6.1) KVL:
\[
\begin{align*}
V_o &= r_o i_o + g_m r_o V_1 + i_o r_o + (-g_m r_o V_o) \\
V_1 &= r_o i_o - g_m r_o V_o \\
\end{align*}
\]
\[
\Rightarrow V_o = 2r_o i_o + g_m r_o i_o - g_m^2 r_o^2 V_o - g_m r_o V_o
\]
\[
\Rightarrow r_{out} = \frac{V_o}{i_o} = \frac{r_o (2 + g_m r_o)}{g_m^2 r_o^2 + g_m r_o + 1}
\]
Assuming \( g_m r_o \gg 1 \Rightarrow g_m^2 r_o^2 \gg g_m r_o \)
\[
\Rightarrow r_{out} \approx \frac{r_o (g_m r_o)}{g_m^2 r_o^2} \approx \frac{1}{g_m}
\]

6.2) For transistors Q_1 to Q_4:
\[
50 \, \mu A = \frac{\mu_n C_{ox}}{2} \left( \frac{W}{L} \right)_i (0.2)^2 \quad \text{for } i=1 \text{ to } 4
\]
\[
\Rightarrow \left( \frac{W}{L} \right)_i = 27.2 \quad \text{for } i=1 \text{ to } 4
\]
Also, \( V_{ass} = V_{as4} + V_{DS3} = V_{eff4} + V_{tn} + V_{eff3} + 0.15 = 1.35 \, V \)
\[
\Rightarrow V_{eff5} = V_{ass} - V_{tn} = 0.55 \, V
\]
Using \( I_{BIAS} = 50 \, \mu A \Rightarrow 
\[
50 \, \mu A = \frac{\mu_n C_{ox}}{2} \left( \frac{W}{L} \right)_5 (0.55)^2 \Rightarrow \left( \frac{W}{L} \right)_5 = 3.6
\]
6.3) Reducing $L$ by 1.6 will decrease $V_{\text{eff}}$ by a factor of $\sqrt{1.6}$. Therefore,

$$V_{DS3} = V_{GrS5} - V_{GrS4} = 1.35 - (V_{tn} + V_{\text{eff}})$$

$$= 1.35 - (0.8 + 0.16) = 0.39 \text{ V}$$

$$V_{DS4} = V_{D4} - V_{DS3} = V_{GrS3} - V_{DS3} = (V_{tn} + V_{\text{eff}}) - 0.39$$

$$= (0.8 + 0.16) - 0.39 = 0.57 \text{ V}$$

6.4) $I_{D3} = I_{D2} \Rightarrow \left(\frac{W}{L}\right)_2 V_{\text{eff}2}^2 = \left(\frac{W}{L}_3\right) V_{\text{eff}3}^2 \Rightarrow V_{\text{eff}3} = 2 V_{\text{eff}2}$

Also, $V_{GrS3} = V_{GrS2} + R_B I$

$$\Rightarrow V_{\text{eff}3} = \frac{V_{\text{eff}3}}{2} + R_B \frac{\mu n C}{2} \left(\frac{W}{L}_3\right) V_{\text{eff}3}^2$$

$$\Rightarrow 10 \mu n C R_B V_{\text{eff}3} = 1$$

Using $V_{\text{eff}3} = 0.2 \Rightarrow R_B = 5.43 \text{ k}\Omega$

6.5) Using the result of Problem 6.4, we have:

$$10 \mu n C R_B V_{\text{eff}3} = 1$$

Also, $\frac{\mu n (100^\circ C)}{\mu n (20^\circ C)} = \left(\frac{373}{293}\right)^{-1.5} = 0.7$

Therefore, $V_{\text{eff}3}$ will be increased by $\frac{1}{0.7}$.

Equivalently, $V_{\text{eff}3} = \frac{1}{0.7} \times 0.2 = 0.29 \text{ V}$
6.6) \[ R_B (100 \, ^\circ C) = (1 + 80 \times \frac{0.3}{100}) R_B (20 \, ^\circ C) = 6.73 \, k\Omega \]

Using the result of Problem 6.5, \( R_B \) is decreased at 100 \( ^\circ C \) by a factor of 0.7 \( \times \) 1.24 = 0.868.

Therefore, \( V_{eff3} \) will increase by \( \frac{1}{0.868} \).

\[ \therefore V_{eff3} = \frac{1}{0.868} \times 0.2 = 0.23 \, V \]

6.7) The equivalent circuit of Fig. 6.3 can be simplified:

\[ g_{m1} V_g \quad i_o \quad V_o \]

\[ r_{ds1} \quad V_1 \]

\[ V_g = -A V_1 \quad r_{ds2} \]

\[ \Rightarrow \frac{V_o}{i_o} = r_{ds1} + r_{ds2} + (1+A) g_{m1} r_{ds1} r_{ds2} \]

ignoring the first two terms:

\[ r_o \approx (1+A) g_{m1} r_{ds1} r_{ds2} \]
6.8) The equivalent circuit of Fig. P6.8 can be simplified as:

\[-A V_1 \rightarrow V_{\pi} \rightarrow V_1 \rightarrow V_0\]

Noting that \(-A V_1 = V_{\pi} + V_1 \Rightarrow V_{\pi} = -(1+A)V_1\)

The circuit can be simplified further as:

\[\Rightarrow R_{out} = \frac{V_o}{i_o} = r_o + \left( R_E || \frac{r_{\pi}}{1+A} \right)(1+(1+A)g_m r_o)\]

Assuming \(R_E \gg r_{\pi} \Rightarrow R_E || \frac{r_{\pi}}{1+A} = \frac{r_{\pi}}{1+A}\)

\[\Rightarrow R_{out} = r_o + \frac{r_{\pi}}{1+A} (1+A)g_m r_o = r_o (1+ \frac{r_{\pi} g_m}{1+A}) = r_o (1+\beta)\]

This result is independent of \(A\).
6.9) Using (6.10): \( R_{out} = g_{m1} r_{ds1} r_{ds2} (1 + A) \)

where \( A = g_{m3} r_{ds3} \) for the circuit of Fig. 6.6.

\[ R_{out} = g_{m1} r_{ds1} r_{ds2} (1 + g_{m3} r_{ds3}) \]

\[ I_{BIAS} = 50 \, mA \Rightarrow I_{D1} = I_{D2} = 350 \, mA \] \( \& \) \( I_{D3} = 200 \, mA \)

\[ \Rightarrow r_{ds1} = r_{ds2} = \frac{8 \times 1.6}{0.35} = 36.57 \, k\Omega \]

\[ r_{ds3} = \frac{8 \times 1.6}{0.2} = 64 \, k\Omega \]

\[ g_{m1} = \sqrt{2 I_{D1} m_n C_{ox} \frac{70}{1.6}} = 1.68 \, mA/V \]

\[ g_{m3} = \sqrt{2 I_{D3} m_n C_{ox} \frac{10}{1.6}} = 0.48 \, mA/V \]

\[ \Rightarrow R_{out} = 2.27 M\Omega (1 + 30.72) = 7.13 \times 10^7 \, \Omega \]

The impedance is 32 times larger than that of a wide-swing cascade current mirror given by

\[ R_{out} = g_{m1} r_{ds1} r_{ds2} = 2.25 M\Omega \]

6.10) \( 2 I_{D3} = \frac{1 mW}{4V} = 250 \, mA \Rightarrow I_{D3} = I_{D4} = 125 \, mA \).

Also, \( 5 I_{D5} = 125 \, mA \Rightarrow I_{D5} = I_{D6} = 25 \, mA \)

\[ I_{D1} = I_{D2} = 100 \, mA \]

\[ g_{m1} = \sqrt{2 I_{D1} m_n C_{ox} (W/L)_1} = 1.9 \, mA/V \]

\[ \omega_t = g_{m1}/C_L = \frac{1.9 \, mA/V}{10 \, pF} = 1.9 \times 10^8 \, \text{rad/s} \Rightarrow f_t = 30.2 \, MHz \]

(cont.)
6.10) (cont.) the slew rate without the clamp transistors:

\[ SR = \frac{I_{D4}}{C_L} = 12.5 \ \text{V/\mu s} \]

With the clamp transistors:

\[ I_{D11} = I_{\text{bias2}} + \frac{125 \mu}{30} = \frac{200 \mu + \frac{125 \mu}{30}}{31} = 6.6 \text{mA} \]

\[ \Rightarrow I_{D3} = 30 \ I_{D11} = 198 \text{ mA} \]

\[ \Rightarrow SR = \frac{198 \text{ mA}}{10 \text{ pF}} = 19.8 \text{ V/\mu s} \]

6.11) Ignoring the junction capacitances, the total capacitance at the drain of Q2 can be calculated as:

(Also ignore \( q_{13} \) since it is small)

\[ C_{p2} = C_{dq2} + C_{dq4} + C_{sq5} = C_{gd(overlap)} (W_2 + W_4 + W_5) + \frac{2}{3} W_5^L C_{ox} \]

\[ = 0.2 \text{ fF/\mu m} (300 + 300 + 60) + \frac{2}{3} \times 60 \times 1.6 \times 1.9 \text{ fF} = 254 \text{ fF} \]

The total conductance at this node is dominated by \( g_{m5} \). Using the result of problem 6.10, we have:

\[ g_{m5} = \sqrt{2 I_D5 \mu p C_{ox} (W/L)_5} = 0.237 \text{ mS} \]

The second pole (half-circuit concept):

\[ \omega_2 = \frac{g_{m5}}{C_{p2}} = 9.33 \times 10^8 \text{ rad/s} \]

\[ \Rightarrow f_2 = \frac{\omega_2}{2\pi} = 148.5 \text{ MHz} \]

(cont.)
6.11) Using (5.52), for a 70° phase margin, we must have: \( \frac{f_t'}{f_2} = \tan 70° \Rightarrow f_t' = 54.05 \text{ MHz} \)

Using (6.30): \( C'_L = \frac{9m_1}{\omega_t'} = \frac{1.9m}{2\pi f_t'} = 5.59 \text{ pF} \)

Finally, using (16.32):

\[
SR' = \frac{I_{Dy}}{C'_L} = \frac{125\mu}{5.59p} = 22.3 \text{ V/ms}
\]

With the clamp transistors: \( SR' = \frac{198\mu}{5.59p} = 35.4 \text{ V/ms} \)

6.12) This is equivalent to a 40° phase margin in the original design. Using (5.52), we have:

\( \frac{f_t'}{f_2} = \tan 50° \Rightarrow f_t' = 1.19f_2 \approx 176.7 \text{ MHz} \)

\( f_2 = 1.2f_t' = 212 \text{ MHz} \)

The final unity-gain frequency: \( f_t = 1.2f_t' = 212 \text{ MHz} \)

\[
\Rightarrow C_L = \frac{9m_1}{\omega_t} = 1.43 \text{ pF}
\]

\[
\omega_z = \frac{1}{RC_L} \Rightarrow R_C = \frac{1}{\omega_z C_L} = 525 \Omega \quad , \quad SR = \frac{I_{Dy}}{C_L} = 87 \text{ V/ms}
\]

With the clamp transistors: \( SR = \frac{198\mu}{1.43p} = 138 \text{ V/ms} \)
6.13) \[ I_{D1} = I_{D2} = K I_{D5} = K I_{D6} \]

\[ I_{total} = 2 \left( I_{D1} + I_{D5} \right) = 2 \left( 1 + K \right) I_{D5} \]

\[ \Rightarrow I_{D5} = \frac{I_{total}}{2 \left( K+1 \right)} \quad , \quad I_{D1} = \frac{K}{K+1} \cdot \frac{I_{total}}{2} \]

\[ g_{m1} = \frac{2 I_{D1}}{V_{eff1}} = \frac{K}{K+1} \cdot \frac{I_{total}}{V_{eff1}} \quad , \quad \omega_t = \frac{K}{K+1} \cdot \frac{I_{total}}{V_{eff1} \cdot C_L} \]

Assuming a constant \( I_{total} \) \& \( V_{eff1} \), both \( g_{m1} \) \& \( \omega_t \) increase with increasing \( K \).

6.14) The dc gain is given by \( A_v = g_{m1} \cdot r_{out} \)

where, approximately: \( r_{out} = g_{m8} \cdot \frac{r_{ds8}}{2} \)

and \( g_{m8} = \frac{2 I_{D8}}{V_{eff8}} \quad , \quad r_{ds8} = \frac{\alpha L}{I_{D8}} \quad , \quad \alpha \) is constant!

\[ \Rightarrow r_{out} = \frac{\alpha^2 L^2}{V_{eff8} \cdot I_{D8}} \]

\[ \Rightarrow A_v = \frac{2 I_{D1}}{V_{eff1}} \cdot \frac{\alpha^2 L^2}{V_{eff8} \cdot I_{D8}} = \left( \frac{I_{D1}}{I_{D8}} \right) \left( \frac{2 \cdot \alpha^2 L^2}{V_{eff1} \cdot V_{eff8}} \right) \]

Noting \( I_{D8} = I_{D5} \) and \( I_{D1} = K I_{D5} \) results in:

\[ A_v = K \left( \frac{2 \cdot \alpha^2 L^2}{V_{eff1} \cdot V_{eff8}} \right) \]

\( A_v \) increases with \( K \)!
6.15) For the folded-cascade amplifier of Fig. 6.9, we have:

\[ \omega_t = \frac{g_{m1}}{C_L} = \frac{\sqrt{2I_D1 \mu_n C_{ox} (w/l)}}{C_L} \]

Also, \( 2I_D1 + \frac{2}{K} I_D1 = I_{total} \Rightarrow I_D1 = \frac{K}{2(K+1)} I_{total} \)

\[ \Rightarrow \omega_t (\text{folded-cascade}) = \frac{1}{C_L} \sqrt{\frac{\mu_n C_{ox} I_{total} (w/l)}{K+1}} \]

For the current-mirror opamp, \( \omega_t \) is given by (6.48).

Therefore,

\[ \frac{\omega_t (\text{folded-cas.})}{\omega_t (\text{current-mir.})} = \sqrt{\frac{K+3}{2K(K+1)}} \]

<table>
<thead>
<tr>
<th>( K )</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_t ) Ratio</td>
<td>1</td>
<td>0.645</td>
<td>0.418</td>
</tr>
</tbody>
</table>

6.16) From Example 6.4: \( C_P = 0.46 \) pF

\[ C_L = 5 + 5 + \frac{5(1+0.46)}{1+0.46+5} = 11.13 \text{ pF} \]

\[ \omega_t = \frac{Kg_{m1}}{C_L} = \frac{2 \times 1.7 \text{ mA/V}}{11.13 \text{ pF}} = 3.05 \times 10^8 \text{ rad/s or } f_t = 48.6 \text{ MHz} \]

\[ \beta = \frac{C_2}{C_1 + C_P + C_2} = \frac{5}{1 + 0.46 + 5} = 0.77 \]

\[ \Rightarrow T = \frac{1}{\beta \omega_t} = 4.24 \text{ nsec} \]

For a 1% accuracy, we need 4.6 \( T \) or \( 19.5 \) nsec.
6.17) Using (6.53) & (6.68): \( SR = 25.4 \text{ V/\mu s} \).

Assuming a high-gain opamp:

\[
A_v \frac{\Delta V_o}{\Delta V_{in}} = -\frac{C_1}{C_2} = -\frac{5 \text{ pF}}{5 \text{ pF}} = -1 \Rightarrow \Delta V_o = -1 \text{ V}.
\]

\( \Rightarrow \) the output voltage rate of change would be

\[
\frac{\Delta V_o}{\Delta t}|_{\text{max}} = \frac{1}{\tau} = \frac{1}{7.8 \text{ ns}} = 128 \text{ V/\mu s} > 25.4 \text{ V/\mu s}
\]

Therefore, the output will be limited by the slew rate.

The output voltage will ramp up with 25.4 V/\mu s until the exponential-curve derivative is equal to the slew rate:

At time \( t_0 \), we have:

\[
\frac{1}{\tau} e^{-t_0/\tau} = 25.4 \text{ V/\mu s}
\]

\( \Rightarrow t_0 = 12.63 \text{ ns} \) & \( (1 - e^{-t_0/\tau}) = 0.8 \text{ V} \)

\( \Rightarrow \) at time \( t = t_1 \), the output has reached to 80% of its final value. \( (t_1 = 31.5 \text{ ns}) \)

For \( t \geq t_1 \): \( \Delta V_o(t) = 0.8 + 0.2 \left( -e^{-(t-t_1)/\tau} + 1 \right) \)

For \( \Delta V_o(t) = 0.99 \Rightarrow t - t_1 = 23.37 \text{ ns} \)

This is the time required after \( t_1 \) for the output to settle to 1% of its final value.
6.18) Fully diff. folded-cas:

- positive SR = \( \frac{I_3 - I_q}{C_L} = \frac{(K+1)I_q - I_q}{C_L} = \frac{KI_q}{C_L} \)
- negative SR = \( \frac{I_q}{C_L} \)

.. for \( K=2 \) and \( I_q = 40 \mu A \):

- positive SR = 8 V/us
- negative SR = 4 V/us

Fully diff. current-mir:

- positive SR = negative SR = \( \frac{KI_{Bias}}{2C_L} \)

.. for \( K=2 \) and \( I_{Bias} = 160 \mu A \), we have:

- positive SR = negative SR = 16 V/us

6.19) \( \frac{dV_{out+}}{dt} \bigg|_{max} = \frac{dV_{out-}}{dt} \bigg|_{max} = \frac{KI_{Bias}}{C_L} \)

Note that both maximums occur simultaneously.

6.20) \( W_t = \frac{Kg_m}{C_L} \) where \( g_m = \sqrt{2I_{D4} \cdot \mu_n \cdot Cox \cdot (W/L)_4} \)

Also, \( I_{total} = (2+K)I_{D4} \Rightarrow I_{D4} = \frac{I_{total}}{2+K} \)

.. \( W_t = \frac{K}{C_L} \cdot \frac{2}{2+K} \cdot I_{total} \cdot \mu_n \cdot Cox \cdot (W/L)_4 \)
6.21) The maximum value of $V_{eff}$ without the $I_b$-transistor going into the triode region is $2 - |V_{TP}| = 1.1\,V$. At this voltage, all the bias current will flow through either $Q_1$ or $Q_2$. Therefore, $I = K(1.1)^3$

For the bias condition ($V_{out+} = 0$), we have:

$$I/2 = K V_{eff}^2 \Rightarrow V_{eff} = \frac{1.1}{\sqrt{2}} = 0.78\,V$$

Transistor $Q_1$ will shut off when $V_{S_1} = |V_{TP}| = 0.9\,V$. At this point, $V_{S_1} = 2\,V$ and $I_B$ flows through $Q_2$.

$$V_{out+}\bigg|_{max} = 2 - 0.9 = 1.1\,V$$

When $V_{out+}$ goes below zero, $V_{S_1}$ starts to fall off from its bias value (i.e. 1.68\,V) until it reaches 0.9\,V. At this point, $Q_2$ shuts off and $I_B$ will pass through $Q_1$.

$$V_{out+}\bigg|_{min} = 0.9 - 2 = -1.1\,V$$

Note that a $V_{eff}$ (bias) that is higher than the optimum value (i.e. 0.78\,V) will cause $I_B$-transistor to enter the triode region at a lower $V_{out+}$ voltage. Also, a $V_{eff}$ (bias) that is lower than the optimum will shut off $Q_1$ at a lower $V_{out+}$ voltage. In both cases, the voltage range of $V_{out+}$ for linear operation is reduced.
6.22) \[ |V_{tp}| = |V_{tpo}| + \gamma \left( \sqrt{|V_{sb}|^2 + |2\Phi_F|^2} - \sqrt{2|\Phi_F|^2} \right) \]

For \( V_s = 2 \, \text{V} \) and \( V_b = 2.5 \, \text{V} \), \( 12\Phi_F = 0.7 \, \text{V} \)

\[ \Rightarrow |V_{tp}| = 1.1 \, \text{V} \Rightarrow I = K(2-1.1)^2 \]

For the bias condition \( (V_{out+} = 0) \), we have

\[ I_{I/2} = K (V_{eff})^2 \Rightarrow V_{eff} = \frac{0.9}{\sqrt{2}} = 0.63 \, \text{V} \]

\[ V_{out+ max} = 2 - 1.1 = 0.9 \, \text{V} \]

\[ V_{out+ min} = |V_{tp}| - V_{s\alpha 1} = |V_{tp}| - (|V_{tp}| + 0.9) = -0.9 \, \text{V} \]

6.23) \[ V_{in} = V_0 \frac{\frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}}}{\frac{1}{R_2 + R_1} \parallel \frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}}} \]

\[ \Rightarrow \frac{V_0}{V_{in}} = \frac{R_1 + R_2}{R_1} + \frac{R_2 (g_{m1} + g_{m2})}{\frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}}} \]

The first term can be recognized as the voltage gain when \( g_{m1} = g_{m2} = 0 \)
6.24) Using Eq. (6.62), (6.64), (6.65), and (6.66):

\[
\tau = \frac{1}{\omega_{3dB}} = \frac{1}{\beta \omega_t} \quad \text{where} \quad \beta = \frac{C_t/M}{C_{1/M} + C_t + C_p} \quad \text{and} \quad \omega_t = \frac{Kg_m}{C_0 + \frac{C_{VM}(C_t+C_p)}{C_{VM}+C_t+C_p}}
\]

\[
\Rightarrow \tau = \frac{1}{Kg_m} \left[ (M+1)C_0 + C_p + \frac{MCoC_p}{C_1} + C_1 \right]
\]

\[
\frac{\partial \tau}{\partial C_1} = 0 \Rightarrow -\frac{CoC_p}{C_1^2} + \frac{1}{M} = 0 \Rightarrow C_{1,\text{opt}} = \sqrt{MC_0C_p}
\]

6.25) \( \tau = \frac{1}{Kg_m} \left[ 2C_0 + C_p + \frac{CoC_p}{C_1} + C_1 \right] = \frac{1}{Kg_m} \left[ 2.05 + 0.05 \frac{C_1}{C_1} + C_1 \right] \) ps

where \( C_1 \) must be expressed in pF unit.

\( C_{1,\text{opt}} = \sqrt{CoC_p} = 0.22 \) pF

\[
\Rightarrow Kg_m \tau \bigg|_{\text{min}} = 2.5 \text{ ps}
\]

The following table shows \( Kg_m \tau \) for some other values of \( C_1 \):

<table>
<thead>
<tr>
<th>( C_1 ) (pF)</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Kg_m \tau ) [ps]</td>
<td>2.65</td>
<td>2.52</td>
<td>2.65</td>
<td>2.82</td>
<td>3</td>
<td>3.1</td>
</tr>
</tbody>
</table>

The graph shows \( Kg_m \tau \) vs. \( C_1 \) (pF)