Adaptive Impedance Matching

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ABSTRACT

In many applications, it is desirable to synthesize an adaptive impedance to match some "real world" system, so that when the target impedance changes, the matching impedance tracks this change and maintains a close match.

In this paper we present a scheme for adaptive impedance matching formulated in the Model Reference Adaptive Systems' Control (MRAS) framework [1-3]. The Model Reference adaptive controller can be implemented using Adaptive Delta-Sigma Filters [4] to improve the performance-to-cost ratio due to their inherent oversampling. We will refer to this combined framework for adaptive impedance matching as MRAS/AIM.

INTRODUCTION

The following echo-cancellation application from telephony is used to motivate and develop the key concepts in this paper.

An important function of the telephone company's linecard is to extract the far-end signal \( V_F \) in the presence of a near-end signal \( V_{RX} \) from the voltage at the tip/ring. This is presently accomplished by employing adaptive echo-cancellers operating at near-Nyquist rates. Another means of implementing the same function, with potentially several advantages, is to have the telephone company's linecard present an output impedance that is matched to the impedance of the subscriber-line in use (which varies considerably from call to call as the impedance of the line and the load presented by the subscriber's equipment changes). From figure 1, if the output impedance of the linecard, \( Z_L \), matches the impedance of the line, \( Z_L \), the far-end signal can be extracted from the voltage at tip/ring by means of a simple subtraction as shown.

The potential advantage of adaptive impedance matching for this application derives from the observation that transmission lines are adequately modelled by first or second order lumped circuits in the band of interest. This translates into adaptive IIR filters of equivalent order within the MRAS/AIM setup. For the same task, echo cancelling adaptive FIR filters are of much higher order, and cannot recover a distortion-free \( V_F \). Also, as they are implemented in conjunction with a fixed digital impedance generator, both functions can be combined with the MRAS/AIM approach presented here.

FORMULATION

The MRAS/AIM formulation is based on figure 2. The pas-

Figure 2: Impedance Synthesis Circuit

![Figure 2: Impedance Synthesis Circuit](image)

\[ V_F = H_g I_g \]

(1)

Figure 3: Transconductance SFG

![Figure 3: Transconductance SFG](image)
By adapting $H_{L}$, $Z_{L}$ can be made to match $Z_{f}$. The signal-flow graph of the impedance generator is shown in figure 3. The transconductance of the loop, $G_{x}$, and the generated impedance are:

$$G_{x} := \frac{I_{x}}{V_{x}} = (1 - G_{p} H_{x})^{-1} G_{p}; \quad G_{p} := P^{-1}$$

$$Z_{x} := \frac{V_{x}}{I_{x}} = P - H_{x}$$

Adaptation is carried out during a training or call-setup phase when there is no far-end signal present. The mismatch between $Z_{x}$ and $Z_{f}$ is reduced by adapting the transresistance $H_{x}$ to minimize the 2-norm of the difference between $I_{x}$ and $I_{f}$ using the LMS algorithm within the MRAS framework.

In the MRAS scheme (figure 4), the controller is adapted to make the plant-controller feedback loop approximate the reference model by minimizing the error between the loop and reference outputs. In the MRAS/AIM formulation, the "plant" is the passive admittance $1/P$, the "controller" is the transresistance $H_{x}$, and the reference model is the loop admittance $1/Z_{f}$. Under adaptation, the generated impedance $Z_{x}$ would converge towards the loop impedance $Z_{f}$, as $I_{x}$ converged towards $I_{f}$.

To make the plant-controller feedback loop approach the reference model in a minimum variance sense, we require that the adaptation rule minimize the error function:

$$J_{E} = \frac{1}{2} E(\eta \hat{e}^2 + (1 - \eta) V_{x}^2), \quad \eta \in (0, 1]$$

The scalar $\eta$ sets the relative importance of minimizing the size of the error signal versus minimizing the size of the control signal. To use the LMS algorithm, we replace the expectation operator in (4) with its operand and obtain the stochastic performance measure:

$$J_{LMS} = \frac{1}{2} (\eta \hat{e}^2 + (1 - \eta) V_{x}^2), \quad \eta \in (0, 1].$$

Since the controller is implemented in the discrete-time domain, we introduce antialiasing and reconstruction filters into the loop. Thus, the transresistance $H_{x}$ becomes a composite of LTI functions:

$$H_{x} = H_{RF} H_{DSP} H_{DSM} H_{AA}.$$  

The four LTI components are: the analog antialiasing filter, $H_{AA}$; the Delta-Sigma A/D converter approximated as $H_{DSM}$; the adaptive IIR filter, $H_{DSP}$; and the analog reconstruction filter, $H_{RF}$.

For notational brevity we introduce the convention of hatted symbols and operators. A hatted signal is the antialiased and sampled version of its hatless namesake, viz:

$$0 = H_{DSM} H_{AA} \hat{v},$$

while a hatted operator is a z-domain approximation of its s-domain hatless namesake over the band of interest (300 Hz to 4 kHz for telephony):

$$T(z) |_{s=j2\pi f, f \in [f_{l}, f_{u}]} = T(s) |_{s=j2\pi f, f \in [f_{l}, f_{u}]}.$$  

A final modification is made to the error measure to reflect time-discretization:

$$J = \frac{1}{2} (\eta \hat{\hat{e}}^2 + (1 - \eta) \hat{V}_{x}^2).$$

Each parameter, $p_{i}$, of $H_{DSP}$ is updated to minimize the instantaneous error measure $J$ according to the rule:

$$p_{i}(n + 1) = p_{i}(n) - \mu \frac{\partial J(n)}{\partial p_{i}(n)}, \quad \mu > 0.$$  


devoting the Gradient Signals

The derivations in this section closely follow [3], which uses results from [5].

The gradients of $J$ with respect to parameters $p_{i}$ are:

$$\frac{\partial J(n)}{\partial p_{i}(n)} := \nabla J = \eta \hat{\hat{e}} \nabla \hat{e} + (1 - \eta) \hat{V}_{x} \nabla \hat{V}_{x}.$$  

If the adaptive controller is implemented using an adaptive delta-sigma IIR filter, the oversampling ratio permits several approximations that minimize implementation complexity. $\hat{e}$ is approximated by noting that

$$\hat{e} = H_{DSM} H_{AA} (G_{t} - G_{x}) V_{s} = (\hat{G}_{t} - \hat{G}_{x}) \hat{V}_{s}.$$  

$$\text{(12)}$$
Therefore, the gradients of $\hat{e}$ with respect to the adaptive filter's coefficients are:

$$\nabla \hat{e} = - \nabla G_p \hat{V}_s.$$  

(13)

Combining (13) and (2), we get

$$\nabla \hat{e} = (1 - G_p H_e)^{-1} G_p \nabla H_e (1 - G_p H_e) (1 - G_p H_e) \nabla H_e \hat{V}_s$$

$$= (1 - G_p H_e)^{-1} G_p \nabla H_e \hat{I}_s.$$  

(14)

$\hat{V}_s$ can be approximated as

$$\hat{V}_s = (1 - G_p H_e) \nabla H_e \hat{I}_s \hat{V}_s = G_p \nabla H_e \hat{V}_s.$$  

(15)

The gradients of $\hat{V}_s$ are therefore:

$$\nabla \hat{V}_s = (\nabla G_p \hat{I}_s + \nabla H_e \hat{I}_s) \hat{V}_s = \hat{H}_s \hat{V}_s + \nabla H_e \hat{I}_s.$$  

(16)

And finally,

$$\nabla \hat{H}_s = (\hat{H}_s H_{RF} H_{DSP} H_{DSM} H_{AA}) = \hat{H}_s \nabla H_{RF} H_{DSP} H_{DSM} H_{AA}$$

(17)

Adaptive delta-sigma IIR filters are described in [4].

The relevant block diagrams are shown below in figure 4A–D. Broken lines indicate sampled data signals and solid lines continuous signals. Figure 5A is the block diagram of the actual analog-digital system; note that $I_s$ is sampled and filtered into $\hat{V}_s$, the process producing $\hat{I}_s$. Figure 5B shows that $I_s$ is sampled and digitally subtracted from $\hat{I}_s$ to produce the error signal $\hat{e}$, and $V_s$ is sampled to produce $\hat{V}_s$. Figure 5C shows the computation of $\nabla \hat{e}$, and figure 5D shows the computation of $\nabla \hat{V}_s$. A number of block-diagram rearrangements were possible because the system under consideration is scalar (SISO). Further optimization, particularly of the implementation-specific kind, is still possible.

**REALIZABILITY OF IMPEDANCE MATCH**

For a given finite dimensional, real-rational and proper target impedance, $Z_i$, to be realizable, it is necessary that $H_e$ as well as the feedback loop (figure 3) be asymptotically stable. The first requirement (internal stability) can be seen from (3), where it is clear that $P$ can be purely resistive and still yield a stable $H_e$ if $Z_i$ is stable. Loop stability, for a purely resistive $P$, is guaranteed if, in addition to $Z_i$ being stable, it is dissipative ($Z_i$ is Strictly Positive Real [8]):

$$\text{Re}(Z_i) > 0, \forall \omega \in \mathbb{R}.$$  

(18)

A proof based on the Argument Principle [7] is sketched here. Since $H_e$ is stable, the contour plot of $1 - G_p H_e$ must not encircle or pass through the origin for the loop to be stable. Sufficient for this is that $\text{Re}(1 - G_p H_e) > 0$, which, from (3), is synonymous with $\text{Re}(Z_i/P) > 0$. Since $P$ is purely resistive, the sufficiency condition in (18) follows.

Loop stability of the starting point is easy to ensure in practise because it depends only on $P$ and $H_e$, both of which are known and controllable. Loop stability of the end-point depends upon (18), which is guaranteed (sufficient but not necessary) if $Z_i$ is dissipative (passive with a non-zero resistive component, for example).

**SIMULATION RESULTS**

A second order IIR $Z_i$ was matched to the Australian linecard impedance specification shown in figure 6.

The antialiasing filter was a first order all-pole filter with the pole at 110 kHz. The reconstruction filter was a second order Butterworth with the poles at 80 kHz. The sampling rate was 4 MHz, which represented an oversampling ratio of 128 relative to a passband of 16 kHz. All delta-sigma filters were based on first order delta-sigma modulators. Bandpass filtered pseudorandom noise was used as the excitation signal; the noise shaping filter's passband sloped upwards from 300 Hz to 4 kHz to maximize Insertion Return Loss (IRL) at 4 kHz. $P = 10002$, $n = 1$. The starting point of the adaptation was pervers:
\[ Z = 220\Omega + 8200\Omega/120\mu F = \frac{1.15 \times 10^7 + 220}{11087 + s} \]  

**Figure 6:** Australian linecard impedance specification

\[ |Z|, \Omega \]

\[ Z \]

\[ f, \text{Hz} \]

\[ 500, 1000, 2000, 5000 \]

**Figure 7:** Matching impedance before and after adaptation

\[ H_{DSP} = 0. \] Plots of \( Z_s \) and \( Z_t \) versus frequency before and after adaptation are shown in figure 7. IRL curves before and after adaptation are plotted in figure 8.

\[ IRL = 20 \log \left( \frac{|Z_s + Z_t|}{|Z_s - Z_t|} \right) \]  \hspace{1cm} (1)

A plot of the averaged error signal versus iteration number is shown in figure 9. A characteristic feature of this adaptation is the sudden decrease in error in the vicinity of a minimum, associated with steep gradients due to abrupt, deep local minima. Better starting points end up in deeper minima.

**CONCLUSION**

This paper presented a scheme for adaptive impedance matching formulated within the Model Reference Adaptive Systems’ Control framework. Simulation results show good performance. Although the ideas were illustrated with an application from telephony, the basic principles are widely applicable.

**ACKNOWLEDGMENT**

The authors wish to express their gratitude to Messrs. Laurie Jones and Dave Foster of Bell-Northern Research, Ottawa for useful discussions that lead to this research.

**REFERENCES**

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[6] Bell Canada Loop Survey conducted in 1981. Private communication with Bell-Northern Research Ltd. These data are measurements of complex impedance over the telephone band of several hundred transmission lines of varying lengths. The data show a variation of \( |Z| \) in the range [270 \( \Omega \), 1250 \( \Omega \)] at 4000 Hz and [400 \( \Omega \), 1700 \( \Omega \)] at 500 Hz.
