V^2}}(12.5)} + 0.5 \text{ V} = 0.641 \text{ V} \]
\[ V_S = V_G - V_{GS} = -0.1 \text{ V} - 0.641 \text{ V} = -0.741 \text{ V} \]


\[ V_{D1} = 1.0 \text{ V} - (0.05 \text{ mA})(10 \text{ k}\Omega) = +0.5 \text{ V} \]
\[ V_{D2} = 1.0 \text{ V} - (0.05 \text{ mA})(10 \text{ k}\Omega) = -0.5 \text{ V} \]
\[ V_{D2} - V_{D1} = -1.0 \text{ V} \]

(c) \( i_{D1} = 0 \text{ mA}, \ i_{D2} = 0.2 \text{ mA} \) is the opposite of (b):
\[ u_{id} = -\sqrt{2}(V_{OV}) = -\sqrt{2}(0.2 \text{ V}) = -0.283 \text{ V} \]

For \( i_{D2} = 0.2 \text{ mA}, \ V_{GS2} = 0.783 \text{ V}, \) So that
\[ V_s = -0.783 \text{ V} \]
\[ V_{D1} = 1.0 \text{ V} \]
\[ V_{D2} = -1.0 \text{ V} \rightarrow V_{D2} - V_{D1} = -2 \text{ V} \]

The results are summarized in the following table:

<table>
<thead>
<tr>
<th>Case</th>
<th>( V_{id} ) (V)</th>
<th>( i_{D1} ) (mA)</th>
<th>( i_{D2} ) (mA)</th>
<th>( V_s ) (V)</th>
<th>( V_{D1} ) (V)</th>
<th>( V_{D2} ) (V)</th>
<th>( V_{D2} - V_{D1} ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0.1</td>
<td>0.15</td>
<td>0.05</td>
<td>-0.5</td>
<td>0.645</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>(c)</td>
<td>0.283</td>
<td>0.2</td>
<td>0</td>
<td>-0.5</td>
<td>-1.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>(d)</td>
<td>-0.1</td>
<td>0.05</td>
<td>0.15</td>
<td>-0.5</td>
<td>-0.741</td>
<td>0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>(e)</td>
<td>-0.283</td>
<td>0.2</td>
<td>0</td>
<td>-0.783</td>
<td>1.0</td>
<td>-1.0</td>
<td>-2.0</td>
</tr>
</tbody>
</table>

\[ V_{GS1} = V_t + \frac{2}{\sqrt{2}} \frac{k_{pW}}{L} \]

\[ = V_t + \sqrt{2} V_{OV} \]

and \( V_{sat} \) is reduced to \( V_t \) thus \( V_s = -V_t \).

Then \( u_{id} = u_{GS1} + u_s \)

\[ = V_t + \sqrt{2} V_{OV} - V_t = \sqrt{2} V_{OV} \]

In a similar manner as for the NMOS Differential Amplifier, as \( v_t \) reaches \( -\sqrt{2} V_{OV} \) \( Q_1 \) turns off and \( Q_2 \) on. Thus the steering range is

\[ \sqrt{2} V_{OV} \leq V_t \leq -\sqrt{2} V_{OV} \]

For this particular case

\[ V_{OV} = \frac{0.25 \text{ mA}}{\sqrt{4 \text{ mA}/\text{V}^2}} = 0.25 \text{ V} \]

\[ \sqrt{2} \leq 0.25 \leq -\sqrt{2} \times 0.25 \]

\[-0.35 \leq u_{id} \leq 0.35 \]

when \( V_{id} = 0.35 \text{ V}, \)

\[ i_{D1} = 0.5 \text{ mA}, \ i_{D2} = 0 \]

\[ V_s = -V_{GS} = +0.8 \text{ V} \]

\[ V_{D1} = 4 \text{ k}\Omega \times 0.5 \text{ mA} - 2.5 = -0.5 \text{ V} \]

\[ V_{D2} = -2.5 \text{ V} = -2.5 \text{ V} \]

when \( V_{id} = +0.35 \text{ V}, \)

\[ i_{D1} = 0 \text{ mA}, \ i_{D2} = 0.5 \text{ mA} \]

\[ V_s = u_{id} - u_{GS1} = u_{id} - V_t \]

\[ = 0.35 \text{ V} + 0.8 \text{ V} = 1.15 \text{ V} \]

\[ V_{D1} = -2.5 \text{ V} \]

\[ V_{D2} = -0.5 \text{ V} \]

8.4

\[ V_{GS1} = u_{id} \]

\[ i_{D1} = 0.11 \text{ mA} \]

\[ V_{GS2} = 0 \text{ mA} \]

\[ i_{D2} = 0.09 \text{ mA} \]

\[ I_D = \frac{1}{2} k' \frac{W}{n L} (V_{GS} - V_t)^2 \]

For \( Q_1: \)

\[ 0.11 m = \frac{1}{2} \frac{3}{5} m (V_{GS1} - 0.5)^2 \]

\[ \rightarrow V_{GS1} = 0.71 \text{ V} \]

For \( Q_2: \)

\[ 0.09 m = \frac{1}{2} \frac{3}{5} m (V_{GS2} - 0.5)^2 \]

\[ \rightarrow V_{GS2} = 0.69 \text{ V} \]

\[ V_s = -V_{GS2} = -0.69 \text{ V} \]

\[ u_{id} = V_s + V_{GS1} = -0.69 + 0.71 = 0.02 \text{ V} \]
\( V_{DS} - V_{D1} = 10 \, k\Omega \) \((i_{D1} - i_{D2})\)
\[= 10 \, kV \,(0.11 \, 0.09) \, m \]
\[= 0.2 \, V \]
thus
\[ V_{DS} - V_{D1} = \frac{0.2}{0.02} = 10 \]
when \(i_{D1} = 0.09 \, mA\) and
\(i_{D2} = 0.11 \, mA\)
is the reverse condition from the case we just studied, thus \(v_{id} = -0.02 \, V\)

8.6
\( V_{GS} = V_{n} + V_{OV} = 0.5 \, V + 0.2 \, V = 0.7 \, V \)
\( V_{D4} = V_{G4} = -V_{SS} + V_{GS} = -1.2 \, V + 0.7 \, V \)
\[= -0.5 \, V \]
\( R = \frac{V_{DD} - V_{D4}}{0.1 \, mA} = \frac{1.2 \, V - (-0.5 \, V)}{0.1 \, mA} = 17 \, k\Omega \)
\( R_{D} = \frac{V_{DD} - V_{D1}}{0.4 \, mA} = \frac{1.2 \, V - 0.2 \, V}{0.4 \, mA} = 5 \, k\Omega \)
\( \left( \frac{W}{L} \right)_{1} = \left( \frac{W}{L} \right)_{2} = \frac{0.4 \, mA}{2} = \frac{0.4 \, mA}{2} \, k \, \left[ \frac{m V}{V} \right] \)
\[= 0.2 \, mA \, (0.25 \, mA \, V^2)(0.2 \, V)^2 = 20 \]
\( \left( \frac{W}{L} \right)_{3} = 0.4 \, mA \, (0.01 \, mA)^{-1} = 40 \)
\( \left( \frac{W}{L} \right)_{4} = 0.1 \, mA \, (0.01 \, mA)^{-1} = 10 \)
\( V_{CS} = V_{n} + V_{DD} - (1/2)R_{D} \)
\[= 0.5 \, V + 1.2 \, V - 0.2 \, mA \, (5 \, k\Omega) = 0.7 \, V \]
\( V_{CM} = V_{SS} + V_{OV3} + V_{OV4} \)
\[= -1.2 \, V + 0.2 \, V + 0.5 \, V + 0.2 \, V = -0.3 \, V \]

8.7
\[ |V_{idmax}| = 160 \, mV \]
\[ \left( \frac{V_{id}/2}{V_{OV}} \right)^2 = 0.1 \]
\( V_{OV} = \left( \frac{80 \, mV}{0.1} \right)^2 = 253 \, mV \)
\[ I = 0.4 \, mA \]
\( k'_{n} = 0.2 \, mA \, V^2 \)
\( W = I \left[ \frac{k'_{n} V^2}{V_{OV}} \right]^{-1} \)
\[= 0.4 \, \left[ (0.2)(0.253)^2 \right]^{-1} = 31.2 \]
\( g_{m} = \frac{I}{V_{OV}} = \frac{0.4 \, mA}{0.253 \, V} = 1.58 \, mA \, V \)

8.8
\[ \left( \frac{v_{idmax}/2}{V_{OV}} \right)^2 = K \]
\[ \Rightarrow 2V_{OV} \sqrt{K} = v_{idmax} \]
Q.E.D.
\[ i_{D1} = \frac{l}{2} + \left( \frac{l}{V_{OV}} \right) v_{id} \sqrt{1 - K} \]
\[ i_{D1} = \frac{l}{2} + \frac{l}{V_{OV}} \cdot \frac{2}{2} V_{OV} \sqrt{K} \cdot \sqrt{1 - K} \]
\[ \Rightarrow i_{D1} = \frac{l}{2} + l \sqrt{K} \left( 1 - K \right) \]
thus \( \Delta I = 2l \sqrt{K} \left( 1 - K \right) \)
Q.E.D.
For \( K = 0.01 \)
\[ \Delta I = 2 \sqrt{0.001} \left( 1 - 0.01 \right) \]
\[= 0.198 \times l \]
\( v_{idmax} = 2V_{OV}\sqrt{K} = 0.2 \, V_{OV} \)
For \( K = 0.1 \)
\[ \Delta I = 2 l \sqrt{0.1} (1 - 0.1) = 0.8 l \]
\( v_{idmax} = 2V_{OV}\sqrt{0.2} = 0.894 \, V_{OV} \)

8.9
\( \frac{1}{l} = \frac{1}{2} \mu_{N} C_{ov} \left( \frac{W}{L} \right) (V_{GS} - V_{rn})^2 \)
\[= \frac{1}{2} \left( 200 \right) \left( \frac{20}{0.5} \right) (V_{GS} - 0.5)^2 \]
\[= (V_{GS} - 0.5)^2 = 1/20 \, V \]
\( V_{GS} = 0.724 \, V \)
\( g_{m} = \frac{2l}{V_{GS} - V_{rn}} = \frac{2 \times 400 \, \mu A}{0.724 \, V - 0.5 \, V} \)
\[= 3.57 \, mA \, V^{-1} \]
\( V_{id} \) for full current switching
\[= \sqrt{2} (V_{GS} - V_{rn}) = 0.317 \, V \]
To double this value, \( V_{OV} \), so quadruple \( I_{D} \) to 1.6 mA

8.10
\( V_{OV} = 0.25V \, g_{m} = 1 \, mA \, V / V_{rn} = 0.8 \, V \)
\[ k_{n} = 100 \, \mu A / V^2 \]
\[ I = g_{m} V_{OV} = 0.25 \, mA \]
\[ I_{D} = \frac{l}{2} = \frac{1}{2} k'_{n} \left( \frac{W}{L} \right) V_{OV}^2 \]
\[ W = \frac{l}{l} \left( \frac{k'_{n} V^2}{V_{OV}} \right) \]
\[= 0.25 \, mA / (0.1 \, mA / V^2 \cdot (0.25 \, V)^2) \]
\[= 0.25 \, mA / 0.00625 \, mA = 40 \]
8.11
\[ i_D = \frac{1}{2} k_s \frac{W}{L} (V_{GS} - V_s)^2 \]

50 = \frac{1}{2} \times 400(V_{GS} - 1)^2

\[ \Rightarrow V_{GS} = 1.5 \text{ V} \]

For \( V_{G1} = V_{G2} = 0, V_s = -1.5 \text{ V} \)

For \( V_{G1} = V_{G2} = 2 \text{ V}, V_s = +0.5 \text{ V} \)

The drain currents are equal in both cases.

For \( V_{G2} = 0 \):

To reduce \( i_{D2} \) by 10%,

\[ i_{D2} = 0.9 \times 50 = 45 \mu \text{A} \]

\( i_{D1} = 55 \mu \text{A} \)

\[ V_{GS2} = \frac{2 i_{D2}}{400} + 1 = 1.47 \text{ V} \]

\[ V_{GS1} = \frac{2 \times 55}{400} + 1 = 1.52 \text{ V} \]

Thus, \( V_{G1} = V_{GS1} - V_{GS2} = 0.05 \text{ V} \)

To increase \( i_{D2} \) by 10%

\( i_{D2} = 55 \mu \text{A} \)

\( i_{D1} = 45 \mu \text{A} \)

\[ V_{GS2} = 1.52 \text{ V} \]

\[ V_{GS1} = 1.47 \text{ V} \]

\[ \Rightarrow V_{G1} = -0.05 \text{ V} \]

\[ \frac{i_{D2}}{i_{D1}} \quad \frac{i_{D2}}{i_{D1}} \quad V_{GS2} \quad V_{GS1} \quad V_G - V_{G1} \]

\( (\mu \text{A}) \quad (\mu \text{A}) \quad (\text{V}) \quad (\text{V}) \quad (\text{V}) \)

\[
\begin{array}{cccc}
1 & 50 & 50 & 1.5 \\
0.5 & 33.3 & 66.7 & 1.408 \\
0.8 & 47.4 & 52.6 & 1.487 \\
0.99 & 47.75 & 50.25 & 1.4886 \\
\end{array}
\]

For \( i_{D1} / i_{D2} = 20 \Rightarrow i_{D2} = 4.76 \mu \text{A} \)

\( i_{D1} = 95.24 \mu \text{A} \)

\( V_{GS2} = 1.154 \text{ V}, \quad V_{GS1} = 1.690 \)

Thus \( V_G - V_{G2} = 0.536 \text{ V} \)

8.12

(a) \( V_{sd} = V_{D2} - V_{D1} = \)

\[ (V_{DD} - i_{D1} R_D) - (V_{DD} - i_{D2} R_D) = (i_{D1} - i_{D2}) R_D \]

USING 8.23 AND 8.24

\[ V_{sd} = \left( \frac{i}{V_{OV}} \right) \left( \frac{V_{id}}{2} \right) \left[ 1 - \left( \frac{V_{id} / 2}{V_{OV}} \right)^2 \right] \]

\[ + \left( \frac{1 - \left( \frac{V_{id} / 2}{V_{OV}} \right)^2}{V_{OV}} \right) R_D \]

\[ = 1 R_D \frac{V_{id}}{V_{OV}} \left[ 1 - \left( \frac{V_{id} / 2}{V_{OV}} \right)^2 \right] \]

(b) see plot

slope of linear portion

\[ = \frac{d}{dV_{id}} \left( \frac{1 R_D}{V_{OV}} V_{id} \right) = 1 R_D / V_{OV} \]

(c) see plot

when the bias current is doubled, \( V_{OV} \) so

\[ V_{sd} / V_{id} = \frac{2}{\sqrt{2} V_{OV}} \left[ 1 - \left( \frac{V_{id} / 2}{V_{OV}} \right)^2 \right] \]

increases by a factor of \( \sqrt{2} \) the slope of the linear part has increased by a factor of \( \sqrt{2} \)

(d) see plot

If \( W/L \) is doubled, \( V_{OV} \) reduces by a factor at \( \sqrt{2} \)

so \[ V_{sd} / V_{id} = \frac{2}{\sqrt{2} V_{OV}} \left[ 1 - \left( \frac{V_{id} / 2}{V_{OV}} \right)^2 \right] \]

The slope of the linear part has increased by factor of \( \sqrt{2} \) compared to (b)

8.13

\[ I = 0.4 \text{ mA} \quad W/L = 32 \quad k_s' = \mu_n C_{ox} \]

\[ = 200 \mu \text{A} / \text{V}^2 \]

\[ V_A = 10 \text{ V} \quad R_D = 5 \text{ k\Omega} \]

\[ V_{OV} = \sqrt{\frac{1}{k_s' \frac{W}{L}}} = 0.4 / (0.2 \cdot 32) \]

\[ = 0.25 \text{ V} \]

\[ g_m = \frac{V}{V_{OV}} = 0.4 \text{ mA} / 0.25 \text{ V} = 1.6 \text{ mA} / \text{V} \]

\[ r_o = \frac{V}{I_D} = \frac{10 \text{ V}}{0.2 \text{ mA}} = 50 \text{ k\Omega} \]

\[ A = g_m (R_D || r_o) = 1.6 (5 || 50) \]

\[ = 1.5 (4.54) = 7.3 \text{ V} / \text{V} \]

8.14

\[ \left( \frac{V_{id} / 2}{V_{OV}} \right)^2 = 0.05 \]

\[ \left( \frac{0.1 / 2}{V_{OV}} \right) = \sqrt{0.05} \]

\[ V_{OV} = \frac{0.05}{\sqrt{0.05}} = 0.224 \text{ V} \]

\[ g_m = \frac{V}{V_{OV}} \]
The figure belongs to 8.12

\[ I = g_m V_{ov} = (1 \text{ mA/V})(0.224 \text{ V}) \]
\[ = 0.224 \text{ mA} \]

\[ A_d = g_m R_D = (1 \text{ mA/V})(10 \text{ kΩ}) = 10 \]

\[ V_{ad} = A_d V_{id} = (10)(0.1 \text{ V}) = 1 \text{ V} \]

\[ \frac{W}{L} = \frac{1}{k_n V_{ov}^2} \]
\[ = 0.224 / (0.2 \times 0.224^2) \]
\[ = 22.3 \]

8.15

+1 V supplies not more than 2 mW, \( V_n = 5 \text{ V/V} \)

\[ I = \frac{2 \text{ mW}}{1 \text{ V} - (-1 \text{ V})} = 1 \text{ mA} \]

\[ R_D = \frac{1 \text{ V} - 0.5 \text{ V}}{1/2 I} = \frac{0.5 \text{ V}}{0.5 \text{ mA}} = 1 \text{kΩ} \]

\[ g_n = \frac{A_d}{R_D} = \frac{5 \text{ V/V}}{1 \text{kΩ}} = 5 \text{ mA/V} \]

\[ V_{ov} = \frac{I}{g_n} = \frac{1 \text{ mA}}{5 \text{ mA/V}} = 0.2 \text{ V} \]

\[ \frac{W}{L} = \frac{2(I/2)}{(k_n' V_{ov}^2)} \]
\[ = 1 \text{ mA} / (0.4 \text{ mA/V}^2 \cdot (0.2 \text{ V})^2) = 62.5 \]

Because we picked \( I = 1 \text{ mA} \) this is the solution with the highest allowable power. This solution will also therefore have the widest range of differential mode operation. An infinite number of other solutions exist.

8.16

\[ I = \frac{2 \text{ mW}}{1 \text{ V} - (-1 \text{ V})} = 1 \text{ mA} \]

\[ 0.4 \text{ V} = 2\sqrt{2} V_{ov} \]
\[ V_{ov} = 0.141 \text{ V} \]

\[ R_D = A_d \frac{V_{ov}}{I} = 5 \sqrt{\frac{0.141 \text{ V}}{1 \text{ mA}}} = 705 \text{ Ω} \]

\[ \frac{W}{L} = \frac{1}{(k_n' V_{ov}^{2})} = 125 \]
since \( A_d = g_{m1.2}(r_{O} \parallel R_D) \),
\[
A_d = g_{m1.2} \left[ R_{O1.2} \parallel \left( \frac{r_{O3.4}}{1 + \frac{g_{m3.4}R_{O3.4}}} \right) \right]
\]

(b) If \( r_o \) is ignored,
\[
R_D' = \frac{1}{1 + g_m} \simeq \frac{1}{r_o}
\]
so that
\[
A_d = g_{m1.2} \left( \frac{1}{\frac{g_{m3.4}}{r_{O}}} \right)
\]
since \( g_m = \sqrt{2} \mu C_{OX} (W/L) I_D \),
\[
A_d = \frac{g_{m1.2}}{g_{m3.4}} = \frac{\sqrt{2} \mu_n C_{OX} (W/L)_{1.2} I_D}{\sqrt{2} \mu_p C_{OX} (W/L)_{3.4} I_D} = \frac{\mu_n}{\mu_p} \left( \frac{W/L}_{1.2} \right) \left( \frac{W/L}_{3.4} \right)
\]
(c) If \( \mu_n = 4 \mu_p \) and \( L_1 = L_2 = L_3 = L_d = L \),
\[
A_d = 10 \text{ V/V} = \frac{\sqrt{2} \mu_n}{\sqrt{2} \mu_p} \left( \frac{W/L}_{1.2} \right) \left( \frac{W/L}_{3.4} \right)
\]
\[
\frac{10}{2} = \frac{\sqrt{W_{1.2}}}{\sqrt{W_{3.4}}} = 25
\]

8.21
HALF-CIRCUIT

\[
V_{GD} \quad R_D \quad V_{id} \quad Q \quad V_{ov} \quad R_S \quad \frac{L}{2}
\]
small-signal analysis
\[
V_{gs} = \frac{V_{id}}{2} - g_m V_{gs} R_S \frac{L}{2}
\]
\[
V_{gs} = \frac{V_{id}/2}{1 + g_m R_S \frac{L}{2}}
\]
\[
V_{D1} = -g_m V_{gs} R_D = -g_m \left[ \frac{V_{id}/2}{1 + g_m R_S \frac{L}{2}} \right] R_D
\]
\[
A_d = \frac{V_{id}}{V_{id}} = \frac{g_m R_D}{1 + g_m R_S \frac{L}{2}}
\]
when \( R_S = 0 \) \( A_d = g_m R_D \) (agrees with Eqn. 8.35)
when \( R_S = \frac{L}{2} \) the differential gain is reduced by half

8.22
(a) \( V_{G1} = V_{G2} = OV \)
\[
V_{S1} = V_{S2} \quad \text{assuming matching components}
\]
\[
V_{S1} = V_{G1} - V_{GS1} = OV - (V_t + V_{OV})
\]
\[
= -(V_t + V_{OV})
\]
(b) zero current flows through \( Q_3 \)
\[
V_{OV3} = V_C - V_{S1} - V_t = V_C - (-(V_t + V_{OV})) - V_t
\]
\[
= V_C + V_t = V_C + V_{OV}
\]
(c) \( V_{G1} = -V_{G2} = V_{id} / 2 \)
\[
V_{S1} \text{ is now more negative than in (a) and } V_{S2} \text{ is now less negative than in (a) so there is a voltage across } Q_3. \text{ If this voltage is small and if } V_C \text{ is such that } V_{GS3} > V_t \text{ then } Q_3 \text{ will operate in triode}.
\]
\[
r_{DS3} = \left[ k_n W_L V_{ov3} \right]^{-1}
\]
\[
g_m = \frac{1/2 k_n W_L V_{ov}^2}{V_{ov}} = 1/2 k_n W_L V_{ov}
\]
so \( r_{DS3} = \left[ g_m \right]^{-1} \frac{V_{ov}}{V_{ov3} g_m} \)

8.23
(a) \( V_{G1} = V_{G2} = 0V \)
\[
V_{S1} = V_{S2} = -(V_t + V_{OV})
\]
Zero current flows through \( Q_3 \) and \( Q_4 \)
\( Q_3 \) and \( Q_4 \) have the same overdrive voltage as \( Q_1 \) and \( Q_2 \)
\[
r_{DS3} = r_{DS4} = \left[ k_n \left( \frac{W}{L} \right)_{3,4} V_{OV3,4} \right]^{-1}
\]
\[
\begin{array}{cccccc}
| Q_1 | Q_2 | Q_3 | Q_4 | Q_5 | Q_6 | Q_7 | \\
<table>
<thead>
<tr>
<th></th>
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<td>74.1</td>
<td>12.3</td>
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<td>74.1</td>
<td>24.7</td>
</tr>
<tr>
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<td>-0.95</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>
\end{array}
\]

Since \( g_m = \frac{|I_D|}{|V_{OV}|/2} \),

\[
|V_{OV1}| = |V_{OV2}| = |V_{OV3}| = \frac{2I_D}{g_m} = \frac{2(50\ \mu A)}{400 \mu A/V} = 0.25 \text{ V}
\]

so,

\[
V_{G3} = V_{G2} + V_{sp} = -0.25 - 0.7 = -0.95 \text{ V}
\]

For \((W/L)\) ratios

\[
I_D = \frac{1}{2} \mu C_{ox} (\frac{W}{L}) (V_{OV})^2
\]

So that

\[
\frac{W}{L} = \frac{2I_D}{\mu C_{ox} V_{OV}^2}
\]

For \(Q_7\),

\[
\left(\frac{W}{L}\right)_7 = \frac{2(100\ \mu A)}{60 \mu A/V^2(0.3 \text{ V})^2} = 24.7
\]

For \(Q_4\) and \(Q_5\),

\[
\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = \frac{2(50\ \mu A)}{90 \mu A/V^2(0.3 \text{ V})^2} = 12.3
\]

For \(Q_1\) and \(Q_2\),

\[
\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2(50\ \mu A)}{30 \mu A/V^2(0.25 \text{ V})^2} = 53.3
\]

For \(Q_6\) and \(Q_3\),

\[
\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_3 = \frac{2(100\ \mu A)}{30 \mu A/V^2(0.3 \text{ V})^2} = 74.1
\]

In summary, the results are as follows:

\[8.25\]

(a) \(I_{D1} = \frac{1}{2} \frac{2}{V_{G31} - V_i}^2\)

\(I_{D2} = \frac{1}{2} \frac{2}{V_{G32} - V_i}^2\)

Since \(V_{GS} - V_i\) is equal for both transistors:

\[
\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2}; I_{D2} = 2I_{D1}
\]

but \(I = I_{D1} + I_{D2} = 3I_{D1}\)

(b) \(V_{OV} = V_{G5} - V_i\)

\[
V_{OV1} = V_{OV2} = V_{OV}
\]

For \(Q1\):

\[
\frac{I}{3} = \frac{1}{2} \frac{g_m}{V_{OV}} \frac{V_{OV}^2}{W/L}
\]

\[8.26\]

By equation 8.38 \(A_d = g_{m1} (R_m || R_{ep})\)

\[= g_{m1} [(g_{m3} r_{G3}) r_{O1} || (g_{m3} r_{G3}) r_{O2}] \]

If all transistors have the same channel length and the same \(|V_{OV}|\) and \(|V_A|\) Since \(g_m = \frac{2I_D}{V_{OV}}\) and

\[r_o = \frac{V_A}{I_D}\]

and with \(g_m\) and \(r_o\) the same for all devices,

\[
A_d = \frac{2I_D}{V_{OV}} \left(\frac{2I_D}{V_{OV}} \frac{V_A}{I_D}\right) \left(\frac{2I_D}{V_{OV}} \frac{V_A}{I_D}\right) I_D
\]

\[= \frac{2I_D^2}{V_{OV}^2 I_D} \frac{2V_A^2}{V_{OV}^2 I_D} \frac{2V_A^2}{V_{OV} I_D}\]
\[
\frac{2V_A^2}{V_{OV}^2} = 2 \left( \frac{V_A}{V_{OV}} \right)^2
\]
\[
A_d = 1000 \text{ V/V and } |V_{OV}| = 0.2 \text{ V}
\]
\[
1000 = 2 \left( \frac{V_A}{V_{OV}} \right)^2
\]
\[
V_A = \sqrt{500} \cdot 0.2 \text{ V} = 4.47 \text{ V}
\]
\[
\text{If } |V_A| = 10 \text{ V/μA}
\]
\[
L = \frac{4.47 \text{ V}}{10 \text{ V/μM}} = 0.447 \text{ μm}
\]
For high \(g_m\) the bias current should be high, but
with ±0.9 V Supplies the bias current must not
exceed 1 mW
\[
\text{1 mW} = 0.556 \text{ mA to keep power dissipation at 1 mW}
\]
\[8.27\]
\[
I = 0.2 \text{ mA, } R_{SS} = 100 \text{ kΩ, } R_D = 10 \text{ kΩ}
\]
\[
k' W W_n = \frac{3 \text{ mA}}{V^2} \text{ 1% mismatch in drain resistances}
\]
\[
V_{OV} = \sqrt{I/k' W W_n} = \sqrt{\frac{0.2 \text{ mA}}{3 \text{ mA/V}^2}} = 0.258 \text{ V}
\]
\[
g_m = \frac{I}{V_{OV}} = \frac{0.2 \text{ mA}}{0.258 \text{ V}} = 0.755 \text{ mA/V}
\]
Equation 8.35
\[
|A_d| = g_m R_D = 0.775 \text{ mA/V} \cdot 10 \text{ kΩ} = 7.75 \text{ V/V}
\]
Equation 8.49a \[|A_{CM}| = \frac{R_D}{2 R_{SS}} \cdot \frac{\Delta R_D}{R_D} = \frac{0.2}{2} \cdot \frac{2}{100} = 0.0005 \text{ V/V}
\]
\[8.28\]
\[
k' W W_n = 4 \text{ mA/V}^2 \text{ Current source resistance}
\]
\[
30 \text{ kΩ}
\]
\[
|V_{OV}| = \sqrt{I/k' W W_n} = \sqrt{0.5 \text{ mA} \cdot 4 \text{ mA/V}^2} = 0.353 \text{ V}
\]
\[
g_m = \frac{I}{|V_{OV}|} = \frac{0.5 \text{ mA}}{0.418 \text{ V}} = 1.42 \text{ mA/V}
\]
\[
|A_d| = g_m R_D = (1.42)(4 \text{ kΩ}) = 5.68 \text{ V/V}
\]
8.29
\[(a) \; I_{D1} = I_{D2} = \frac{1 \text{ mA}}{2} = 0.5 \text{ mA}
\]
\[
\frac{1}{2} k' W W_n V_{OV} = \frac{1}{2} \left( 2.5 \text{ mA/V}^2 \right) V_{OV}^2
\]
\[
V_{OV} = \left[ \frac{2 \cdot 0.5 \text{ mA}}{2.5 \text{ mA/V}^2} \right]^{1/2} = \sqrt{0.4 \text{ V}^2} = 0.632 \text{ V}
\]
\[
V_{GS} = V_i, \; V_{OV} = 0.7 \text{ V} + 0.632 \text{ V} = 1.332 \text{ V}
\]
\[
V_S = (1 \text{ mA})(1 \text{ kΩ}) = 1 \text{ V}
\]
\[
V_{cn} = V_S + V_{GS} = 1 \text{ V} + 1.332 \text{ V} = 2.332 \text{ V}
\]
(b) \[A_d = g_m R_D = I_{OV}/R_D
\]
\[
R_D = A_d \cdot \frac{V_{OV}}{I} = 8 \text{ V/V} \cdot \frac{0.632 \text{ V}}{1 \text{ mA}} = 5.06 \text{ kΩ}
\]
(c) \[V_D = V_{DD} - I_D R_D
\]
\[
= 5 \text{ V} - (0.5 \text{ mA})(5.06 \text{ kΩ})
\]
\[
= 2.47 \text{ V}
\]
(d) From Equation 8.43
\[
\frac{\Delta V_{D1}}{V_{CM}} = -\frac{1}{g_m} \cdot 
\]
\[8.30\]
\[(a) \; R_{D1} = R_D + \frac{\Delta R_D}{2} \; R_{D2} = R_D - \frac{\Delta R_D}{2}
\]
\[
g_{m1} = g_m + \frac{\Delta g_m}{2} \; g_{m2} = g_m - \frac{\Delta g_m}{2}
\]
\[
i_{d1} = \frac{g_{m1} V_{icm}}{g_{m1} R_{SS}} \; i_{d2} = \frac{g_{m2} V_{icm}}{2 g_{m2} R_{SS}}
\]
\[
i_{d1} - i_{d2} = (g_{m1} - g_{m2}) \frac{V_{icm}}{2 g_{m1} R_{SS}}
\]
8.70
The bins current I will split between the two differential transistors according to their base-emitter areas. So, the larger device will carry \( \frac{2I}{3} \)
Amperes, while the second transistor will carry \( \frac{I}{3} \) A.
Assuming, for example, that \( Q_1 \) has the larger area, \( r_{ce1} = \frac{V_T}{2I/3} \) and \( r_{ce2} = \frac{V_T}{I/3} \).
Normally, we could apply the common-mode half circuit symmetry. But here, the amplifier is not symmetrical.
If \( r_{ce1} = \frac{3V_T}{2I} \) and \( r_{ce2} = \frac{3V_T}{I} \),
\[ i_{c1} \neq i_{c2} \]
so, \( V_{os} = -i_{c2} R_c (-i_{c1} R_c) = (i_{c1} - i_{c2}) R_c \)
with \( \alpha \geq 1 \)
\[ i_{c1} + i_{c2} = i_{cm} \text{ (since } R_{st} \gg r_s) \]
\[ A_{cm} = \frac{V_{os}}{V_{cm}} = \frac{1}{3} \frac{R_C}{R_{EE}} = \frac{12 \text{ K}}{3(500 \text{ K})} = 0.008 \text{ V/V} \]

8.71
For \( I = 160 \mu A \),
\[ I_D = \frac{1}{2} \frac{160 \mu A}{2} = 80 \mu A \]
\[ g_m = \frac{2k_n W}{I_D} = \sqrt{2(4 \text{ mA/V}^2)(80 \mu A)} \]
= 0.8 m\text{A/V}
\[ R_D = 10 \text{ k\Omega} \text{, so that } \]
\[ A_v = g_m R_D = \text{(0.8 m\text{A/V})(10 k)} = 8 \text{ V/V} \]
using Eq. (8.108),
\[ V_{OS} = \frac{V_{ov}}{2} \left( \frac{\Delta R_D}{R_D} \right) \text{ where } \frac{\Delta R_D}{R_D} = 0.02 \]
worst-case
since \( g_m = \frac{I_D}{V_{ov}/2} \).
\[ V_{ov} = \frac{I_D}{g_m} = \frac{80 \mu A}{0.8 \text{ m\text{A/V}}} = 0.1 \text{ V} \]
Therefore, \( V_o = (0.1 \text{ V})(0.02) = 2 \text{ mV} \)
For \( I = 360 \mu A \), \( I_D = \frac{360 \mu A}{2} = 180 \mu A \)
\[ g_m = \frac{2k_n W}{I_D} = \sqrt{2(4 \text{ mA/V}^2)(0.18 \text{ mA})} \]
= 1.2 mA/V
\[ A_v = g_m R_D = \text{(1.2 mA/V)(10 k)} = 12 \text{ V/V} \]
\[ V_{ov} = \frac{I_D}{g_m} = \frac{180 \mu A}{1.2 \text{ mA/V}} = 0.15 \text{ V} \]
so that

\[ V_{os} = \frac{V_{os}}{2} \left( \frac{\Delta R_D}{R_D} \right) = (0.15 \text{ V})(0.02) = 3 \text{ mV} \]
So both \( A_v \) and \( V_o \) increase at the same ratio since both are proportional to \( \sqrt{I_D} \).

8.72
Worst-case \( \Delta V_o = 10 \text{ mV} \)
Worst-case \( \frac{\Delta R_D}{R_D} = 0.04 \)
Worst-case \( \frac{\Delta(W/L)}{(W/L)} = 0.04 \)
From Eq. (8.108),
\[ V_{OS} \text{(d}_{ov} \text{ to } \Delta R_D) = \frac{V_{ov}}{2} \cdot \frac{\Delta R_D}{R_D} = \frac{0.2 \text{ V}}{2} \cdot (0.04) \]
= 4 mV
From Eq. (8.113),
\[ V_{OS} \text{(d}_{ov} \text{ to } \Delta \frac{W}{L}) = \frac{V_{ov}}{2} \cdot \frac{\Delta(W/L)}{(W/L)} \]
= \frac{0.2 \text{ V}}{2} \cdot (0.04) = 4 \text{ mV} \]
From Eq. (8.116),
\[ V_{OS} \text{(d}_{ov} \text{ to } \Delta V_o) = \Delta V_o = 10 \text{ mV} \]
The absolute worst-case total offset would be \( 4 + 4 + 10 = 18 \text{ mV} \) However, since these offset sources are not correlated, a realistic observation might be [from Eq. (8.117)]
\[ V_{os} = \sqrt{\left( V_{os1} \right)^2 + \left( V_{os2} \right)^2 + \left( V_{os3} \right)^2} \]
= \sqrt{(4)^2 + (4)^2 + (10)^2} = 11.5 \text{ mV} \]
The major contribution is from the variation in \( V_o \).
If we attempt to compensate for \( V_{OS} \) by Changing \( R_D \),
We have, 11.5 mV = \frac{V_{ov}}{2} \cdot \frac{\Delta R_D}{R_D}
or
\[ \frac{\Delta R_D}{R_D} = \frac{11.5 \text{ mV}}{0.2 \text{ V}/2} = 0.115 \text{ or } 11\% \]
If \( V_{OS} \) is reduced by a factor of 10,
\[ V_{os} = \sqrt{(4)^2 + (4)^2 + (10)^2} = 5.74 \text{ mV} \]
So that
\[ \frac{\Delta R_D}{R_D} = \frac{5.74 \text{ mV}}{0.2 \text{ V}/2} = 5.74 \% \]

8.73
\[ I_D = \frac{I}{2} = \frac{100 \mu A}{2} = 50 \mu A \]
\[ I_D = \frac{1}{2} K_n \left( \frac{W}{L} \right) V_{OV} \text{ or } \]
\[ V_{OV} = \sqrt{\frac{2 I_D}{K_n(W/L)}} = \sqrt{\frac{2(50 \mu A)}{250 \mu A / V^2(10)}} \]
\[ V_{os1} = 0.2 \text{ V} \]

From Eq. (8.108),
\[ V_{os1} = \frac{V_{ov}}{2} \cdot \frac{\Delta R_D}{R_D} = 0.2 \text{ V} \cdot (0.05) \]

\[ = 5 \text{ mV} \]

From Eq. (8.113),
\[ V_{os2} = \frac{V_{ov}}{2} \cdot \frac{\Delta W/\text{L}}{W/\text{L}} = \frac{V_{ov}}{2} \cdot \frac{\Delta (W/\text{L})}{(W/\text{L})} \]

\[ = 0.2 \text{ V} \cdot (0.05) = 5 \text{ mV} \]

From Eq. (8.116),
\[ V_{os3} = \Delta V_i = 5 \text{ mV} \]

The worst-case offset is
\[ V_{os} = V_{os1} + V_{os2} + V_{os3} = 5 + 5 + 5 \]

\[ = 15 \text{ mV} \]

The root-sum-square value from Eq. (8.117) is
\[ V_{os} = \sqrt{V_{os1}^2 + V_{os2}^2 + V_{os3}^2} \]

\[ = \sqrt{(5)^2 + (5)^2 + (5)^2} = 8.66 \text{ mV} \]

### 8.74
The output offset voltage is \( \Delta V_C = \Delta R_C \cdot \frac{1}{2} \)

\[ A_d = \frac{g_m R_C}{V_T} = \frac{I_C}{V_T} \cdot R_C = \frac{I R_C}{2 V_T} \]

\[ |V_{os}| = \frac{|\Delta V_C|}{A_d} = \frac{\Delta R_C}{2} \cdot \frac{I R_C}{2 V_T} \]

\[ = V_T \cdot \left( \frac{\Delta R_C}{R_C} \right) = 25 \text{ mV} (0.08) \]

\[ |V_{os}| = 2 \text{ mV} \]

### 8.75
From Eq. (8.126),
\[ |V_{os}| = V_T \left( \frac{\Delta I_S}{I_S} \right) \]

\[ |V_{os}| = 25 \text{ mV} (0.10) = 2.5 \text{ mV} \]

### 8.76
\[ \Delta V_E = \Delta R_C \cdot \frac{1}{2} \]

\[ A_d = \frac{R_C}{r_e + R_C} = \frac{2 V_T}{V_T + R_C} = \frac{I R_C}{2 V_T + I R_E} \]

\[ V_{os} = \frac{\Delta V_C}{A_d} = \frac{\Delta R_C}{R_C} \cdot \left( V_T + \frac{I R_E}{2} \right) \]

### 8.77
\[ \Delta V_C = \alpha_1 I_C - \alpha_2 \frac{I R_C}{2} \]

\[ = \frac{1}{2} R_C (\alpha_1 - \alpha_2) \]

\[ = \frac{1}{2} R_C \left( \frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1} \right) \]

For \( \beta_1, \beta_2 >> 1 \)
\[ \Delta V_C = \frac{1}{2} R_C \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right) \]

\[ A_d = \frac{R_C}{r_e} = \frac{I R_C}{2 V_T} \]

\[ V_{os} = \frac{\Delta V_C}{A_d} = V_T \left( \frac{1}{\beta_2} - \frac{1}{\beta_1} \right) \text{ Q.E.D.} \]

For \( \beta_1 = 100 \) and \( \beta_2 = 200 \)
\[ V_{os} = 25 \left( \frac{1}{200} - \frac{1}{100} \right) \]

\[ = -125 \text{ mV} \]

### 8.78
CASE 1: BJT Diff. Amp.
From Eq. (8.121),
\[ |V_{os}| = V_T \left( \frac{\Delta R_C}{R_C} \right) = 25 \text{ mV} (0.04) = 1 \text{ mV} \]

CASE 2: MOSFET Diff. Amp.
From Eq. (8.108),
\[ V_{os} = \left( \frac{V_{ov}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) = \frac{300 \text{ mV}}{2} (0.04) = 6 \text{ mV} \]

If the MOSFET widths, are increased by a factor of 4, and since \( I_D \) must remain constant, we see that since
\[ I_D = \frac{1}{2} K_n \left( \frac{W}{L} \right) V_{ov}^2 \]

The new \( V_{ov} = \sqrt{\frac{2I_D}{K_n \left( \frac{W}{L} \right)}} \) which is \( \frac{1}{2} \) or \( \frac{1}{4} \) of its original value.

So, the new offset voltage is
\[ V_{os} = \left( \frac{150 \text{ mV}}{2} \right) (0.04) = 3 \text{ mV} \]

### 8.79
Since the two transistors are matched except for their \( V_T \) value, we can express the collector currents when the input terminals are grounded as,
\[ I_{C1} = I_C \left( 1 + \frac{V_{CE}}{V_A} \right) \]
However, if we use KVL,

\[ i_6 = \frac{v_o - v_i}{r_o} = \frac{1}{2} \frac{g_m r_o v_{id} - V_{id}}{r_o} \]

\[ = \frac{1}{2} g_m V_{id} - \frac{V_{id}}{4r_o} \text{ inconsistent} \]

\[ i_7 = i_5 - i_6 = \frac{3}{4} g_m V_{id} - \frac{g_m V_{id}}{2} = \frac{g_m V_{id}}{4} \]

(which is the same as \( i_3 \))

\[ i_8 = g_m v_{id} = g_m \left( \frac{V_{id}}{2} - \frac{V_{id}}{4} \right) = \frac{1}{4} g_m v_{id} \]

\[ i_9 = i_8 = \frac{1}{4} g_m V_{id} \]

\[ i_{10} = i_8 - i_7 = \frac{g_m V_{id}}{4} - \frac{g_m V_{id}}{4} = 0 \]

\[ i_{11} + i_{10} = i_9 \text{ or} \]

\[ i_{11} = i_9 - i_{10} = i_9 = \frac{g_m V_{id}}{4} \]

(which is the same as \( i_1 \))

\[ i_{12} = g_m v_{id} = \frac{1}{4} g_m V_{id} \]

\[ i_{13} = i_{11} - i_{12} = \frac{1}{4} g_m V_{id} - \frac{1}{4} g_m V_{id} = 0 \]

Note, through, that this is inconsistent with KVL. If \( i_{13} = 0, V_{D3} = 0, \) but \( V_{D3} = -V_{d4}/4. \)

If \( i_{10} = 0, V_{D1} = \frac{V_{id}}{4} \), but this conflicts with \( V_{D3} \)

being \( -\frac{V_{id}}{4} \).

It appears that the approximations for \( v_{gs} \) and \( v_{g2} \) prevent a clean solution. If these were more exact, all current and voltage relationships should be consistent.

### 8.91

Assuming a configuration similar to Fig. 8.32(a),

\[ I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{100 \mu A}{2} = 50 \mu A \]

\[ g_{m1} = g_{m2} = \frac{I_{D}}{V_{ov}/2} = \frac{50 \mu A}{0.2 V/2} = 0.5 \text{ mA/V} \]

\[ G_m = g_{m1} = 0.5 \text{ mA/V} \]

\[ r_{ov1} = r_{ov2} = \frac{V_{A2}}{I_{D}} = \frac{20 \text{ V}}{0.05 \text{ mA}} = 400 \text{ k}\Omega \]

\[ r_{ov3} = r_{ov4} = \frac{|V_{A11}|}{I_{D}} = \frac{12 \text{ V}}{0.05 \text{ mA}} = 240 \text{ k}\Omega \]

From Eq. (8.140),

\[ R_o = r_{ov} \parallel r_{ov4} = 400 \text{ k} \parallel 240 \text{ k} = 150 \text{ k}\Omega \]

\[ A_d = G_m R_o = (0.5 \text{ mA/V})(150 \text{ k}) = 75 \text{ V/V} \]

Gain will be reduced by a factor of 2 if

\[ R_L = R_o = 150 \text{ k}\Omega \]
8.110

\[ I_{DS} = \frac{W_n}{W_s} I_{REF} = \frac{50}{40} \times 90 \mu A \]
\[ = 112.5 \mu A \]
Output offset current \( I_{OD} - I_{OB} \)
\[ = 112.5 - 90 = 22.5 \mu A \]
\[ \Rightarrow V_o = 22.5 \mu (r_{oB}) \]
\[ r_{oB} = \frac{10}{112.5 \mu} = 88.9 \text{ k}\Omega \]
\[ \Rightarrow V_o = 22.5 \mu \text{ (111 k||88.9 k) } \]
\[ = 1.11 \text{ V} \]
\[ V_{OS} = \frac{V_o}{A_o} = \frac{1.11 \text{ V}}{1109} = 1 \text{ mV} \]

8.111

Offset current \( I_{O2} - I_{O4} \)
\[ = I_{O3} - I_{O4} \]
\[ I_{O3} = \frac{K}{2} (V_{G3} - V_i)^3 \]
\[ I_{O4} = \frac{K}{2} (V_{G4} - (V_i + \Delta V_i))^3 \]
\[ I_o = I_{O3} - I_{O4} \]
\[ = \frac{K}{2} [(V_{G3} - V_i - V_{G4} + V_i + \Delta V_i) \times \]
\[ (V_{G3} - V_i + V_{G4} - V_i - \Delta V_i)] \]
\[ = \Delta V_i \frac{K}{2} (2V_{G3} - 2V_i - \Delta V_i) \]
\[ \simeq K(V_{G3} - V_i) \cdot \Delta V_i \]
\[ I_o = g_{m3} \Delta V_i \]
Recall \( I_o = g_{m3} \cdot V_{os} \)
and \( g_{m1} = g_{m3} \)
\[ \Rightarrow V_{os} = \frac{g_{m3}}{g_{m1}} \cdot \Delta V_i \]
For \( \Delta V_i = 2 \text{ mV} \)
\[ V_{os} = \frac{0.3 \text{ m}}{0.3 \text{ m}} \times 2 \text{ m} = 2 \text{ mV} \]

8.112

(a) Referring to Fig. P8.112,
\[ I_{E1} = I_{E3} = I_2 = \frac{0.2 \text{ mA}}{2} = 0.1 \text{ mA} \]
\[ I_{E3} \simeq I_{E4} = I_2 = 0.1 \text{ mA} \]
\[ I_{E5} = 0.5 \text{ mA} \]
since the output voltage is held at 0 V,
\[ I_{E6} = 1 \text{ mA} \]
(b) considering the first stage,
\[ G_{m1} = g_{m1} = \frac{|I_{E1}|}{I_T} = \frac{0.1 \text{ mA}}{25 \text{ mV}} = 4 \text{ mA/V} \]
since all \( r_{oa} = \infty \), the load of the differential stage is just the input of \( Q_5 \).
\[ r_{oa5} = \frac{V_T}{I_{T5}} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega \]
\[ R_{o1} = (\beta + 1)r_{oa5} = (100)(50) = 5.05 \text{ k}\Omega \]
\[ A_1 = G_{m1}R_{o1} = (4 \text{ mA/V})(5.05 \text{ k}\Omega) = 20.2 \text{ V/V} \]
For the common-emitter stage \( Q_5 \),
\[ g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5 \text{ mA}}{25 \text{ mA}} = 20 \text{ mA/V} \]
The load is essentially
\[ (R_L + r_{oB})(\beta + 1) \leq R_L(\beta + 1) \]
\[ A_5 = -g_{m5}(R_L)_{(\beta + 1)} \]
\[ = -20 \text{ mA/V} \times (10 \text{ k})(101) = -20,200 \text{ V/V} \]
\[ A_E \approx 1 \text{ so, } \]
\[ A_v = A_1 \cdot A_5 \cdot A_6 = (20.2)(-20,200)(1) \]
\[ = 408,040 \text{ V/V} \]
or
\[ -A_v(dB) = 20 \log_{10}(408,040) = 112 \text{ dB} \]

8.113

\[ I_g = 225 \mu A \]
\[ \mu_+ C_{ox} = 180 \mu A/V^2 \]
\[ \mu_- C_{ox} = 60 \mu A/V^2 \]
For \( Q_8 & Q_9 : W/L = 60/0.5 \)
\[ \Rightarrow |V_{os}| = \frac{2I_g}{k_p(W/L)} \]
\[ |V_{os}|_{(60)} = \frac{2 \times 225 \mu}{60 \mu \times 120} = 0.25 \text{ V} \]
then \( g_{m,k,0} = \frac{2I_g}{|V_{os}|} = 2 \times 225 \mu \)
\[ = 2 \times 0.25 \text{ V} \]
\[ = 1.8 \text{ mA/V} \]
Since \( g_m \) of \( Q_{10}, Q_{11} \) & \( Q_{13} \) are identical to \( g_m \) of \( Q_8 \) & \( Q_9 \) then \( V_{OS13} = 0.25 \text{ V} \)
Thus for \( Q_{13} \)
\[ (0.25)^2 = \frac{2 \times 225 \mu}{180 \mu \times (W/L)_{13}} \]
\[ \rightarrow (W/L)_{13} = 40 \text{ i.e., } (20/0.5) \]
Since \( Q_{12} \) is 4 times as wide as \( Q_{13} \), then
\[ (W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5 \]
\[ R_B = \frac{2}{\sqrt{2} k_n (W/L)_{12}} I_B \left( \frac{(W/L)_{12} - 1}{(W/L)_{12} - 1} \right) \]
\[ = \frac{2}{\sqrt{2} \times 180 \mu \times 80/0.5 \times 225 \mu} \left( \frac{80/0.5}{20/0.5} - 1 \right) \]
\[ \rightarrow R_B = 555.6 \Omega \]
The voltage drop on \( R_B \) is :
\[ 555.6 \times 225 \mu = 0.125 \text{ V} \]
To obtain the gate voltages: (assume \( |V_{in}| = |V_{gp}| = 0.7 \text{ V} \))