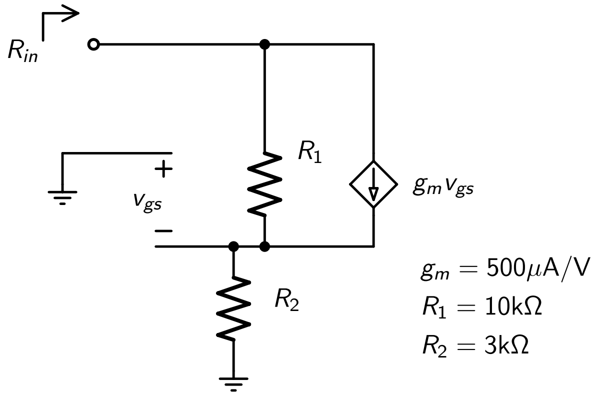
ECE 331 — Oct 22, 2024 — 50 min

Term Test 1

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Notation: 15e3 is equivalent to 15×10^3
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

[5] **Q1.** Derive the input impedance, R_{in} for the circuit below.



Solution

At the R_{in} node, apply a voltage v_x and determine i_x going into that node and by definition, $R_{in} = v_x/i_x$
Define the voltage at the top of R_2 to be v_2

$v_2 = i_x R_2$

$i_x = g_m v_{gs} + (v_x - v_2)/R_1$

$v_{gs} = -v_2$

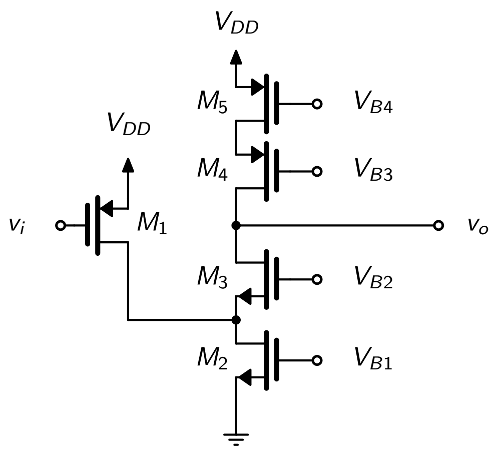
$i_x = g_m(-i_x R_2) + (v_x - i_x R_2)/R_1$

$i_x R_1 + i_x g_m R_1 R_2 + i_x R_2 = v_x$

$R_{in} = v_x/i_x = R_1 + R_2 + g_m R_1 R_2$

$R_{in} = R_1 + R_2 + g_m * R_1 * R_2 = (10e3) + (3e3) + (500e{-}6) * (10e3) * (3e3) = 28k\Omega$

[5] **Q2.** Consider the circuit shown below.



For all transistors
$g_m = 800\mu\text{A/V}$
$r_o = 18\text{k}\Omega$

Find the small-signal output impedance, R_{out} and small-signal gain, v_o/v_i.
For R_{out}, do NOT assume $g_m r_o \gg 1$
For i_{sc}, assume $\lambda = 0$ (in other words, $r_o \to \infty$ for all transistors)

Solution

Define R_{op} to be the impedance looking up into the drain of M_4
Since r_{o4} is the source impedance attached to the source of M_4

$R_{op} = r_{o4} + (1 + g_{m4} * r_{o4}) * r_{o5} = (18e3) + (1 + (800e{-}6) * (18e3)) * (18e3) = 295.2\text{k}\Omega$
Define R_{on} to be the impedance looking down into the drain of M_3
The source impedance attached to M_3 is
$R_x = r_{o1}||r_{o2} = (18e3)||(18e3) = 9\text{k}\Omega$ leading to
$R_{on} = r_{o3} + (1 + g_{m3} * r_{o3}) * R_x = (18e3) + (1 + (800e{-}6) * (18e3)) * (9e3) = 156.6\text{k}\Omega$
and so the output impedance is given by
$R_{out} = R_{op}||R_{on} = (295.2e3)||(156.6e3) = 102.3\text{k}\Omega$

$R_{out} = 102.3\text{k}\Omega$

For i_{sc}, we assume all $r_o \to \infty$ resulting in all of the drain current of M_1 going straight to the short circuit output. As a result,

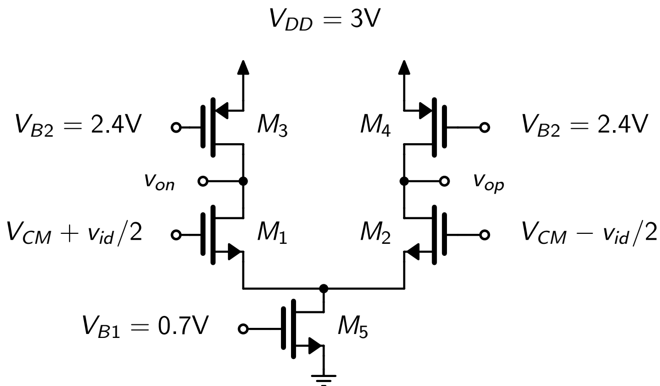
$i_{sc} = G_m v_i$ where $G_m = -g_{m1} = -(800e{-}6) = -800\mu\text{A/V}$

and we have

$v_o/v_i = G_m * R_{out} = (-800e{-}6) * (102.3e3) = -81.86\text{V/V}$

$v_o/v_i = -81.86\text{V/V}$

[5] **Q3.** Consider the circuit shown below where v_o is defined to be $v_{op} - v_{on}$ and the overdrive voltage for M_1 and M_2 is 0.1V. Also, $V_{CM} = 1.5$V and $I_{D5} = 100\mu$A.



$V_{tp} = -0.3$V

$r_{o3} = r_{o4} = 70\text{k}\Omega$

$V_{tn} = 0.25$V

$r_{o1} = r_{o2} = 50\text{k}\Omega$

$r_{o5} = 60\text{k}\Omega$

[3] (a) Find the small-signal gain v_o/v_{id}

[2] (b) Assuming the small-signal gain is so large that you can ignore the voltage swing on the input differential signal, find the max and minimum output voltage for v_{op} such that transistors remain in the active region.

Solution

(a) This a balanced circuit so we can find the gain of the half circuit M_1/M_3 assuming the source of M_1 is grounded.

In this circuit, $v_o/v_{id} = -(v_{on}/(v_{id}/2))$ due to the following...

Define $A_1 = v_{on}/(v_{id}/2)$ (A_1 is a negative gain)

$v_{on} = A_1(v_{id}/2)$ and $v_{op} = A_1(-v_{id}/2)$

$v_o = v_{op} - v_{on} = -A_1(v_{id}/2) - A_1(v_{id}/2) = -A_1 v_{id}$

So $v_o/v_{id} = -A_1$ where A_1 is the negative gain of the half circuit.

Carrying on, we have

$I_{D1} = I_{D5}/2 = (100e{-}6)/2 = 50\mu$A

$g_{m1} = (2 * I_{D1})/V_{ov1} = (2 * (50e{-}6))/(0.1) = 1\text{mA/V}$

$R_o = r_{o1}||r_{o3} = (50e3)||(70e3) = 29.17\text{k}\Omega$

$v_{on}/(v_{id}/2) = -g_{m1} * R_o = -(1e{-}3) * (29.17e3) = -29.17\text{V/V}$

$v_o/v_{id} = -v_{on}/(v_{id}/2) = -(-29.17) = 29.17\text{V/V}$

$$v_o/v_{id} = 29.17\text{V/V}$$

(b) With the assumption of a very large small-signal gain, we can assume the input voltage remains at the common-mode voltage, V_{CM}.

The maximum voltage for v_{op} occurs when M_4 is at the edge of triode/active. This occurs when the drain of M_4 one threshold voltage higher than the gain of M_4 (in other words, higher by $|V_{tp}|$).

$v_{op,max} = V_{B2} + |V_{tp}| = (2.4) + |(-0.3)| = 2.7\text{V}$

$$v_{op,max} = 2.7\text{V}$$

(another approach is to look for V_{SD} of M_4 reaching the overdrive voltage)

The minimum voltage for v_{op} occurs when M_2 is at the edge of triode/active. This occurs when the drain of M_2 is one threshold voltage below the gate of M_2

$v_{op,min} = V_{CM} - V_{tn} = (1.5) - (0.25) = 1.25\text{V}$

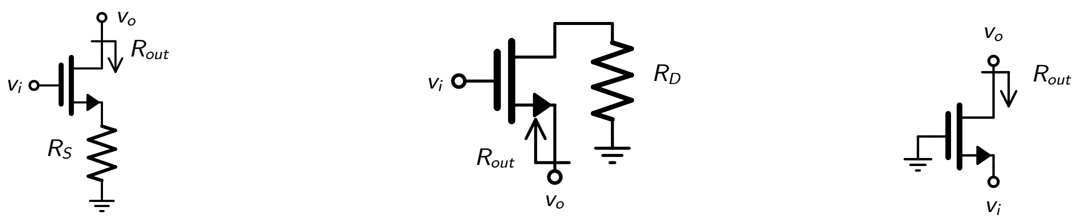
$$v_{op,min} = 1.25\text{V}$$

Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v_{DS}')$; $v_{DS}' = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n{}'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p|v_{SD}')$; $v_{SD}' = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p{}'|I_D)$



Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = ((1 + g_m r_o)/r_o)v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = -v_i/((1/g_m) + R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = g_m v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = v_i$	$v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

SEE NEXT PAGE ...

Feedback: $A_f = A/(1+A\beta)$; $x_i = (1/(1+A\beta))x_s$; $dA_f/A_f = (1/(1+A\beta))dA/A$; $\omega_{Hf} = \omega_H(1+A\beta)$; $\omega_{Lf} = \omega_L/(1+A\beta)$;

Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1+L)) + d/(1+L)$; $Z_{port} = Z_{p^o}((1+L_S)/(1+L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;

Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;

Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;

$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$