## Problem Set 3B - MultiStage

## Question 1

It is desired to create a voltage output from a small current source input (say from a photodetector). The maximum current source amplitude is $10 \mu \mathrm{~A}$. The figure below shows 2 circuits. Circuit (a) does not make use of a transistor while circuit (b) makes use of one transistor. $R_{i}$ is the output impedance of the current source and $i_{i}$ is the input current source. $V_{B}$ is a dc bias voltage. Also, assume the current source $I_{B}$ is ideal.

(a)

(b)

Use small-signal analysis to answer the following
(a) Find the maximum $v_{o, \text { max }}$ for circuit (a)
(b) Find the maximum $v_{o, \max }$ for circuit (b)

## Solution

(a) $R_{o}=R_{i}\left\|R_{D}=(10 e 3)\right\|(100 e 3)=9.091 \mathrm{k} \Omega$
$i_{s c}=i_{i}$ and we have $v_{o}=i_{i} R_{o}$ resulting in
$v_{o} / i_{i}=R_{o}=(9.091 e 3)=9.091 \mathrm{k} \Omega$
For $i_{i, \max }=10 \mu \mathrm{~A}$, we have
$v_{o, \max }=i_{i, \max } *\left(v_{o} / i_{i}\right)=(10 e-6) *((9.091 e 3))=90.91 \mathrm{mV}$
(b) We can start by finding the output impedance, $R_{o}$

Define $R_{d x}$ to be the small signal resistance looking into the drain of $M_{1}$
$R_{d x}=r_{o 1}+\left(1+g_{m 1} * r_{o 1}\right) * R_{i}=(20 e 3)+(1+(1 e-3) *(20 e 3)) *(10 e 3)=230 \mathrm{k} \Omega$
$R_{o}=R_{d x}\left\|R_{D}=(230 e 3)\right\|(100 e 3)=69.7 \mathrm{k} \Omega$
Next, we find the short circuit current, $i_{\text {sc }}$
We have the following small circuit circuit


Defining $R_{S x}$ to be the impedance looking in to the source of $M_{1}$ we have
$R_{s x}=\left(1 / g_{m 1}\right)\left\|r_{o 1}=(1 /(1 e-3))\right\|(20 e 3)=952.4 \Omega$
and we see a current divider, so we have
$i_{s c} / i_{i}=R_{i} /\left(R_{i}+R_{s x}\right)=(10 e 3) /((10 e 3)+(952.4))=0.913 \mathrm{~A} / \mathrm{A}$ leading to
$v_{o} / i_{i}=i_{s c} / i_{i} * R_{o}=(0.913) *(69.7 e 3)=63.64 \mathrm{k} \Omega$
$v_{o, \max }=i_{i, \max } *\left(v_{o} / i_{i}\right)=(10 e-6) *((63.64 e 3))=0.6364 \mathrm{~V}$
So the improvement in signal gain for circuit (b) over circuit (a) is about 7

## Question 2



For the circuit above
(a) Find $v_{o} / v_{i 1}$ assuming $v_{i 2}$ is a dc bias voltage.
(b) Find $v_{o} / v_{i 2}$ assuming $v_{i 1}$ is a dc bias voltage.

## Solution

(a) Define $R_{o p}$ to be the impedance looking up into the drain of $M_{3}$ and define $R_{o n}$ to be the impedance looking down into the drain of $M_{2}$
$R_{o p}=r_{o 3}+\left(1+g_{m 3} * r_{o 3}\right) * r_{o 4}=(10 e 3)+(1+(500 e-6) *(10 e 3)) *(20 e 3)=130 \mathrm{k} \Omega$
$R_{o n}=r_{o 2}+\left(1+g_{m 2} * r_{o 2}\right) * r_{o 1}=(10 e 3)+(1+(500 e-6) *(10 e 3)) *(20 e 3)=130 \mathrm{k} \Omega$
Define $R_{o}$ to be the impedance to ground at node $v_{o}$
$R_{o}=R_{o p}\left\|R_{\text {on }}=(130 e 3)\right\|(130 e 3)=65 \mathrm{k} \Omega$
For $i_{s c}$, we have the following circuit


Define $R_{S 2}$ to be the impedance looking up into the source of $M_{2}$
$R_{S 2}=\left(1 / g_{m 2}\right)\left\|r_{o 2}=(1 /(500 e-6))\right\|(10 e 3)=1.667 \mathrm{k} \Omega$
The drain current of $M_{1}$ current divides between $R_{S 2}$ and $r_{o 1}$ resulting in
$G_{M a}=-g_{m 1} *\left(r_{o 1}\right) /\left(r_{o 1}+R_{S 2}\right)=-(1 e-3) *((20 e 3)) /((20 e 3)+(1.667 e 3))=-923.1 e-6$
and $i_{s c}=G_{M a} * v_{i}$. The resulting gain is
$v_{o} / v_{i 1}=G_{M a} * R_{o}=(-923.1 e-6) *(65 e 3)=-60 \mathrm{~V} / \mathrm{V}$
(b) For $v_{i 2}$, we have the same output impedance of $R_{o}=65 \mathrm{k} \Omega$

However, for $i_{\text {sc }}$, we now have

$G_{M b}=\left(-g_{m 2} * r_{o 2}\right) /\left(r_{o 2}+\left(1+g_{m 2} * r_{o 2}\right) * r_{o 1}\right)=(-(500 e-6) *(10 e 3)) /((10 e 3)+(1+(500 e-6) *(10 e 3)) *(20 e 3))=$ $-38.46 e-6$
and $i_{s c}=G_{M b} * v_{i}$. The resulting gain is
$v_{o} / v_{i 2}=G_{M b} * R_{o}=(-38.46 e-6) *(65 e 3)=-2.5 \mathrm{~V} / \mathrm{V}$
This result is MUCH smaller than the gain found in (a). This reduction is due to the large resistor value of $r_{o 1}$ attached between the source of $M_{2}$ and ground and results in a much smaller short circuit current.

