University of Toronto

Term Test 2

Date — Nov 21, 2022: 12:10pm

Duration — 50 min

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, assume $g_m r_o \gg 1$
- Notation: 15e3 is equivalent to 15×10^3
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.
- If you need more space, write on the back of pages.

Last Name: ______________________________

First Name: ______________________________

Student #: ______________________________

Question	1	2	3	Total
Points:	5	5	5	15
Score:				

Grading Table

Q1. A transfer-function has the equation

$$T(s) = \frac{1e4(1 + s/1.2e5)}{(1 + s/1e2)(1 + s/3e3)}$$

[3] (a) Estimate the gain (in dB) at $\omega = 1.2\text{Mrad/s}$. For this estimate, assume $\omega = 1.2\text{Mrad/s}$ is much greater than all the pole/zero frequencies.

[2] (b) Estimate the phase (in degrees) at $\omega = 1.2\text{Mrad/s}$. For this estimate, take into account the zero at $1.2e5$ while assuming $\omega = 1.2\text{Mrad/s}$ is much greater than the pole frequencies.

Solution

(a) Since $\omega = 1.2\text{Mrad/s}$ is much greater than each of $1.2e5, 1e2, 3e3$ so we can ignore the "1" in each term so

$|T(j\omega)| \approx \frac{|1e4(j\omega/1.2e5)|}{|(j\omega/1e2)(j\omega/3e3)|}$

$|T(j\omega)| \approx \frac{|1e4(j1.2e6/1.2e5)|}{|(j1.2e6/1e2)(j1.2e6/3e3)|}$

$|T(j\omega)| \approx 2.08e{-2}$

and in dB, we have

$T_{dB} = 20 * log_{10}(|T(j\omega)|) = -33.6$ dB

(b) Since $\omega = 1.2\text{Mrad/s}$ is much greater than each of $1e2, 3e3$ we can write

$\angle T(j\omega) = \angle(1e4) + \angle(1 + j\omega/1.2e5) - \angle(1 + j\omega/1e2) - \angle(1 + j\omega/3e3)$

$\angle T(j\omega) \approx 0° + \angle(1 + j1.2e6/1.2e5) - 90° - 90°$

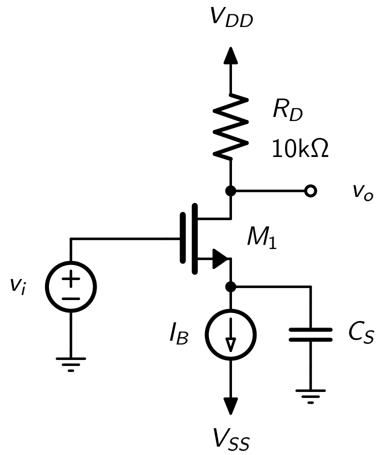
where we have used $\angle(1 + jk) \approx 90°$ when $k \gg 1$

$\angle T(j\omega) \approx \arctan[(1.2e6/1.2e5)/(1)] - 180°$

$\angle T(j\omega) \approx 84.29° - 180°$

$\angle T(j\omega) \approx -95.71°$

Q2. Consider the amplifier shown below where the current source I_B is ideal.



$g_{m1} = 1\text{mA/V}$

$r_{o1} \to \infty$

[4] (a) Find the value for C_S so that the low frequency cutoff is at 1kHz.

[1] (b) Is there a zero in the transfer-function? If so, what is the frequency location for the zero?

Solution

(a) The pole frequency for C_S is

$$F_{3dB} = 1/(2\pi C_S R_x)$$

where R_x is the small-signal resistance seen by C_S.

Since $r_{o1} \to \infty$, the impedance looking into the source of M_1 is $1/g_{m1}$ and since the current source I_B is ideal, we have

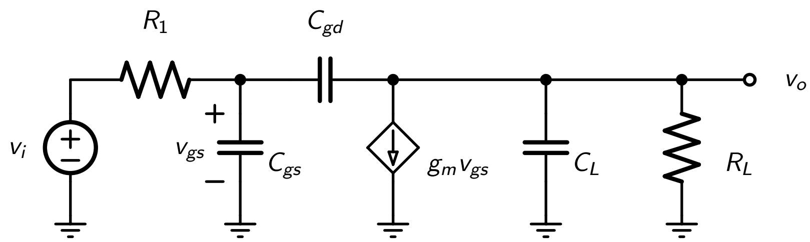
$R_x = 1/g_{m1} = 1/(1e{-}3) = 1\text{k}\Omega$

Using the F_{3dB} equation above, we find C_S as

$C_S = 1/(2 * \pi * F_{3dB} * R_x) = 1/(2 * (3.142) * (1e3) * (1e3)) = 159.2\text{nF}$

(b) Since the current source is ideal (with infinite output impedance), the gain for this circuit is zero at dc. As a result, there is a zero in the transfer-function and the zero frequency is at 0 Hz.

Q3. The small signal model for a common-source amp is shown below.



$R_1 = 10\text{k}\Omega$ $\quad C_{gd} = 100\text{fF}$ $\quad C_L = 1\text{pF}$

$C_{gs} = 1\text{pF}$ $\quad g_m = 2\text{mA/V}$ $\quad R_L = 30\text{k}\Omega$

[2] (a) Find the midBand gain A_M

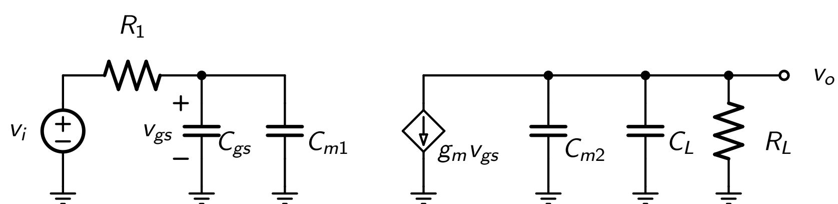
[3] (b) Use Millers Theorem to find the 2 pole locations, F_1 and F_2 in Hz.

Solution

(a) For the midband gain, we assume all the capacitors limiting the high freq gain are open circuited (in this case, all the capacitors in this circuit).

$A_M = -g_m * R_L = -(2e{-}3) * (30e3) = -60\text{V/V}$

(b) Using Millers Theorem, we break C_{gd} into 2 grounded capacitors, C_{m1}/C_{m2}



$C_{m1} = C_{gd} * (1 - A_M) = (100e{-}15) * (1 - (-60)) = 6.1\text{pF}$

$C_{m2} = C_{gd} * (1 - (1/A_M)) = (100e{-}15) * (1 - (1/(-60))) = 101.7\text{fF}$

So we have 2 poles and the 2 nodes in the circuit resulting in

$F_1 = 1/(2 * \pi * (C_{gs} + C_{m1}) * R_1) = 1/(2 * (3.142) * ((1e{-}12) + (6.1e{-}12)) * (10e3)) = 2.242\text{MHz}$

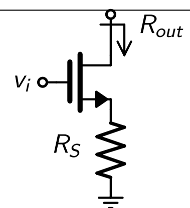
$F_2 = 1/(2 * \pi * (C_{m2} + C_L) * R_L) = 1/(2 * (3.142) * ((101.7e{-}15) + (1e{-}12)) * (30e3)) = 4.816\text{MHz}$

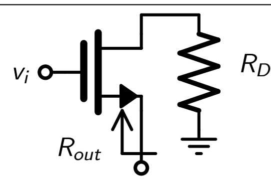
Equation Sheet

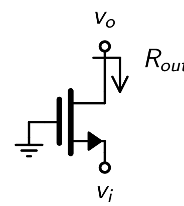
Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n{}'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p|v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p{}'|I_D)$







Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = ((1 + g_m r_o)/r_o)v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = -v_i/((1/g_m) + R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = g_m v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = v_i$	$v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^\circ}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;
Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$