An Approach for
Tuning High-Q Continuous-Time Bandpass Filters

K. A. Kozma, D. A. Johns and A. S. Sedra

University of Toronto
Department of Electrical and Computer Engineering
Toronto, Ontario, M5S 1A4, Canada
karen@eeec.utoronto.edu
(416) 978-3381

ABSTRACT
A new technique for tuning high-Q continuous-time filters
is presented. This technique is a modified version of an
adaptive tuning method where the tuning signals are sinusoids
generated by digital bandpass delta sigma oscillators. As will
be shown, the required building blocks for the tuning system
are simple. Simulation results are presented to show the
feasibility of the proposed approach.

INTRODUCTION
In many communication channels (particularly wireless)
there is a need for accurate high-Q bandpass filters. However,
as with any integrated filter, the transfer-function is not
necessarily exact after fabrication, hence, the need for tuning
[1]-[4]. This paper discusses the tuning of continuous-time
bandpass filters with a modified version of the adaptive tuning
method [5]. Instead of using discrete-time tuning signals,
sinusoidal signals are employed. It will be shown that these
sinusoidal signals are easily generated using a digital bandpass
delta sigma ($\Delta\Sigma$) oscillator. This oscillator is simply
digital oscillator, the circuitry of which is greatly simplified
by using a bandpass $\Delta\Sigma$ modulator. The $\Delta\Sigma$ modulator
converts a multi-bit signal to a single-bit stream so that
complex and time-consuming operations, such as
multiplication, are simplified. The bandpass $\Delta\Sigma$ modulator
lends itself nicely to this application as the inherent noise
shaping, which occurs with this modulator, works well with
high-Q bandpass filters. As will be shown, the tuning input
and reference signals, generated by the proposed oscillator,
are practically noiseless for frequencies within the passband
of the filter; but the noise increases for frequencies further
away from the passband. However, this out-of-band noise is
not detrimental since for high-Q filters, this noise is
significantly attenuated. Therefore, higher-Q bandpass filters
are more attractive when using the bandpass $\Delta\Sigma$ oscillators.

This paper begins by briefly reviewing the adaptive tuning
system and describes the use of sinusoidal tuning signals. The
concept is shown to be valid through a simulation example
where a fourth-order bandpass filter, with a Q-factor of 27, is
tuned. Next, the proposed bandpass $\Delta\Sigma$ oscillator is explained.
Then, the generation of the sinusoidal tuning signals, using
the bandpass $\Delta\Sigma$ oscillator, is described. Lastly, a simulation
element is presented where the poles of the same fourth-order
bandpass filter are tuned using the bandpass $\Delta\Sigma$ oscillator to
generate the tuning signals.

Fig. 1 The adaptive tuning system.

THE ADAPTIVE TUNING TECHNIQUE USING
SINUSOIDAL TUNING SIGNALS
The adaptive tuning technique, which has been shown to be
effective for tuning continuous-time low-pass filters is
illustrated in Fig. 1. Basically, the adaptive approach requires
two signals to be generated: a tuning input and a reference
signal. The reference signal represents the output which the
ideal filter would produce for the given tuning input. As in [5],
the adaptive algorithm used is the least-mean-squared (LMS)
algorithm. The LMS algorithm adjusts coefficients of the filter
that determine the poles and zeros until the mean-squared-
derror value, MSE($e(t)$), is minimized. The LMS algorithm
uses the error signal, $e(t)$, and a gradient signal, to tune
the coefficients. The error signal, is simply the difference between
the reference signal and the output of the filter. Since the
structure chosen for the tunable filter, is an orthonormal ladder
filter, it can be shown that the necessary gradient signals are
easily generated by duplicating the tunable filter and swapping
some coefficients [5]. Therefore, in order to make the system
practical, simple tuning input and reference signals should be
devised.

Although a pseudo-random sequence was used as the
tuning input in previous instances [5] [6], it is not appropriate
tuning bandpass filters as it is spectrally rich over a large
range of frequencies. As a result, most of the useful
information is severely attenuated by a bandpass filter. Thus
an input signal that is spectrally rich in the band of interest,
namely the passband, appears to be a better choice. For the
bandpass case, the authors suggest the use of a sum of
sinusoids, the frequencies of which are placed within the
passband of the desired transfer-function, so that its
information is useful and not greatly attenuated. Given that
the input is a sum of sinusoidal signals, the reference signal
would also be a sum of sinusoidal signals as the filter is a
linear system.

Consider, the maximally-flat fourth-order bandpass filter
centered at 0.7588 rad/s and having a Q-factor of 27:

$$f_{\text{ideal}} = \frac{7.8609 \times 10^{-1}}{\sqrt{(r^2 + 0.0201r + 0.5910)^2 + (0.0196r + 0.5699)^2}}$$

(1)

to be the ideal transfer-function. A plot of this transfer-function is shown in Fig. 2. If the following sum of five sinusoidal waves:

$$\text{input} = \sin (0.7500t) + \sin (0.7566t) + \sin (0.7588t) + \sin (0.7634t) + \sin (0.7667t)$$

(2)

is the input to the filter in (1), the ideal output can be easily calculated to also be a sum of five sinusoids at the same frequencies but with the following gain-factors and phase shifts:

$$y_{\text{ideal}} = 0.9283 \sin (0.7500t + 0.9805) + 0.9997 \sin (0.7566t + 0.2229) + 1.0000 \sin (0.7588t + 0.0000) + 0.9945 \sin (0.7634t - 0.4748) + 0.9542 \sin (0.7667t - 0.8567).$$

(3)

Using the sum of sinusoids in (2) and (3) as the tuning input and reference signals respectively, and arbitrarily initializing the tunable filter to the transfer-function shown in Fig. 2 the adaptive tuning system was simulated. The coefficients which determine the poles and the finite zeros in the lower stopband were adapted while the two zeros at infinity were fixed. As expected the passband is accurately matched; however, the lower stopband is not exact since the tuning input focused on frequencies within the passband.

This simulation shows that the adaptive tuning system can accurately tune the passband of a bandpass filter when both the tuning input and reference signals are sinusoids. Therefore an oscillator is required to generate the tuning signals.

**BANDPASS DELTA SIGMA OSCILLATOR**

Since both of the required signals for the adaptive tuning system are sinusoids, a bandpass $\Delta\Sigma$ oscillator may be used to generate both signals. A bandpass $\Delta\Sigma$ oscillator, which is based on a low-pass $\Delta\Sigma$ oscillator [7], is depicted in Fig. 3. It uses the same principle as the low-pass $\Delta\Sigma$ oscillator, the only difference being that the fourth-order bandpass $\Delta\Sigma$ modulator shown at (a) in Fig. 3, replaces a second-order low-pass $\Delta\Sigma$ modulator. Hence, the bit stream produced by the modulator has a band-reject noise transfer-function instead of a high-pass noise transfer-function.

![Fig. 3 Digital bandpass $\Delta\Sigma$ oscillator.](image)

It can be shown that the frequency of oscillation is set by the product of the coefficients $a1$ and $a2$ and that the amplitude of oscillation is set by the initial value in register 1 [7]. Specifying the coefficient $a2$ to be a power of 2 makes this multiplication a simple shift operation and the difficult multiplication by the multi-bit coefficient, $a1$, is simplified as the bandpass $\Delta\Sigma$ modulator produces a single-bit output. Consequently, the multi-bit multiplication is reduced to a simple multiplexer as shown in Fig. 3.

To simplify the required circuitry of the $\Delta\Sigma$ modulator, the oscillator is defined to have its oscillation frequency around $f/4$. In other words, the sampling rate of the oscillator is defined to be four times the passband of the filter. Also, a fourth-order bandpass $\Delta\Sigma$ modulator was chosen so that there is at least second-order noise-shaping. The $\Delta\Sigma$ modulator used in this paper is illustrated in Fig. 4. Notice that the $\Delta\Sigma$

![Fig. 4 Fourth-order bandpass $\Delta\Sigma$ modulator.](image)

modulator consists of simple circuitry: delays, multi-bit adders and a latched comparator.

A plot of the spectrum of the output of the bandpass $\Delta\Sigma$ oscillator, $y_\text{out}$, is shown in Fig. 5. The frequency of oscillation here is $f/4$, but this can be changed somewhat by changing $a1$. However, the frequency of oscillation cannot be varied too
much as it becomes buried in the noise added by the ΔΣ modulator. Hence, the bandpass ΔΣ oscillator can be used to produce sinusoidal signals that are very close together in frequency which is exactly what is required for tuning high-Q bandpass filters.

GENERATION OF THE SINUSOIDAL TUNING SIGNALS

For the fourth-order bandpass filter discussed in this paper, five bandpass ΔΣ oscillators are employed to generate five sinusoidal signals at different frequencies within the passband of the filter. Thus the tuning input is a sum of the $y_\nu$ outputs of the bandpass ΔΣ oscillators. Notice that this sum is simple as it is just the addition of five single-bit streams.

Given this tuning input, a suitable reference signal must be generated. Although five oscillators are used, consider the input of a single sinusoid, $y_\nu$. As shown before, the ideal output should also be a sinusoidal signal with a specific gain and phase shift. However, this is not exactly the case here as the ΔΣ modulators do add noise to the sinusoidal signals, so that $y_\nu$ can be written as $\sin(\omega t) + e_n(t)$, where $e_n(t)$ is the noise added to the pure sinusoidal wave. Assuming that $e_n(t)$ is zero in the band of interest, the output of the ideal filter, $y_{\text{ideal}}(t)$, can be shown to be $k \sin(\omega t + A)$, where $k$ is the gain and $A$ is the phase shift of the filter at the frequency $\omega$ rad/s. From trigonometry, this is equivalent to:

$$y_{\text{ideal}}(t) = k \sin(\omega t) \cos(A) + k \cos(\omega t) \sin(A),$$

where $\cos(A)$ and $\sin(A)$ are simply scalars. Denoting $k \cos(A)$ by $\alpha$ and $k \sin(A)$ by $\beta$, (4) can be rewritten as:

$$y_{\text{ideal}}(t) = \alpha \sin(\omega t) + \beta \cos(\omega t).$$

It can be shown that the output, $y_\nu$, at (b) in Fig. 3 is approximately 90 degrees out of phase to $y_\nu$. Thus the $\cos(\omega t)$ component of (5), can be obtained from $y_\nu$ so that the reference signal can be generated as follows:

$$y_{ref}(t) = \alpha y_\nu + \beta y_\nu.$$  

Now, only $\alpha$ and $\beta$ have to be determined. Since $y_\nu$ is not exactly 90 degrees out of phase with $y_\nu$, and $y_\nu$ and $y_\nu$ do have an added noise content, the calculation of $\alpha$ and $\beta$ is not obvious. Hence the approach used in [5], to determine the reference signal, is utilized here.

As in [5], simulations are performed using the adaptive tuning system to tune the $\alpha$ and $\beta$ coefficients. The setup used in the simulations is illustrated in Fig. 6. In this case the reference signal is $y_{\text{ideal}}$, and the signal being adapted is $y_{\text{ref}}$. The coefficients being adjusted are $\alpha$ and $\beta$. As mentioned before, the LMS algorithm adjusts the coefficients by using the error signal, $e(t)$, and gradient signals. The gradient signals are the gradients $\partial y_{\text{ref}}/\partial \alpha$ and $\partial y_{\text{ref}}/\partial \beta$. It is obvious from (6), that these gradients are:

$$\frac{\partial y_{\text{ref}}}{\partial \alpha} = y_\nu; \quad \text{and} \quad \frac{\partial y_{\text{ref}}}{\partial \beta} = y_\nu.$$  

Hence, all the signals required for adapting the reference signals are readily available. The dotted boxes in Fig. 6, represent filters that are used in the simulations to reduce the noise added by the ΔΣ modulators and thus isolate the useful signals in the band of interest. This does not interfere with the tuning of $\alpha$ and $\beta$ as any gain or phase shift caused by the filters is applied to both paths: $y_{\text{ref}}$ and $y_{\text{ideal}}$. Therefore, the filters only aid in speeding up the simulations and their removal should not yield different results. For simplicity the transfer-function in (1) is used for $T$.

Performing this simulation on each of the five bandpass ΔΣ oscillators, used to generate the tuning input, give the specific $\alpha$’s and $\beta$’s, required by (6). Therefore, only five bandpass ΔΣ oscillators are needed to create both the tuning input and reference signals. Fig. 7 illustrates sample waveforms of the tuning input and reference signals.

Example

A simulation example was performed to validate the use of sinusoidal signals, generated by bandpass ΔΣ oscillators, for tuning the passband of the fourth-order bandpass filter. Since the adaptive algorithm places more emphasis on tuning the
passband and hence the poles of a filter, for a white noise input [6], only the coefficients that directly affect the poles of the filter are tuned in this example. Also, it is not expected to have accurate stopband matching as the reference signal is only valid for frequencies within the passband of the filter. Any information at frequencies in the stopband of the filter is buried in noise added by the ΔΣ modulators.

The ideal, initial and tuned transfer-functions are shown in Fig. 8. As in the previous example, the same ideal transfer-function is used and the tunable filter is similarly initialized. After simulating the adaptive tuning system where both the tuning input and reference signals are generated by five ΔΣ oscillators the filter was tuned to the transfer-function shown in Fig. 8. Notice the accurate matching achieved in the passband.

CONCLUSIONS

A modified version of the adaptive tuning system was presented for tuning high-Q continuous-time bandpass filters. Instead of using discrete-time tuning input and reference signals, sinusoidal signals that are generated by bandpass ΔΣ oscillators are employed. The inherent noise shaping, due to the bandpass ΔΣ oscillators, works well with high-Q bandpass filters as they are used to produce sinusoidal signals close together in frequency and well within the passband of the filter. Furthermore, it has been shown that the circuitry required to realize these bandpass ΔΣ oscillators is simple. Simulation results were presented to show the feasibility of the proposed tuning approach.

REFERENCES


