Thevenin/Norton Equivalent

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Thevenin’s theorem is used to replace a multiple element circuit with a single voltage source and resistor while Norton’s theorem is used to replace a multiple element circuit with a single current source and resistor.

Specifically, Thevenin’s theorem states:

"Any linear electrical network containing only voltage sources, current sources and resistances can be replaced at terminals A–B by an equivalent combination of a voltage source $V_{oc}$ in a series connection with a resistance $R_o$.”

Norton’s theorem is almost identical except that the replacement is a single current source $I_{sc}$ in parallel with a resistor. The resistor $R_o$ is the same value for both the Thevenin and Norton equivalent circuits.

The voltage and current sources may be either independent or dependent sources. An independent source is one where the value does not depend on anything else in the circuit. A dependent source is a source where the value of the source changes based on a voltage or current somewhere in the circuit.

Consider a linear circuit shown below with a port A-B as shown.

![Original Circuit Diagram]

The above circuit can be replaced with either the Thevenin Equivalent or the Norton Equivalent shown below.

![Thevenin and Norton Equivalents]

If more circuit elements are attached to port A-B, the Thevenin and Norton equivalent circuits will behave exactly the same as the original circuit.
Since either of the above equivalent circuits can replace the original circuit, one might ask, “is it better to use one instead of the other?”. The short answer is that either will work however, the more complete answer is that if you are doing hand analysis, you will get a better intuitive understanding of the circuit if you use the one best suited for the load that is attached to the circuit.

For example, if the original circuit is attached to a large resistive load, then the Thevenin equivalent circuit should be used for analysis. If the original circuit is attached to a small resistive load, then the Norton equivalent circuit will give better intuitive understanding of the circuit. In this case, small and large are relative terms and are relative to the Thevenin/Norton output resistance, $R_o$.

**Finding $R_o$**

To find the value of $R_o$, an arbitrary voltage $V_x$ can be applied to port A-B as shown below.

The Thevenin/Norton resistance is then

$$R_o = \frac{V_x}{I_x}$$

When finding $I_x$, all independent sources should be set to zero. This is equivalent to shorting all independent voltage sources and opening all independent current sources. However, dependent sources remain in the circuit while finding $I_x$.

**Finding $V_{oc}$**

To find the value of $V_{oc}$, find the open-circuit voltage at port A-B. In other words, disconnect any other elements connected to port A-B and find the voltage from B to A.

**Finding $I_{sc}$**

To find the value of $I_{sc}$, find the short-circuit current at port A-B. In other words, short port A-B and find the short-circuit current flowing through the shorting wire from A to B.
Relationship between $V_{oc}$ and $I_{sc}$

Since the Thevenin equivalent and Norton equivalent are both equal to the original circuit, they too must be equivalent. As a result,

$$I_{sc} = \frac{V_{oc}}{R_o}$$

Example 1

A 32V audio amplifier has a 0.1Ω ohm output resistance and is driving an 8Ω ohm loudspeaker. Show the Thevenin and Norton Equivalent models and explain why one is not intuitively satisfying.

The Thevenin equivalent model is shown below.

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R_o = 0.1Ω

V_{oc} = 32V

RL = 8Ω
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Thevenin Equivalent

The Norton equivalent model is shown below.

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I_{sc} = 320A

R_o = 0.1Ω

RL = 8Ω
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Norton Equivalent

The Norton equivalent model is not intuitively satisfying since it includes a 320A current source and may give the false impression that some wires may have to be able to carry 320A of current. In fact, almost all that current goes through $R_o$ and there is no large amount of 320A current anywhere in the amplifier.

Example 2

Find the Thevenin and Norton Equivalent circuits for the circuit below
To find $R_o$, we set $V_1 = 0$ and $I_1 = 0$ resulting in the circuit below.

We could do nodal analysis but in this case, it is clear that the resistance looking into port A-B is $R_2$ in series with $R_1$ in parallel with $R_3$ so

$$R_o = R_2 + R_1 \parallel R_3 = (1e3) + (1e3) \parallel (1e3) = 1.5k\Omega$$

To find $V_{oc}$, we use superposition since the circuit is linear and we have 2 independent sources, $V_1$ and $I_1$. Find $V_{ocv}$ due to $V_1$ with $I_1 = 0$. Next, find $V_{oci}$ due to $I_1$ with $V_1 = 0$. Finally, add the two results to find $V_{oc}$.

$$V_{ocv} = (R_3/(R_1 + R_3)) \times V_1 = ((1e3)/((1e3) + (1e3))) \times (10) = 5V$$

$$V_{oci} = (R_1\parallel R_3) \times (-I_1) = ((1e3)\parallel(1e3)) \times (-1e-3) = 0.5V$$

Note that the voltage across $R_2$ is zero since no current flows across $R_2$ when the port is open.

$$V_{oc} = V_{ocv} + V_{oci} = (5) + (-0.5) = 4.5V$$

To find $I_{sc}$ we could calculate the short circuit output current but since we already know $V_{oc}$ and $R_o$, we can find $I_{sc}$ as

$$I_{sc} = V_{oc}/R_o = (4.5)/(1.5e3) = 3mA$$

So the equivalent circuits are ...
Example 3

Find the Thevenin and Norton Equivalent circuits for the circuit below

![Circuit Diagram]

To find $R_o$, we set $V_1 = 0$ and have the following circuit

![Circuit Diagram]

We have $V_y = -V_x$ which leads to

$$I_x = \frac{V_x - (10V_y)}{R_1} = \frac{V_x + 10V_x}{R_1} = \frac{11V_x}{R_1}$$

$$R_o = \frac{V_x}{I_x} = \frac{R_1}{11} = 90.91\Omega$$

To find $I_{sc}$, we use the following circuit

![Circuit Diagram]

$$V_y = V_1 = (10) = 10V$$
\[ I_{sc} = \frac{10V_y - 0}{R_1} = 0.1A \]

And we can find \( V_{oc} \) from
\[ V_{oc} = I_{sc} \times R_o = (0.1) \times (90.91) = 9.091V \]

So the equivalent circuits are ...

- **Thevenin Equivalent**
  - \( R_o = 90.91\Omega \)
  - 9.091V

- **Norton Equivalent**
  - 0.1A
  - \( R_o = 90.91\Omega \)