12.1) The total number of switches is \( \sum_{i=1}^{N} 2^i = 2(N-1) \)

12.2) In the equivalent circuit shown, \( r = \frac{400}{256} \, \Omega \), \( C = 0.1 \, \text{pF} \).

2C represents the capacitance of the only switch that is ON.

This switch is considered to be in the middle of the string for the worst case time constant.

\[
\tau = \sum_{i=1}^{255} \left[ (ir) || (256-i)r \right] C + C \left( \frac{128 \, r}{128 \, r} \right)
\]

\[
= rc \sum_{i=1}^{255} \frac{i(256-i)}{256} + 64 \, rc
\]

\[
= rc \sum_{i=1}^{255} i - \frac{rc \sum_{i=1}^{255} i^2}{256} + 64 \, rc
\]

\[
= rc \frac{(255)(256)}{2} - \frac{rc \frac{(255)(256)(510+1)}{6}}{256} + 64 \, rc = 10987 \, rc
\]

\[
= 10987 \times \frac{400}{256} \times 0.1 \, \text{pF} = 1.7 \, \text{ns}
\]

The settling time to 0.1% is \( 7 \tau = 12. \, \text{ns} \)
12.3) The total number of switches is \( 2^{N/2} \cdot 2^{N/2} + 2^{N/2} = 2 + 2 \).

12.4) For the output opamp, the offset that can be tolerated is:

\[
\frac{1}{2} \times \frac{0.1}{100} \times V_{ref} = \frac{1}{2} \times \frac{1}{1000} \times 5V = 2.5 \text{ mV}
\]

For the two opamp in the middle, the offset must be less than

\[
\frac{1}{2} \times 2.5 \text{ mV} \times 64 = 80 \text{ mV}
\]

Note that the offset introduced from the middle opamps will be divided by 64 (\(=2^6\)). The \( \frac{1}{2} \) factor accounts for two opamps in the middle.

12.5) The ratio between the largest and the smallest resistor is

\[
\frac{2^{10}}{2^R} = 2^9 = 512
\]

The current ratio is the same as above.

12.6) The matching accuracy required for the \( b_2 \) resistor, \( b_3 \) resistor, and \( b_4 \) resistor is 2 times, 4 times, and 8 times the matching accuracy of \( b_1 \) resistor, respectively.
12.7) The worst case DNL happens at the transition from nominal 7LSB to 8LSB, assuming -0.5\% error for C, 2C, and 4C, and +0.5\% error for 8C.

\[ \text{DNL} = 8\text{LSB} \times (1.05) - 7\text{LSB} = 0.75 \text{LSB} \]

12.8) The largest resistance, in this case, is \(2^{\frac{N}{2}}R\) while the smallest resistance is \(2R\). Therefore, the resistance ratio is \(2^{\frac{N}{2} - 1}\). Noting the resistance ratio for a binary-scaled A/D is \(2^{N-1}\), we have:

Resistance ratio improvement = \(2^{\frac{N}{2}}\) times

12.9)

Resistance Ratio = \(16R/(2R) = 8\)

Resistance Ratio = \(4R/(2R) = 2\)
The equivalent circuit for current calculation in 2R-branches is shown below:

The 100-mV voltage sources model the voltage drop of the switches. One end of 2R-resistor is connected to -100mV. Therefore, they can all be connected together and the 100 mV voltage-source can be moved and added to the Vref source. This will not change the current calculations of 2R-resistors.

\[-V_{\text{ref}} + 100 \text{ mV} = -(V_{\text{ref}} - 100 \text{ mV})\]

In effect, \( V_{\text{ref}} \) has decreased by 100 mV and the circuit still operates as if there is no voltage drop across the switches.
12.11) If \( R_A = 2.01 \, R_B \), the output error is:

\[
(\frac{2}{2.01} - 1) \times 16 \text{ LSB} = 0.08 \text{ LSB}
\]

If \( R_C = 2.01 \, R \), the output error is:

\[
(\frac{2}{2.01} - 1) \times 1 \text{ LSB} = 0.005 \text{ LSB}
\]

It is obvious that precision for \( R_A \) is more important than the precision for \( R_C \).

12.12) Assuming only \( b_1 = 1 \), the current drawn through \( R_f \) is \( \frac{I}{2} \).

" " \( b_2 = 1 \), " " \( b_3 = 1 \), the equivalent circuit to find the current through \( R_f \) is:

\[
R_X = R + \frac{2R}{3} = \frac{5}{3} \, R
\]

\[
I_X = I \times \frac{R}{R + \frac{5}{3} R} = \frac{3}{8} \, I
\]

\[
I_f = \frac{2R}{2R + R} \times I_X = \frac{1}{4} \times I
\]

With similar analysis, it can be shown that the current through \( b_4 \) and \( R_f \) is \( \frac{I}{8} \). Therefore,

\[
V_o = 2R_f I \left( 2^{-1} b_1 + 2^{-2} b_1 b_2 + 2^{-3} b_3 + 2^{-4} b_4 \right)
\]

(Cont.)
12.12) (cont.) The equivalent circuit for the open circuit time-
constant analysis is shown:

where

\[ R = 10 \text{ k}\Omega, \; C = 0.5 \text{ pF} \]

\[ \tau = C_3 \frac{R}{2} + C_2 \frac{5R}{8} + C_1 \frac{21R}{32} = 1.78125 \text{ RC} = 8.9 \text{ nsec} \]

\[ \Rightarrow \omega_{3dB} = \frac{1}{\tau} = 2\pi \times 17.9 \text{ MHz} \]

12.13) Assuming \( R_f = 2 \text{ k}\Omega \) \( \Rightarrow \) \( 8 \text{ LSB} = 2 \text{ V} \) \( \Rightarrow \) \( 1 \text{ LSB} = 0.25 \text{ V} \)

For "0000" input, \( V_o = (0 + 0.15) \text{ LSB} = 0.0375 \text{ V} \)

For "1000" input, \( V_o = (8 + 0.15 + \frac{0.2}{2}) \text{ LSB} = 2.0625 \text{ V} \)

For "1111" input, \( V_o = (15 + 0.15 + 0.2) \text{ LSB} = 3.8375 \text{ V} \)

12.14) \[ \frac{0.5}{\frac{2(4-1)}{8}} = \frac{0.5}{8} = 62.5 \text{ mV} \]
12.15) Let's denote the voltage of the node connecting $Q_1$, $Q_2$, and $Q_3$ by $V_x$. The following table shows $V_x$ as a function of $d_i$ and $\bar{d}_i$.

<table>
<thead>
<tr>
<th>$d_i$</th>
<th>$\bar{d}_i$</th>
<th>$V_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>$-V_{ss}$</td>
<td>$-V_{gs2}$</td>
</tr>
<tr>
<td>$-V_{ss}$</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$-V_{ss}$</td>
<td>$-V_{ss}$</td>
<td>$-V_{ss}$</td>
</tr>
</tbody>
</table>

The $V_x$ waveforms:

overlapping $d_i$ and $\bar{d}_i$

12.16) Let's denote the voltage of the node connecting $Q_1$, $Q_2$, and $Q_3$ by $V_x$.

$V_x$ waveforms:

non-overlapping $d_i$ and $\bar{d}_i$
Using (1.67) with $I_D = 0.1 I_{ref} = 5 \mu A$, we have

$$5 \mu = \frac{M_n C_ox}{2} \frac{W}{L} (V_{GS} - V_t)^2 = (46 \mu) \frac{W}{L} (3 - 1)^2$$

$$\Rightarrow \frac{W}{L} = 0.027$$

$$\Delta I_D = g_m \Delta V_{GS}$$

where $g_m = \frac{M_n C_ox}{2} \frac{W}{L} (V_{GS} - V_t) = 96 \mu \times 0.027 \times 2 = 5 \frac{\mu A}{V}$

and $\Delta V_{GS} = 1 mV$

$$\Rightarrow \Delta I_D = 5 \frac{\mu A}{V} \times 1 mV = 5 nA$$

12.18) \hspace{1cm} 50 \mu = (46 \mu) \frac{W}{L} (3 - 1)^2 \Rightarrow \frac{W}{L} = 0.27$

$$g_m = \frac{2 I_D}{V_{GS} - V_t} = \frac{100 \mu}{2V} = 50 \frac{\mu A}{V}$$

$$\Rightarrow \Delta I_D = g_m \Delta V_{GS} = 50 \frac{\mu A}{V} \times 1 mV = 50 nA$$