University of Toronto

Term Test 2

Date — Nov 20, 2023: 4:10pm

Duration — 50 min

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Notation: 15e3 is equivalent to 15×10^3
- Non-programmable calculator is allowed; No other aids are allowed
- Write using a non-erasable ink.
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: ______________________________

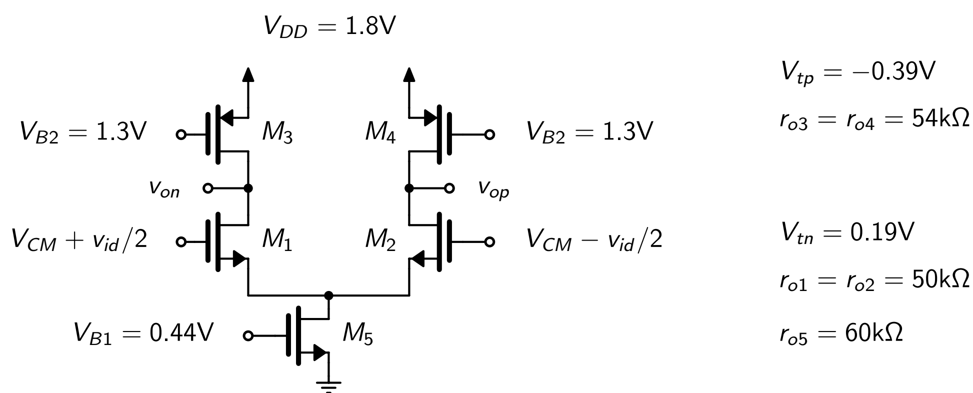
First Name: ______________________________

Student #: ______________________________

Question	1	2	3	Total
Points:	5	5	5	15
Score:				

Grading Table

Q1. Consider the circuit shown below where v_o is defined to be $v_{op} - v_{on}$ and all pmos transistors have $V_{ovp} = 0.11\text{V}$ while all nmos transistors have $V_{ovn} = 0.25\text{V}$. Also, $V_{CM} = 0.88\text{V}$ and $I_{D5} = 100\mu\text{A}$.



[3] (a) Find the small-signal gain v_o/v_{id}

[2] (b) Assuming the small-signal gain is so large that you can ignore the voltage swing on the input differential signal, find the max and minimum output voltage for v_{op} such that transistors remain in the active region.

Solution

(a) This a balanced circuit so we can find the gain of the half circuit M_1/M_3 assuming the source of M_1 is grounded.

In this circuit, $v_o/v_{id} = -(v_{on}/(v_{id}/2))$ due to the following...

Define $A_1 = v_{on}/(v_{id}/2)$ (A_1 is a negative gain)

$v_{on} = A_1(v_{id}/2)$ and $v_{op} = A_1(-v_{id}/2)$

$v_o = v_{op} - v_{on} = -A_1(v_{id}/2) - A_1(v_{id}/2) = -A_1 v_{id}$

So $v_o/v_{id} = -A_1$ where A_1 is the negative gain of the half circuit.

Carrying on, we have

$I_{D1} = I_{D5}/2 = (100e{-}6)/2 = 50\mu\text{A}$

$g_{m1} = (2 * I_{D1})/V_{ovn} = (2 * (50e{-}6))/(0.25) = 400\mu\text{A/V}$

$R_o = r_{o1}||r_{o3} = (50e3)||(54e3) = 25.96\text{k}\Omega$

$v_{on}/(v_{id}/2) = -g_{m1} * R_o = -(400e{-}6) * (25.96e3) = -10.38\text{V/V}$

$v_o/v_{id} = -v_{on}/(v_{id}/2) = -(-10.38) = 10.38\text{V/V}$

$$v_o/v_{id} = 10.38\text{V/V}$$

(b) With the assumption of a very large small-signal gain, we can assume the input voltage remains at the common-mode voltage, V_{CM}.

The maximum voltage for v_{op} occurs when M_4 is at the edge of triode/active. This occurs when the drain of M_4 one threshold voltage higher than the gate of M_4 (in other words, higher by $|V_{tp}|$).

$v_{op,max} = V_{B2} + |V_{tp}| = (1.3) + |(-0.39)| = 1.69\text{V}$

$$v_{op,max} = 1.69\text{V}$$

(another approach is to look for V_{SD} of M_4 reaching the overdrive voltage)

The minimum voltage for v_{op} occurs when M_2 is at the edge of triode/active. This occurs when the drain of M_2 is one threshold voltage below the gate of M_2

$v_{op,min} = V_{CM} - V_{tn} = (0.88) - (0.19) = 0.69\text{V}$

$$v_{op,min} = 0.69\text{V}$$

Q2. A transfer-function has the equation

$$T(s) = \frac{8.8e3(1 + s/1.19e5)}{(1 + s/8.8e1)(1 + s/2.29e3)}$$

[3] (a) Estimate the gain (in dB) at $\omega = 1.19\text{Mrad/s}$. For this estimate, assume $\omega = 1.19\text{Mrad/s}$ is much greater than all the pole/zero frequencies.

[2] (b) Estimate the phase (in degrees) at $\omega = 1.19\text{Mrad/s}$. For this estimate, take into account the zero at $1.19e5$ while assuming $\omega = 1.19\text{Mrad/s}$ is much greater than the pole frequencies.

Solution

(a) Since $\omega = 1.19\text{Mrad/s}$ is much greater than each of $1.19e5, 8.8e1, 2.29e3$ so we can ignore the "1" in each term so

$|T(j\omega)| \approx \frac{|8.8e3(j\omega/1.19e5)|}{|(j\omega/8.8e1)(j\omega/2.29e3)|}$

$|T(j\omega)| \approx \frac{|8.8e3(j1.19e6/1.19e5)|}{|(j1.19e6/8.8e1)(j1.19e6/2.29e3)|}$

$|T(j\omega)| \approx 1.25e{-2}$

and in dB, we have

$T_{dB} = 20 * log_{10}(|T(j\omega)|) = -38$ dB

(b) Since $\omega = 1.19\text{Mrad/s}$ is much greater than each of $8.8e1, 2.29e3$ we can write

$\angle T(j\omega) = \angle(8.8e3) + \angle(1 + j\omega/1.19e5) - \angle(1 + j\omega/8.8e1) - \angle(1 + j\omega/2.29e3)$

$\angle T(j\omega) \approx 0° + \angle(1 + j1.19e6/1.19e5) - 90° - 90°$

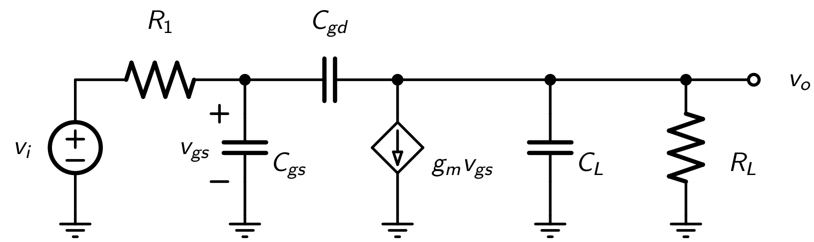
where we have used $\angle(1 + jk) \approx 90°$ when $k \gg 1$

$\angle T(j\omega) \approx \arctan[(1.19e6/1.19e5)/(1)] - 180°$

$\angle T(j\omega) \approx 84.29° - 180°$

$\angle T(j\omega) \approx -95.71°$

Q3. The small signal model for a common-source amp is shown below.



$R_1 = 8.8\text{k}\Omega$ $\quad C_{gd} = 77\text{fF}$ $\quad C_L = 770\text{fF}$

$C_{gs} = 1.3\text{pF}$ $\quad g_m = 1.8\text{mA/V}$ $\quad R_L = 26\text{k}\Omega$

[2] (a) Find the midBand gain A_M

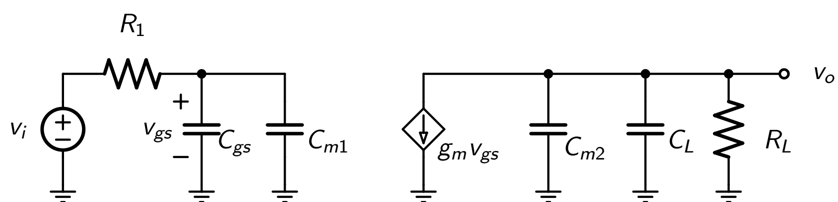
[3] (b) Use Millers Theorem to find the 2 pole locations, F_1 and F_2 in Hz.

Solution

(a) For the midband gain, we assume all the capacitors limiting the high freq gain are open circuited (in this case, all the capacitors in this circuit).

$A_M = -g_m * R_L = -(1.8e{-}3) * (26e3) = -46.8\text{V/V}$

(b) Using Millers Theorem, we break C_{gd} into 2 grounded capacitors, C_{m1}/C_{m2}



$C_{m1} = C_{gd} * (1 - A_M) = (77e{-}15) * (1 - (-46.8)) = 3.681\text{pF}$

$C_{m2} = C_{gd} * (1 - (1/A_M)) = (77e{-}15) * (1 - (1/(-46.8))) = 78.65\text{fF}$

So we have 2 poles and the 2 nodes in the circuit resulting in

$F_1 = 1/(2 * \pi * (C_{gs} + C_{m1}) * R_1) = 1/(2 * (3.142) * ((1.3e{-}12) + (3.681e{-}12)) * (8.8e3)) = 3.631\text{MHz}$

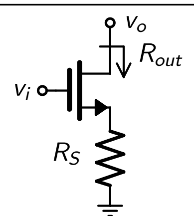
$F_2 = 1/(2 * \pi * (C_{m2} + C_L) * R_L) = 1/(2 * (3.142) * ((78.65e{-}15) + (770e{-}15)) * (26e3)) = 7.213\text{MHz}$

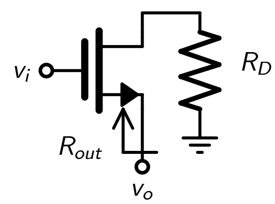
Equation Sheet

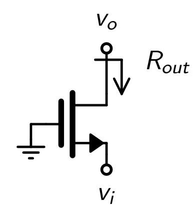
Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p| v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p'|I_D)$







Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$ $i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$R_{out} = r_o$ $i_{sc} = ((1 + g_m r_o)/r_o)v_i$ $v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$ $i_{sc} = -v_i/((1/g_m) + R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = v_i$	$R_{out} = r_o$ $i_{sc} = g_m v_i$ $v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^\circ}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;
Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$