University of Toronto

Faculty of Applied Science & Engineering

Final Exam

Date — Dec 19, 2023: 2pm

Duration — 2hr 30min

ECE 331 — Analog Electronics

Exam Type: A

Non-programmable calculator is allowed

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Notation: 15e3 is equivalent to 15×10^3
- Grading indicated by [ ].
- Attempt all questions since a blank answer will certainly get 0.
- If you need more space, write on the back of pages.
- Assume $g_m r_o \gg 1$ unless otherwise stated

Last Name: ______________________________

First Name: ______________________________

Student #: ______________________________

Question	1	2	3	4	5	6	Total
Points:	6	6	6	6	6	6	36
Score:							

Grading Table

[2] **Q1.** (a) Assume the NMOS transistor parameters shown below. (Note: aF is ato Farads which is $10e-18$ Farads) Consider a transistor of size $W = 3\mu$m and $L = 180$nm that is biased in the active region with $V_{DB} = 0$. Find the values of C_{gs}, C_{gd}, and C_{db} in units of fF.

V_{tn}	0.3V
$u_n C_{ox}$	250μA/V^2
λ'_n	50nm/V
C_{ox}	8fF/μm^2
t_{ox}	4nm
L_{ov}	40nm
C_{db0}/W	300aF/μm

Solution

Note that C_{ox} is per sq μm so we need to leave W and L in μm units.

$C_{gs} = (2/3) * C_{ox} * W' * L' + C_{ox} * W' * L'_{ov} = (2/3) * (8e{-}15) * (3) * (0.18) + (8e{-}15) * (3) * (40e{-}3) = 3.84\text{fF}$

$C_{gd} = C_{ox} * W' * L'_{ov} = (8e{-}15) * (3) * (40e{-}3) = 960\text{aF}$

$C_{db} = C_{db0}/W * W' = (300e{-}18) * (3) = 900\text{aF}$

[2] (b) For the transistor in part (a), if $V_{ov} = 0.2$, find the unity gain freq of the transistor in Hz (do not ignore C_{gd})

Solution

$g_m = \mu_n C_{ox} * (W'/L') * V_{ov} = (250e{-}6) * ((3)/(0.18)) * (0.2) = 833.3\mu\text{A/V}$

$w_t = g_m/(C_{gs} + C_{gd}) = (833.3e{-}6)/((3.84e{-}15) + (960e{-}18)) = 173.6\text{Grad/s}$

$f_t = w_t/(2 * \pi) = (173.6e9)/(2 * (3.142)) = 27.63\text{GHz}$

If instead, you use the formula $f_t = 2\mu_n V_{ov}/(4\pi L^2)$, then you have

$f_t = 3 * (\mu_n C_{ox}/(C_{ox} * 10^12)) * V_{ov}/(4 * 3.14159 * (L^2)) = 3 * ((250e{-}6)/((8e{-}15) * 10^12)) * (0.2)/(4 * 3.14159 * ((180e{-}9)^2)) = 46.05\text{GHz}$

but in this case, C_{gd} is ignored as well as the gate-source overlap capacitance.

[2] (c) Besides noise, give 2 reasons why bias voltages for current mirrors are NOT sent a long distance in a microchip. (Explain clearly).

Solution

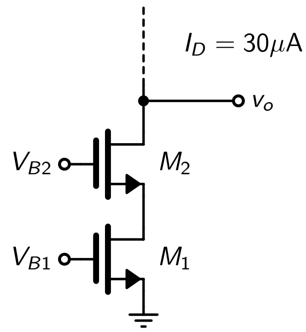
The current mismatch could be large due to 2 main reasons

1) transistor V_t mismatch can be large due to matching transistors being a long distance away from each other

2) dc currents in the ground or power lines can cause the transistor source voltages to be different for matched transistors that are located far away from each other.

[4] **Q2.** (a) Consider the wide-swing current mirror shown below where the desired output current is 30μA. Given that the minimum desired output voltage is 0.5V and the $L_1 = 2L_2$ while $W_1 = 2W_2$.

Find the lengths, L_1 and L_2 such that the current mirror output resistance is $60\text{M}\Omega$



V_{tn}	0.3V
$u_n C_{ox}$	160μA/V^2
λ'_n	40nm/V

Solution

Since M_1 and M_2 both have the same current, $I_{D1} = I_{D2}$, and $W_1/L_1 = W_2/L_2$, we then have $V_{ov1} = V_{ov2}$. So we have

$V_{o,min} = 0.5\text{V} = V_{ov1} + V_{ov2} = 2V_{ov1}$

$V_{ov1} = V_{o,min}/2 = (0.5)/2 = 0.25\text{V} = V_{ov2}$

We can find the transconductance as

$g_{m2} = 2 * I_{D2}/V_{ov2} = 2 * (30e{-}6)/(0.25) = 240\mu\text{A/V}$

We also have

$r_{o1} = L_1/(\lambda'_n I_{D1})$ and $r_{o2} = L_2/(\lambda'_n I_{D2})$

and since $L_1 = 2L_2$ and the currents are the same, we have

$r_{o1} = 2r_{o2}$

We also have

$R_{out} \approx g_{m2} r_{o2} r_{o1} = g_{m2}(2r_{o2}^2)$

$r_{o2} = sqrt(R_{out}/(2 * g_{m2})) = sqrt((60e6)/(2 * (240e{-}6))) = 353.6\text{k}\Omega$

And from the r_o formula, we have

$L_2 = r_{o2} * \lambda'_n * I_{D2} = (353.6e3) * (40e{-}9) * (30e{-}6) = 424.3\text{nm}$

$L_1 = 2 * L_2 = 2 * (424.3e{-}9) = 848.5\text{nm}$

[2] (b) For the above circuit, assume the bodies of both transistors are connected to ground.

When body effect is taken into account, do you expect the output resistance for this current mirror to increase or decrease?

(assume the dc bias current remains unchanged and both transistors remain in the active region)

Explain your answer.

Solution

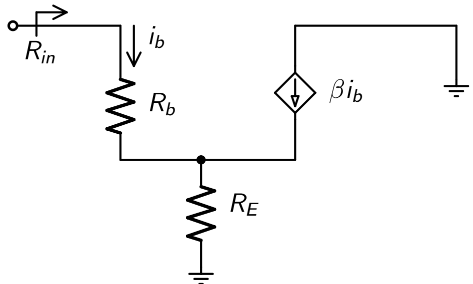
For M_1, $V_{SB} = 0$ so the body has no effect. For M_2, $V_{SB} > 0$ resulting in a body effect. If ΔV_{S2} is negative, I_{D2} increases due to $g_m 2$ and I_{D2} also increases due to g_{mb2} which can be modelled as an INCREASE in g_{m2} for M_2.

As a result, the output impedance will increase since $R_{out} \approx g_{m2} r_{o2} r_{o1}$.

r_{o2} and r_{o1} do not change due to the body effect.

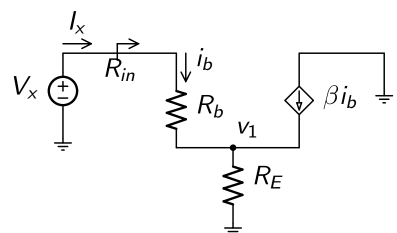
Q3.

[3] (a) Find an equation for the input impedance R_{in} for the circuit shown below.
β is a constant and β is the current gain for the current controlled current source.



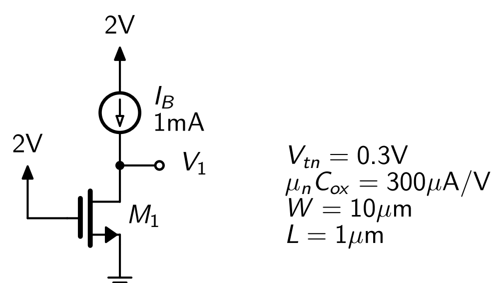
Solution

Apply a voltage V_x and determine the current I_x and $R_{in} \equiv V_x/I_x$



$i_b + (0 - v_1)/R_E + \beta i_b = 0$
$v_1 = (1+\beta)R_E i_b$
$i_b = (V_x - v_1)/R_b = (V_x - (1+\beta)R_E i_b)/R_b$
$V_x = i_b(R_b + (1+\beta)R_E)$
$i_b = I_x$
$R_{in} = V_x/I_x = V_x/i_b = R_b + (1+\beta)R_E$

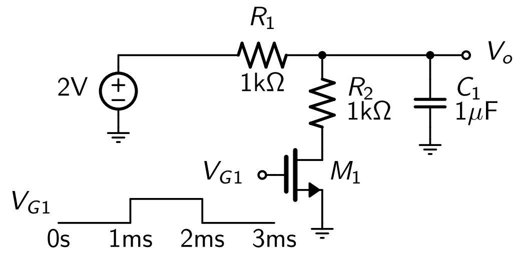
[3] (b) Find the value of V_1 by recognizing that M_1 is deep into the triode region so small-signal analysis can be used.



Solution

In triode, $r_{ds} = 1/(\mu_n C_{ox}(W/L)V_{ov})$
$V_G = 2$V
$V_{ov} = V_G - V_{tn} = (2) - (0.3) = 1.7$V
$r_{ds} = 1/(\mu_n C_{ox} * (W/L) * V_{ov}) = 1/((300e{-}6) * ((10e{-}6)/(1e{-}6)) * (1.7)) = 196.1\Omega$
$v_1 = I_B * r_{ds} = (1e{-}3) * (196.1) = 0.1961$V

[6] **Q4.** For the circuit below, treat M_1 as a switch with an infinite off resistance and an on-resistance of $R_{ds} = 200\Omega$. Assume that V_{G1} is initially 0V and goes high at time $t_{on} = 1\text{ms}$ and low again at time $t_{off} = 2\text{ms}$. Find the output voltage, V_o, at times $t_1 = 2\text{ms}$ and $t_2 = 3\text{ms}$.



Solution

When switch M_1 turns on, the initial voltage, V_{0+}, is

$V_{0+} = V_B = (2) = 2\text{V}$

and the final voltage, V_∞ is

$V_\infty = V_B * (R_2 + R_{ds})/(R_2 + R_{ds} + R_1) = (2) * ((1e3) + (200))/((1e3) + (200) + (1e3)) = 1.091\text{V}$

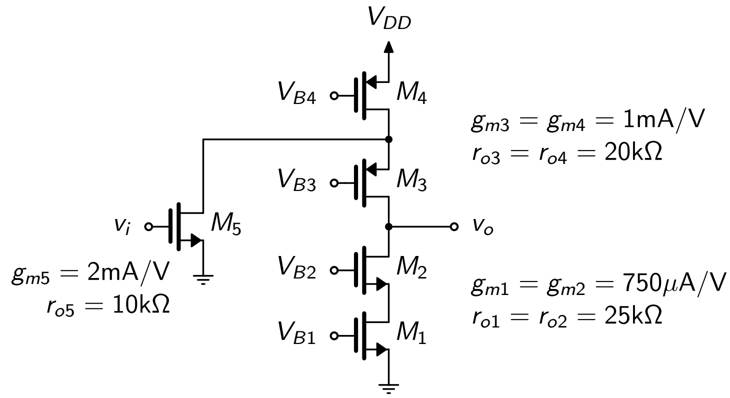
$\tau_{on} = (R_1||(R_2 + R_{ds})) * C_1 = ((1e3)||((1e3) + (200))) * (1e{-6}) = 545.5\mu\text{s}$

$V_{o,t1} = V_\infty - (V_\infty - V_{0+}) * e^(- (t_1 - t_{on})/\tau_{on}) = (1.091) - ((1.091) - (2)) * (2.718)^(- ((2e{-3}) - (1e{-3}))/(545.5e{-6})) = 1.236\text{V}$

$\tau_{off} = R_1 * C_1 = (1e3) * (1e{-6}) = 1\text{ms}$

$V_{o,t2} = 2 - (2 - V_{o,t1}) * e^(- (t_2 - t_1)/\tau_{off}) = 2 - (2 - (1.236)) * (2.718)^(- ((3e{-3}) - (2e{-3}))/(1e{-3})) = 1.719\text{V}$

Q5.



For the circuit above, find the small-signal gain v_o/v_i in 2 ways as described below.

[3] (a) Find v_o/v_i by assuming $g_m r_o \gg 1$

Solution

Define R_{on} to be the impedance seen looking into the drain of M_2

$R_{on} = g_{m2} * r_{o2} * r_{o1} = (750e{-}6) * (25e3) * (25e3) = 468.8\text{k}\Omega$

Define R_{op} to be the impedance seen looking into the drain of M_3

$R_{op} = g_{m3} * r_{o3} * (r_{o4}||r_{o5}) = (1e{-}3) * (20e3) * ((20e3)||(10e3)) = 133.3\text{k}\Omega$

$R_{out} = R_{op}||R_{on} = (133.3e3)||(468.8e3) = 103.8\text{k}\Omega$

Since $g_m r_o \gg 1$, all of the i_{D5} current goes to the short circuit output at v_o when finding i_{sc} so we have

$i_{sc}/v_i = g_{m5}$

leading to

$v_o/v_i = -g_{m5} * R_{out} = -(2e{-}3) * (103.8e3) = -207.6\text{V/V}$

[3] (b) Find v_o/v_i WITHOUT assuming $g_m r_o \gg 1$

Solution

Assume the output node v_o is shorted and find the drain voltage at v_{d5} relative to v_i

The impedance looking into the source of M_3, R_{S3} is given by

$R_{S3} = (1/g_{m3})||r_{o3} = (1/(1e{-}3))||(20e3) = 952.4\Omega$

The impedance at the drain of M_5 to ground is

$R_{o5} = r_{o5}||r_{o4}||R_{S3} = (10e3)||(20e3)||(952.4) = 833.3\Omega$

$v_{d5}/v_i = -g_{m5} * R_{o5} = -(2e{-}3) * (833.3) = -1.667\text{V/V}$

We now have a common-gate amplifier to get to the output, v_o, and the short circuit current is given by

$i_{sc} = ((1 + g_{m3}r_{o3})/r_{o3})v_{d5}$

So defining $G_m = i_{sc}/v_i$, we have

$G_m = ((1 + g_{m3} * r_{o3})/r_{o3}) * v_{d5}/v_i = ((1 + (1e{-}3) * (20e3))/(20e3)) * (-1.667) = -1.75\text{mA/V}$

To find R_{out}, we have

$R_{on} = r_{o2} + (1 + g_{m2} * r_{o2}) * r_{o1} = (25e3) + (1 + (750e{-}6) * (25e3)) * (25e3) = 518.8\text{k}\Omega$

$R_{op} = r_{o3} + (1 + g_{m3} * r_{o3}) * (r_{o5}||r_{o4}) = (20e3) + (1 + (1e{-}3) * (20e3)) * ((10e3)||(20e3)) = 160\text{k}\Omega$

$R_{out} = R_{on}||R_{op} = (518.8e3)||(160e3) = 122.3\text{k}\Omega$

We can now find v_o/v_i as

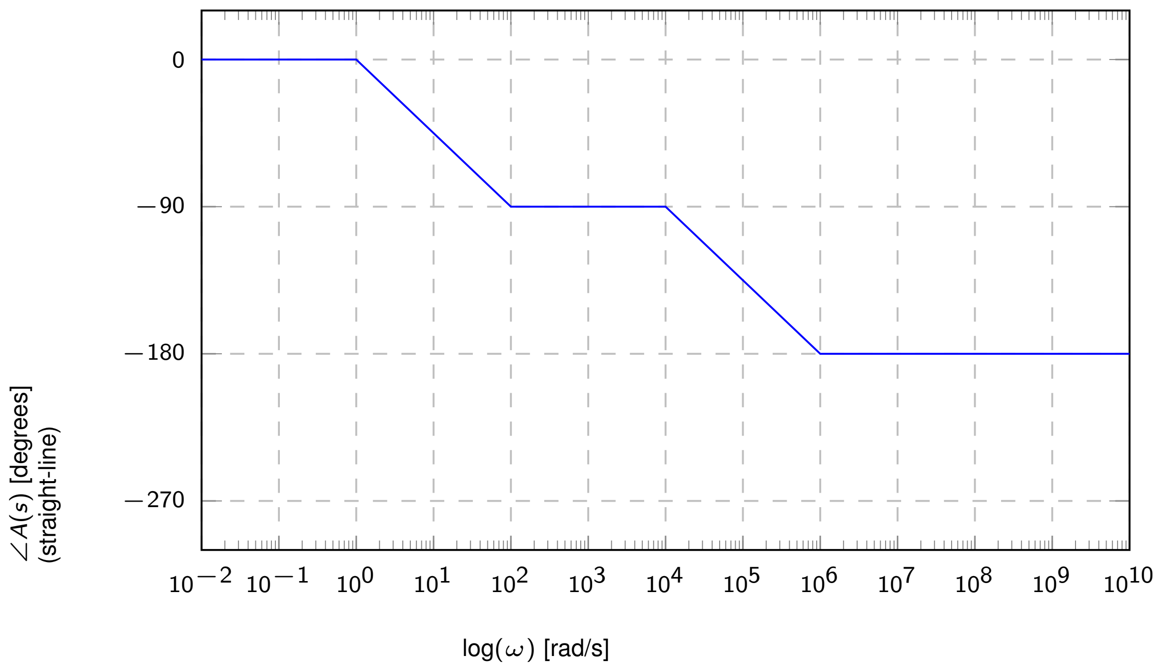
$v_o/v_i = G_m * R_{out} = (-1.75e{-}3) * (122.3e3) = -214\text{V/V}$

Q6.

Assume an opamp is ideal with the transfer-function,

$$A(s) = \frac{k_{dc}}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

The straight-line Bode PHASE plot for the amplifier is shown below and the dc gain is given by $k_{dc} = 20e3$



[2] (a) Find the values for ω_{p1} and ω_{p2} in rad/s

Solution

The pole frequencies can be found by looking at the straight-line Bode phase plot.
Assuming the poles are widely spaced apart, the phase at where the poles occur are at $-45°$, $-135°$
Therefore, the pole locations are
$\omega_{p1} = 10\text{rad/s}$
$\omega_{p2} = 100\text{krad/s}$

[4] (b) If this opamp is used in an NON-INVERTING configuration, what is the smallest dc closed-loop amplifier gain that will result in a phase margin of 65°? (use the actual $A(s)$ and NOT the straight-line Bode plot).

Solution

Given, we want $\text{PM} = 65°$, we can find ω_1
$PM = 90 - tan^{-1}(\omega_1/\omega_{p2})$
$\omega_1 = \omega_{p2} * tand(90 - \text{PM}) = (100e3) * tand(90 - (65)) = 46.63\text{krad/s}$
and define $K_{PM} = \omega_1/\omega_{p2} = 0.4663$
We can also use the following equation to find the dc loop gain that would result in this PM with these 2 poles ω_{p1} and ω_{p2}.
$\omega_{p1} = (\omega_1(1 + K_{PM}^2)^{1/2})/L_0$ and solve for L_0
$L_0 = (\omega_1 * sqrt(1 + {K_{PM}}^2))/\omega_{p1} = ((46.63e3) * sqrt(1 + (0.4663)^2))/(10) = 5.145e3$
However, the opamp dc gain is $k_{dc} = 20e3$ so it should be reduced by β given by
$\beta = L_0/k_{dc} = (5.145e3)/(20e3) = 0.2573$

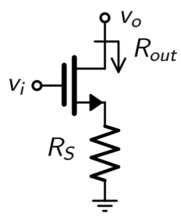
This value of β would result in a dc closed-loop gain of $1/\beta$ which is
$A_{cl} = 1/\beta = 1/(0.2573) = 3.887\text{V/V}$
A larger gain would have a phase-margin greater than $65°$ while a lower gain would have a phase-margin less than $65°$.

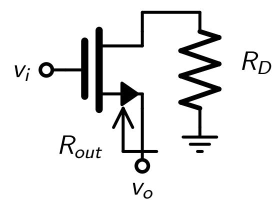
Equation Sheet

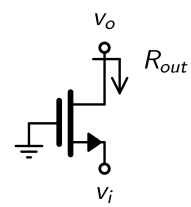
Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_n C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n{}'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p|v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p{}'|I_D)$







Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$ $i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$R_{out} = r_o$ $i_{sc} = ((1 + g_m r_o)/r_o)v_i$ $v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$ $i_{sc} = -v_i/((1/g_m) + R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = v_i$	$R_{out} = r_o$ $i_{sc} = g_m v_i$ $v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\dots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\dots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^\circ}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$;
$GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Phase Margin: $PM = 90^\circ - tan^{-1}(\omega_1/\omega_{p2})$; $K_{PM} \equiv \omega_1/\omega_{p2}$; $\omega_{p1} \approx (K_{PM}\omega_{p2}(1 + K_{PM}^2)^{1/2})/L_0$

Body Effect: $V_t = V_{t0} + \gamma(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})$; $\gamma = \sqrt{2qN_A\epsilon_s}/C_{ox}$; $g_{mb} = \chi g_m$; $\chi = \gamma/(2\sqrt{2\phi_f + V_{SB}})$