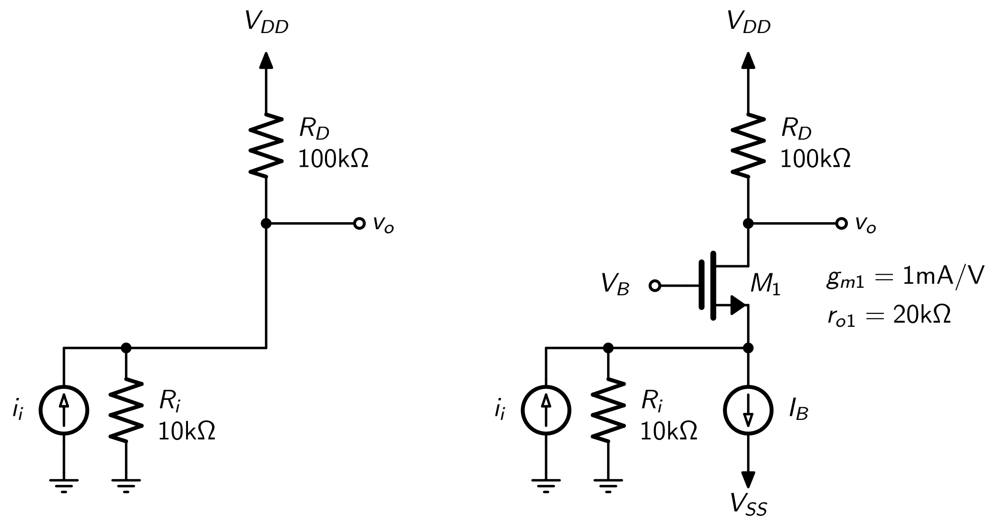
Problem Set 3C - MultiStage

Question 1

It is desired to create a voltage output from a small current source input (say from a photodetector). Shown below, the small current source input and its output impedance is shown as i_i and R_i, respectively. The figure below shows 2 circuits. Circuit (a) does not make use of a transistor while circuit (b) makes use of one transistor. V_B is a dc bias voltage. Also, assume the current source I_B is ideal.



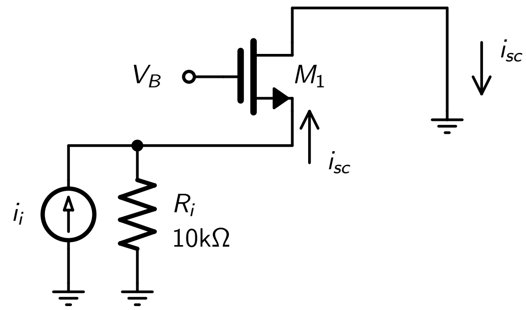
(a) (b)

(a) For circuit (a), find the small-signal gain, v_o/i_i. Next, find the change in v_o when $i_{i,max} = 10\mu\text{A}$.
(b) For circuit (b), find the small-signal gain, v_o/i_i. Next, find the change in v_o when $i_{i,max} = 10\mu\text{A}$
(c) What is the small-signal gain improvement for circuit (b) over circuit (a)?

Solution

(a) $R_o = R_i||R_D = (10e3)||(100e3) = 9.091\text{k}\Omega$
$i_{sc} = i_i$ and we have $v_o = i_i R_o$ resulting in
$(v_o/i_i)_a = R_o = (9.091e3) = 9.091\text{k}\Omega$
For $i_{i,max} = 10\mu\text{A}$, we have
$v_{o,max} = i_{i,max} * ((v_o/i_i)_a) = (10e{-}6) * ((9.091e3)) = 90.91\text{mV}$

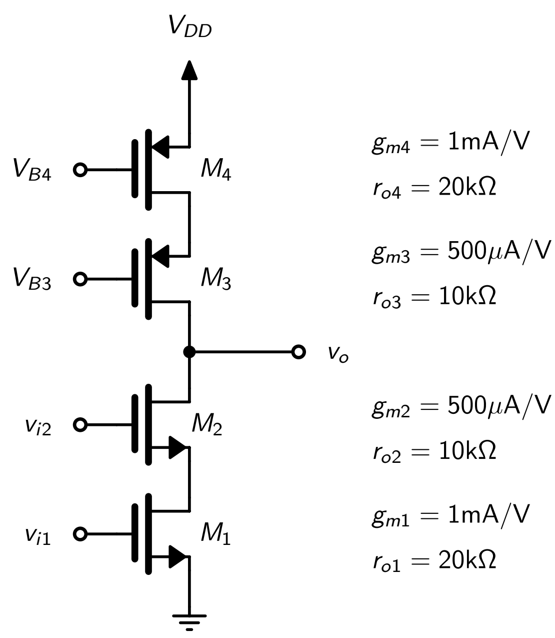
(b) We can start by finding the output impedance, R_o
Define R_{dx} to be the small signal resistance looking into the drain of M_1
$R_{dx} = r_{o1} + (1 + g_{m1} * r_{o1}) * R_i = (20e3) + (1 + (1e{-}3) * (20e3)) * (10e3) = 230\text{k}\Omega$
$R_o = R_{dx}||R_D = (230e3)||(100e3) = 69.7\text{k}\Omega$
Next, we find the short circuit current, i_{sc}
We have the following small circuit circuit



Defining R_{sx} to be the impedance looking in to the source of M_1 we have
$R_{sx} = (1/g_{m1})||r_{o1} = (1/(1e{-}3))||(20e3) = 952.4\Omega$
and we see a current divider, so we have
$i_{sc}/i_i = R_i/(R_i + R_{sx}) = (10e3)/((10e3) + (952.4)) = 0.913\text{A/A}$ leading to
$(v_o/i_i)_b = i_{sc}/i_i * R_o = (0.913) * (69.7e3) = 63.64\text{k}\Omega$
$v_{o,max} = i_{i,max} * ((v_o/i_i)_b) = (10e{-}6) * ((63.64e3)) = 0.6364\text{V}$

(c) The improvement, k, in small-signal gain is
$k = (v_o/i_i)_b/(v_o/i_i)_a = (63.64e3)/(9.091e3) = 7$

Question 2



For the circuit above
(a) Find v_o/v_{i1} assuming v_{i2} is a dc bias voltage.
(b) Find v_o/v_{i2} assuming v_{i1} is a dc bias voltage.

Solution

(a) Define R_{op} to be the impedance looking up into the drain of M_3 and define R_{on} to be the impedance looking down into the drain of M_2

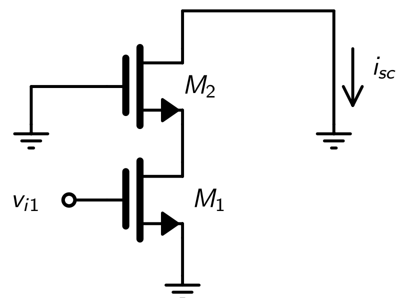
$R_{op} = r_{o3} + (1 + g_{m3} * r_{o3}) * r_{o4} = (10e3) + (1 + (500e{-}6) * (10e3)) * (20e3) = 130\text{k}\Omega$

$R_{on} = r_{o2} + (1 + g_{m2} * r_{o2}) * r_{o1} = (10e3) + (1 + (500e{-}6) * (10e3)) * (20e3) = 130\text{k}\Omega$

Define R_o to be the impedance to ground at node v_o

$R_o = R_{op}||R_{on} = (130e3)||(130e3) = 65\text{k}\Omega$

For i_{sc}, we have the following circuit



Define R_{S2} to be the impedance looking up into the source of M_2

$R_{S2} = (1/g_{m2})||r_{o2} = (1/(500e{-}6))||(10e3) = 1.667\text{k}\Omega$

The drain current of M_1 current divides between R_{S2} and r_{o1} resulting in

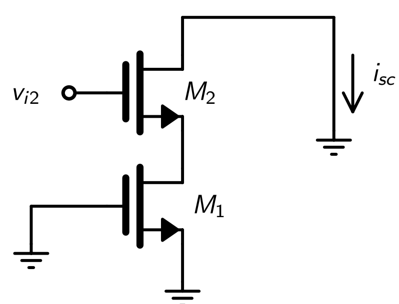
$G_{Ma} = -g_{m1} * (r_{o1})/(r_{o1} + R_{S2}) = -(1e{-}3) * ((20e3))/((20e3) + (1.667e3)) = -923.1e{-}6$

and $i_{sc} = G_{Ma} * v_i$. The resulting gain is

$v_o/v_{i1} = G_{Ma} * R_o = (-923.1e{-}6) * (65e3) = -60\text{V/V}$

(b) For v_{i2}, we have the same output impedance of $R_o = 65\text{k}\Omega$

However, for i_{sc}, we now have



$G_{Mb} = (-g_{m2}*r_{o2})/(r_{o2}+(1+g_{m2}*r_{o2})*r_{o1}) = (-(500e{-}6)*(10e3))/((10e3)+(1+(500e{-}6)*(10e3))*(20e3)) = -38.46e{-}6$

and $i_{sc} = G_{Mb} * v_i$. The resulting gain is
$v_o/v_{i2} = G_{Mb} * R_o = (-38.46e{-}6) * (65e3) = -2.5\text{V/V}$
This result is MUCH smaller than the gain found in (a). This reduction is due to the large resistor value of r_{o1} attached between the source of M_2 and ground and results in a much smaller short circuit current.