10.1) The clock period is \( \frac{1}{10 \text{ MHz}} = 100 \text{ ns} \). Assuming 50% duty cycle, we have the following timing diagram:

(not to scale)

10.2) \( C_2 V_o(n) = C_2 V_o(n-1) - C_1 V_i(n) \)

\[ \Rightarrow C_2 V_o(z) = C_2 z^{-1} V_o(z) - C_1 V_i(z) \Rightarrow \frac{V_o(z)}{V_i(z)} = \frac{-C_1/C_2}{1 - z^{-1}} \]

10.3) \( C_{p_2} \) is always discharged since its voltage is virtually ground.

During \( \phi_1 \), \( C_{p_1} \) is charged to \( V_i(n) C_{p_1} \).

This charge will be transferred to \( C_2 \) during \( \phi_2 \). Therefore:

\[ C_2 V_o(n) = C_2 V_o(n-1) - C_{p_1} V_i(n-1) - C_1 \]

\[ \Rightarrow \frac{V_o(z)}{V_i(z)} = \frac{-C_1/C_2 + C_{p_1}/C_2}{1 - z^{-1}} \]
10.4) A finite gain of $A$ implies the voltage of the inverting terminal is $\frac{1}{A} V_{o}(n)$ if the output voltage is $V_{0}(n)$.

$$C_{2} V_{0}(n) \left[1 + \frac{1}{A}\right] = C_{2} V_{0}(n-1) \left[1 + \frac{1}{A}\right] - C_{1} \left[V_{i}(n-1) + \frac{V_{0}(n)}{A}\right]$$

Note that at the end of $\Phi_{2}$, the voltage across $C_{1}$, which is $V_{i}(n-1)$, reduces to $-\frac{V_{0}(n)}{A}$, not ground.

Also, assuming $\frac{1}{A} \ll 1$, we have

$$C_{2} V_{0}(n) = C_{2} V_{0}(n-1) - C_{1} \left[V_{i}(n-1) + \frac{V_{0}(n)}{A}\right]$$

$$\Rightarrow \quad \frac{V_{0}(Z)}{V_{i}(Z)} = \frac{-C_{1}/C_{2}}{Z \left(1 + \frac{C_{1}}{C_{2}A}\right) - 1}$$

At dc ($z=1$), this gain is equal to $\frac{-C_{1}/C_{2}}{1 + \frac{C_{1}}{C_{2}A}} = -A$

$$Z_{p} = \frac{1}{1 + \frac{C_{1}}{C_{2}A}} = 1 - \frac{C_{1}}{C_{2}A} \quad \text{assuming} \quad C_{1} \ll C_{2}A.$$  

10.5) $H(z) = \frac{KZ}{Z - 0.53327}$

Setting $H(1) = 1 \Rightarrow H(z) = \frac{0.46673Z}{Z - 0.53327}$

Equating $H(z)$ with that of Eq. (10.33) and assuming

$C_{A} = 10\,pF \Rightarrow C_{1} = 0 \quad C_{2} = -8.752\,pF \quad C_{3} = 8.752\,pF$

the new gain at 50 kHz = $H(-1) = -0.304 = -10.3\,dB$
10.6) \[ W_{3dB} = \frac{1}{50kHz} \times 2\pi = 0.04\pi \text{ RAO/sample} \]

Since the zero is at 0 rather than -1, we cannot use a bilinear transform. From example 9.5 in chapter 9, we have
\[ W_{3dB} = \cos^{-1} \left( 2 - \frac{a}{2} - \frac{1}{2a} \right) \]
This example assumed a zero at \( \infty \) which has the same magnitude response as a zero at 0.
\[ \cos \left( 0.4\pi \right) = 2 - \frac{a}{2} - \frac{1}{2a} \Rightarrow a = 0.8821 \]
\[ H(z) = \frac{k z}{z - 0.8821} \]
Forcing \( H(1) = 1 \Rightarrow k = 0.1179 \)
Equating coeff with (10.33) results in
\[ C_1 = 0, \hspace{1cm} -C_2 = C_3 = 6.683 \text{ pF} \]
25 kHz corresponds to \( z = -1 \), \( H(-1) = 0.0626 = -24.1 \text{ dB} \)

10.7) The transfer function is given by (10.33). Substituting the given capacitances, we have:
\[ H(z) = \frac{0.1z}{1.1z - 1} \]
At dc: \( z = 1 \), \( H(1) = 1 \), \( \angle H(1) = 0 \)
At \( f_s/4 \): \( z = j \), \( H(j) = 0.673 \angle -42.27^\circ \)
At \( f_s/2 \): \( z = -1 \), \( H(-1) = 0.0476 \angle 0^\circ \)
10.8) The equivalent signal flow graph is:

\[ V_i(z) \frac{1}{C_2 z^{-1}} + \frac{1}{C_A} \frac{1}{1-z^{-1}} \rightarrow V_o(z) \]

\[ \left\{ V_i(z) \left[ C_2 z^{-1} - C_1 + C_1 z^{-1} \right] - C_3 V_o(z) \right\} \frac{1}{C_A} \frac{1}{1-z^{-1}} = V_o(z) \]

\[ \Rightarrow H(z) = \frac{V_o(z)}{V_i(z)} = -\frac{C_1 - (c_1 + c_2) z^{-1}}{(c_A + c_3) - c_A z^{-1}} \]

10.9) The transfer function poles are the zeros of the denominator.

Using (10.49) with \( k_6 = 0 \), we have

\[ z^2 + (k_4 k_5 - 2) z + 1 = 0 \]

Since all the coefficients are real numbers, this equation has, in general, two complex conjugate roots, \( z_1 \) & \( z_1^* \), with their product equal to the constant term of the equation (i.e. 1).

\[ z_1 z_1^* = 1 \Rightarrow |z_1|^2 = 1 \Rightarrow |z_1| = 1 \]

\[ \therefore \text{both } z_1 \text{ & } z_1^* \text{ lie on the unit circle.} \]
\[ 10.10 \quad \omega_0 = \frac{2\pi}{100} = 0.062832 \text{ rad/sample} + Q = 20 \]

Equivalent \(Q_0 = \tan\left(\frac{\omega_0}{2}\right) = 0.0314263 \text{ rad/s} \]

\[ H_0(s) = \frac{\omega_0}{Q_0 \ s} \cdot \frac{s}{s^2 + \frac{\omega_0}{Q_0} s + \omega_0^2} = \frac{0.0015713 \ s}{s^2 + 0.0015713 s + 0.00098761} \]

Transform using the bilinear transform \(s = \frac{2 \ z - 1}{z + 1}\)

\[ H(z) = \frac{0.001572 z^2 - 0.001572}{1.003145 z^2 - 1.991875 z + 1} \]

**For low-\(Q\) biquad from (10.51) - (10.56)**

\[ K_3 = -0.001572 \quad K_2 = 0.003144 \]

\[ K_6 = 0.003145 \quad K_4 = K_5 = 0.06291 \]

\[ K_1 = 0 \]

**For high-\(Q\) biquad from (10.69) - (10.74)**

\[ H(z) = \frac{0.001567 z^2 - 0.001567}{z^2 - 1.992922 z + 0.99687} \]

\[ K_3 = 0.001567 \quad K_1 = 0 \quad K_2 = 0.0499 \]

\[ K_4 = K_5 = 0.06285 \quad K_6 = 0.0498 \]

Ignoring \(K_3\) (since it only sets zero at -1)

**Cap ratio for low-\(Q\) is 318 while it is 20 for high-\(Q\) biquad.**
10.12) Assuming $V_{out} = -V_{ss}$,

$$\Delta V_x = -K_2 V_{ss} = -K_{in} V_{in}$$

The positive jump of $V_x$ is still $k_1(V_{ss} + V_{DD})$ as before.

$$\Rightarrow T_{osc} = 2 \frac{k_1 (V_{ss} + V_{DD})}{K_2 V_{ss} + K_{in} V_{in}} T$$

Or, equivalently (assuming $V_{ss} = V_{DD}$)

$$f_{osc} = \frac{1}{4} \left( \frac{K_2}{K_1} + \frac{K_{in} V_{in}}{K_1 V_{DD}} \right)f$$