Noise

Interference Noise

- Unwanted interaction between circuit and outside world
- May or may not be random
- Examples: power supply noise, capacitive coupling

Improvement by ...

- Reduced by careful wiring or layout

- These notes do not deal with interference noise.

Inherent Noise

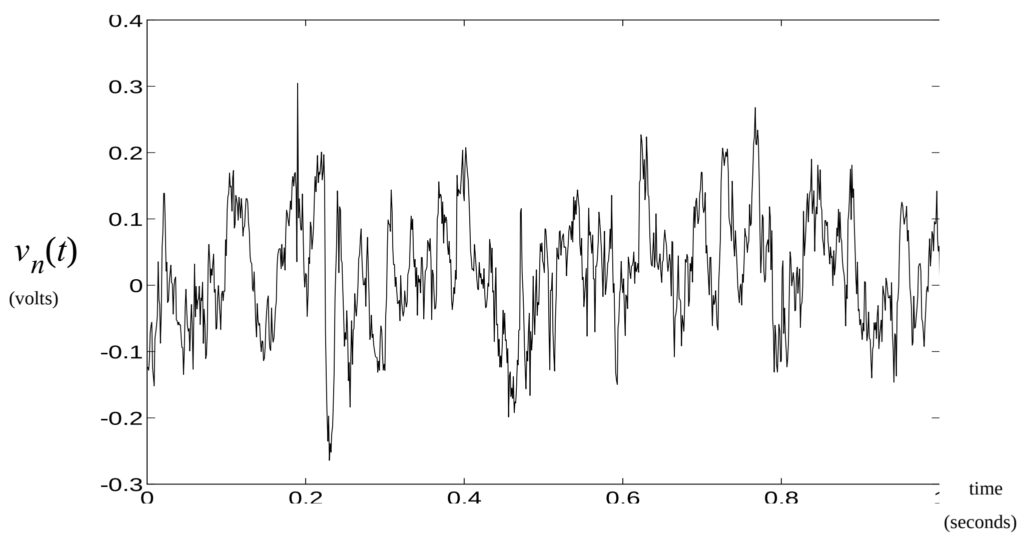
- Random noise — can be reduced but NEVER eliminated
- Examples: thermal, shot, and flicker

Improvement by ...

- Not strongly affected by wiring or layout
- Reduced by proper circuit DESIGN.

- These notes discuss noise analysis and inherent noise sources.

Time-Domain Analysis



- ***Assume all noise signals have zero mean***

RMS Value

$$V_{n(rms)} \equiv \left[\frac{1}{T}\int_0^T v_n^2(t)dt\right]^{1/2} \quad (1)$$

- T — suitable averaging time interval
- Indicates ***normalized noise power***.
- If $v_n(t)$ applied to 1Ω resistor, average power dissipated, P_{diss},

$$P_{diss} = \frac{V_{n(rms)}^2}{1\Omega} = V_{n(rms)}^2 \quad (2)$$

SNR

$$\mathrm{SNR} \equiv 10\log\left[\frac{\text{signal power}}{\text{noise power}}\right] \quad (3)$$

- If signal node has normalized signal power of $V_{x(rms)}^2$, and noise power of $V_{n(rms)}^2$,

$$\mathrm{SNR} = 10\log\left[\frac{V_{x(rms)}^2}{V_{n(rms)}^2}\right] = 20\log\left[\frac{V_{x(rms)}}{V_{n(rms)}}\right] \quad (4)$$

- When mean-squared values of noise and signal are same, $\mathrm{SNR} = 0dB$.

Units of dBm

- Often useful to know signal's power in dB on absolute scale.
- With dBm, all power levels referenced 1mW.
- 1mW signal corresponds to 0 dBm
- $1\mu W$ signal corresponds to -30dBm

What if only voltage measured (not power)?

- If voltage measured — reference level to equiv power dissipated if voltage applied to 50 Ω resistor
- Also, can reference it to 75 Ω resistor

dBm Example

- Find rms voltage of 0 dBm signal (50Ω reference)
- What is level in dBm of a 2 volt rms signal?

- $0\ dBm$ signal (50Ω reference) implies

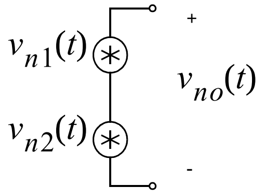
$$V_{(rms)} = \sqrt{(50\Omega) \times 1mW} = 0.2236 \quad (5)$$

- Thus, a 2 volt (rms) signal corresponds to

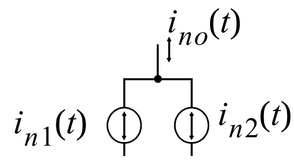
$$20 \times \log\left(\frac{2.0}{0.2236}\right) = 19\ \text{dBm} \quad (6)$$

- Would dissipate $2^2 / 50 = 80\ \text{mW}$ across a 50Ω resistor
- $80\ \text{mW}$ corresponds to $10\log(80) = 19\ \text{dBm}$

Noise Summation



Voltage



Current

$$v_{no}(t) = v_{n1}(t) + v_{n2}(t) \tag{7}$$

$$V^2_{no(rms)} = \frac{1}{T}\int_0^T [v_{n1}(t) + v_{n2}(t)]^2 dt \tag{8}$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + \frac{2}{T}\int_0^T v_{n1}(t)v_{n2}(t)dt \tag{9}$$

Correlation

- Last term relates correlation between two signals
- Define correlation coefficient, C,

$$C \equiv \frac{\frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)dt}{V_{n1(rms)}V_{n2(rms)}} \tag{10}$$

$$V^2_{no(rms)} = V^2_{n1(rms)} + V^2_{n2(rms)} + 2CV_{n1(rms)}V_{n2(rms)} \tag{11}$$

- Correlation always satisfies $-1 \le C \le 1$
- $C = +1$ — fully-correlated in-phase (0 degrees)
- $C = -1$ — fully-correlated out-of-phase (180 degrees)
- $C = 0$ — uncorrelated (90 degrees)

Uncorrelated Signals

- ***In case of two uncorrelated signals, mean-squared value of sum given by***

$$V_{no(rms)}^2 = V_{n1(rms)}^2 + V_{n2(rms)}^2 \qquad (12)$$

- Two rms values add as though they were vectors at right angles

When fully correlated

$$V_{no(rms)}^2 = (V_{n1(rms)} \pm V_{n2(rms)})^2 \qquad (13)$$

- sign is determined by whether signals are in or out of phase
- Here, rms values add linearly (aligned vectors)

Noise Summation Example

- $V_{n1(rms)} = 10\mu V$, $V_{n2(rms)} = 5\mu V$, then

$$V_{no(rms)}^2 = (10^2 + 5^2) = 125 \qquad (14)$$

which results in $V_{no(rms)} = 11.2\mu V$.

- Note that ***eliminating*** $V_{n2(rms)}$ noise source same as ***reducing*** $V_{n1(rms)} = 8.7\ \mu V$ (i.e. ***reducing by 13%***)!

Important Moral

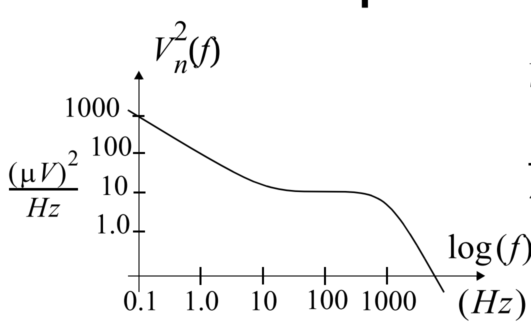
- ***To reduce overall noise, concentrate on large noise signals***.

Frequency-Domain Analysis

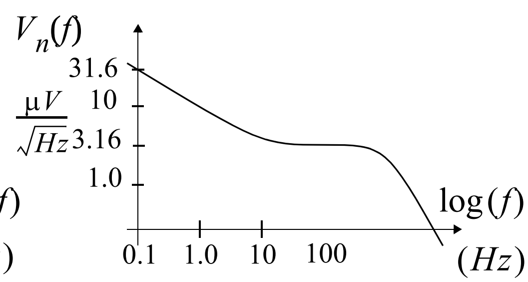
- With deterministic signals, frequency-domain techniques are useful.
- Same true for dealing with random signals like noise.

- This section presents frequency-domain techniques for dealing with noise (or random) signals.

Spectral Density



Spectral Density



Root-Spectral Density

- Periodic waveforms have their power at distinct frequencies.
- Random signals have their power spread out over the frequency spectrum.

Spectral Density

Spectral Density $V_n^2(f)$

- Average normalized power over a 1 hertz bandwidth
- Units are volts-squared/hertz

Root-Spectral Density $V_n(f)$

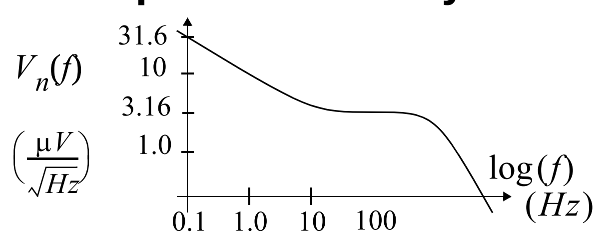
- Square root of vertical axis (freq axis unchanged)
- Units are volts/root-hertz (i.e. $V/\sqrt{Hz}$).

Total Power

$$V_{n(rms)}^2 = \int_0^{\infty} V_n^2(f)df \tag{15}$$

- Above is a one-sided definition (i.e. all power at positive frequencies)

Root-Spectral Density Example



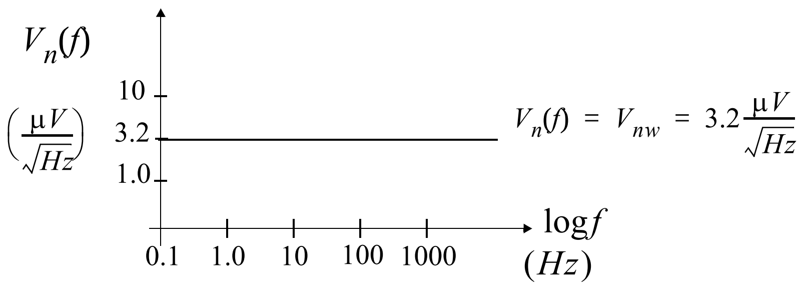
- Around 100 Hz, $V_n(f) = \sqrt{10}\ \mu V/\sqrt{Hz}$
- If measurement used $RBW = 30\ Hz$, measured rms

$$\sqrt{10} \times \sqrt{30} = \sqrt{300}\ \mu V \tag{16}$$

- If measurement used $RBW = 0.1\ Hz$, measured rms

$$\sqrt{10} \times \sqrt{0.1} = 1\ \mu V \tag{17}$$

White Noise

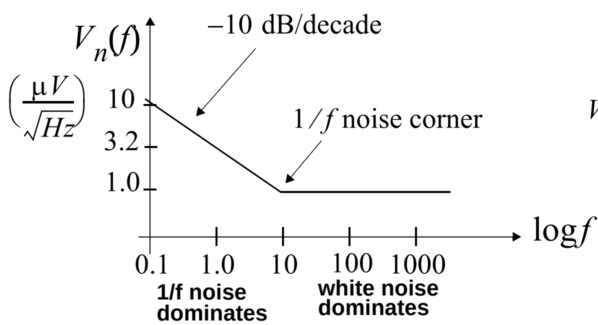


- Noise signal is "white" if a constant spectral density

$$V_n(f) = V_{nw} \quad (18)$$

where V_{nw} is a constant value

1/f Noise



$$V_n^2(f) \approx \frac{(3.2 \times 10^{-6})^2}{f} + (1 \times 10^{-6})^2$$

$$V_n^2(f) = k_v^2/f \quad (k_v \text{ is a constant}) \quad (19)$$

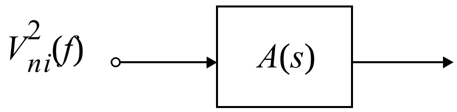
- In terms of root-spectral density

$$V_n(f) = k_v/\sqrt{f} \quad (20)$$

Falls off at -10 db/decade due to $\sqrt{f}$

- Also called ***flicker*** or ***pink*** noise.

Filtered Noise



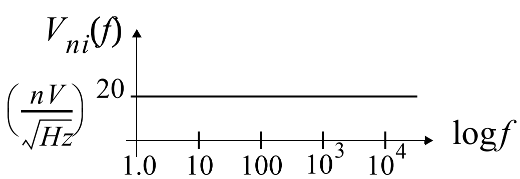
$$V_{no}^2(f) = |A(j2\pi f)|^2 V_{ni}^2(f)$$

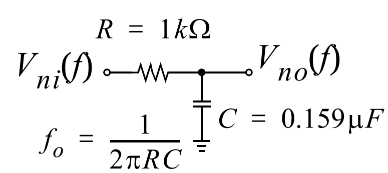
$$V_{no}(f) = |A(j2\pi f)| V_{ni}(f)$$

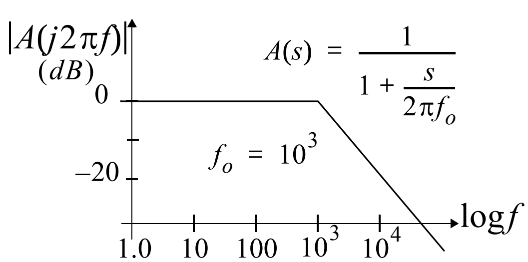
- Output only a function of magnitude of transfer-function ***and not its phase***
- Can always apply an allpass filter without affecting noise performance.
- Total output mean-squared value is

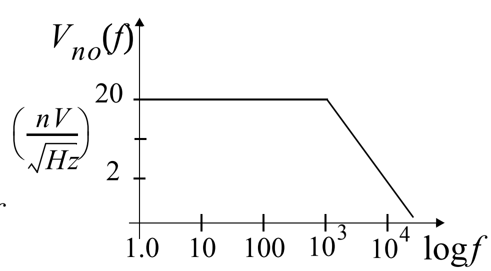
$$V_{no(rms)}^2 = \int_0^{\infty} |A(j2\pi f)|^2 V_{ni}^2(f) df \qquad (21)$$

Noise Example









Noise Example

- From dc to 100 kHz of input signal

$$V_{ni(rms)}^2 = \int_0^{10^5} 20^2 df = 4 \times 10^7 (nV)^2 = (6.3\ \mu\text{V rms})^2 \qquad (22)$$

- Note: for this simple case,

$$V_{ni(rms)} = 20\ nV/\sqrt{Hz} \times \sqrt{100kHz} = 6.3\ \mu\text{V rms} \qquad (23)$$

- For the filtered signal, $V_{no}(f)$,

$$V_{no}(f) = \frac{20 \times 10^{-9}}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2}} \qquad (24)$$

Noise Example

- Between dc and 100kHz

$$V_{no(rms)}^2 = \int_0^{10^5} \frac{20^2}{1 + \left(\frac{f}{f_o}\right)^2} df = 20^2 f_o \operatorname{atan}\left(\frac{f}{f_o}\right)\Bigg|_0^{10^5}$$

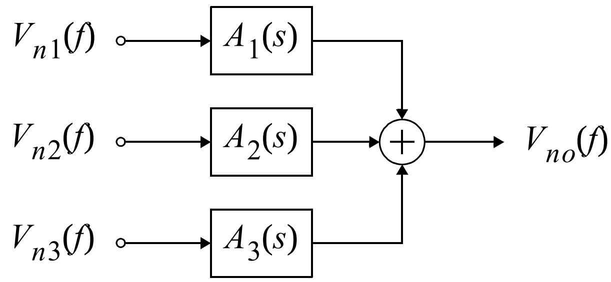
$$= 6.24 \times 10^5 (nV)^2 = (0.79\ \mu V\ \text{rms})^2 \qquad (25)$$

- Noise rms value of $V_{no}(f)$ is almost $1/10$ that of $V_{ni}(f)$ since high frequency noise above 1kHz was filtered.

- ***Don't design for larger bandwidths than required otherwise noise performance suffers.***

Sum of Filtered Noise

uncorrelated noise sources



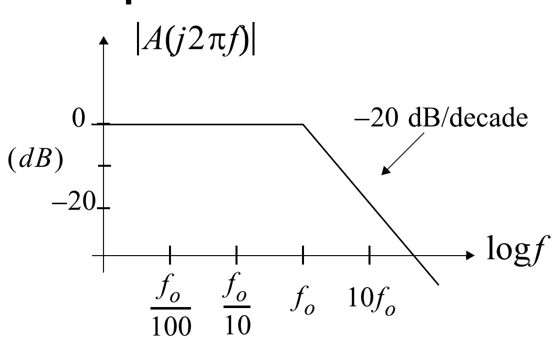
- If filter inputs are uncorrelated, filter outputs are also uncorrelated
- Can show

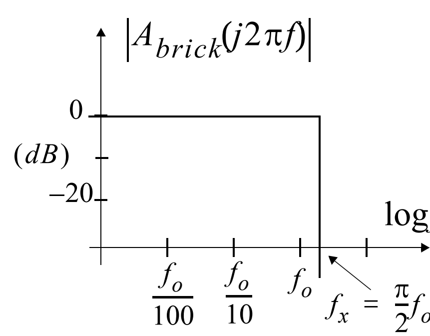
$$V_{no}^2(f) = \sum_{i = 1, 2, 3} |A_i(j2\pi f)|^2 V_{ni}^2(f) \qquad (26)$$

Noise Bandwidth

- Equal to the frequency span of a brickwall filter having the same output noise rms value when white noise is applied to each

Example





- Noise bandwidth of a 1'st-order filter is $\frac{\pi}{2} f_o$

Noise Bandwidth

- Advantage — total output noise is easily calculated for white noise input.
- If spectral density is V_{nw} volts/root-Hz and noise bandwidth is f_x, then

$$V_{no(rms)}^2 = V_{nw}^2 f_x \tag{27}$$

Example

- A white noise input of $100\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$ applied to a 1'st order filter with 3 dB frequency of 1 MHz

$$V_{no(rms)} = 100 \times 10^{-9} \times \sqrt{\frac{\pi}{2} \times 10^6}$$

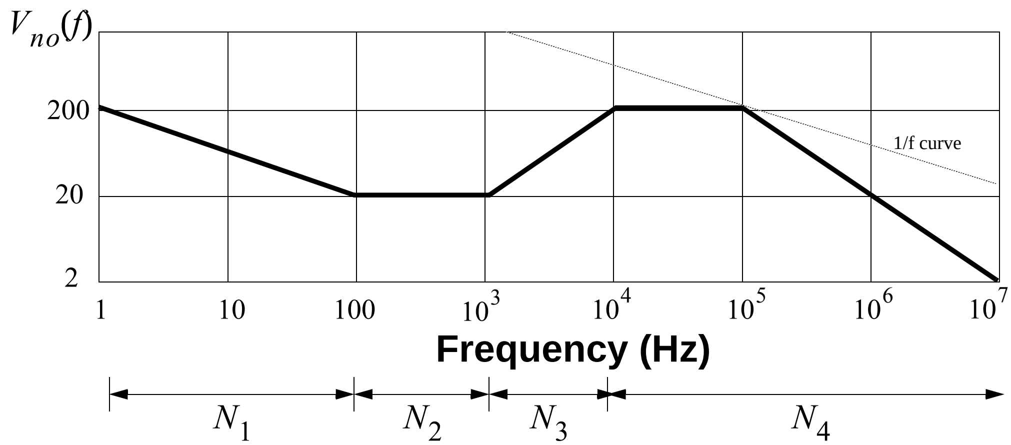
$$V_{no(rms)} = 125\ \mu\mathrm{V} \tag{28}$$

1/f Noise Tangent Principle

- Method to determine the frequency region(s) that contributes the dominant noise
- Lower a 1/f noise line until it touches the spectral density curve
- The total noise can be approximated by the noise in the vicinity of the 1/f line

- Works because a curve proportional to $1/x$ results in equal power over each decade of frequency

1/f Tangent Example



- Consider root-spectral noise density shown above

1/f Tangent Example

$$N_1^2 = \int_1^{100} \frac{200^2}{f} df = 200^2 \ln(f)\Big|_1^{100} = 1.84 \times 10^5 (nV)^2 \qquad (29)$$

$$N_2^2 = \int_{100}^{10^3} 20^2 df = 20^2 f\Big|_{100}^{10^3} = 3.6 \times 10^5 (nV)^2 \qquad (30)$$

$$N_3^2 = \int_{10^3}^{10^4} \frac{20^2 f^2}{(10^3)^2} df = \left(\frac{20}{10^3}\right)^2 \left[\frac{1}{3} f^3 \Big|_{10^3}^{10^4}\right] = 1.33 \times 10^8 (nV)^2 \qquad (31)$$

$$N_4^2 = \int_{10^4}^{\infty} \frac{200^2}{1 + \left(\frac{f}{10^5}\right)^2} df = \int_0^{\infty} \frac{200^2}{1 + \left(\frac{f}{10^5}\right)^2} df - \int_0^{10^4} 200^2 df$$

$$= 200^2 \left(\frac{\pi}{2}\right) 10^5 - (200^2)(10^4) = 5.88 \times 10^9 (nV)^2 \qquad (32)$$

1/f Tangent Example

- The total output noise is estimated to be

$$V_{no(rms)} = (N_1^2 + N_2^2 + N_3^2 + N_4^2)^{1/2} = 77.5\mu V \text{ rms} \qquad (33)$$

- But ...

$$N_4 = 76.7\mu V \text{ rms} \qquad (34)$$

- Need only have found the noise in the vincinity where the 1/f tangent touches noise curve.

- ***Note:*** if noise curve is parallel to 1/f tangent for a wide range of frequencies, then also sum that region.

Noise Models for Circuit Elements

- Three main sources of noise:

Thermal Noise

- Due to thermal excitation of charge carriers.
- Appears as white spectral density

Shot Noise

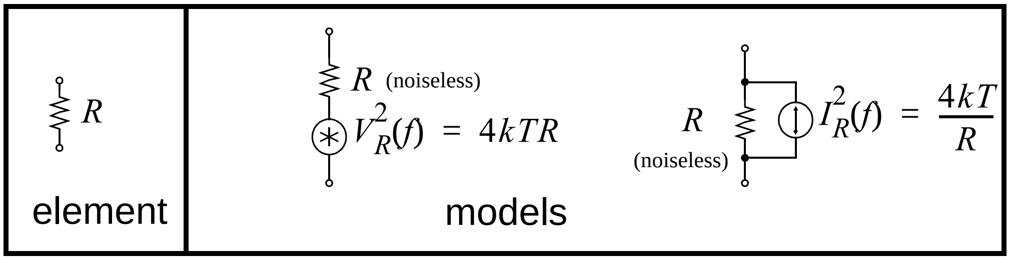
- Due to dc bias current being pulses of carriers
- Dependent of dc bias current and is white.

Flicker Noise

- Due to traps in semiconductors
- Has a 1/f spectral density
- Significant in MOS transistors at low frequencies.

Resistor Noise

- Thermal noise — white spectral density

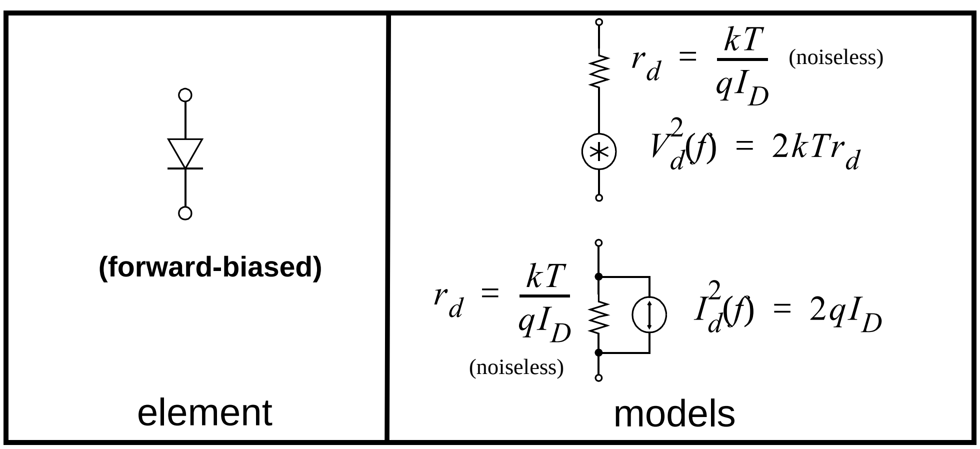


- k is Boltzmann's constant = $1.38 \times 10^{-23}\ JK^{-1}$
- T is the temperature in degrees Kelvin
- Can also write

$$V_R(f) = \sqrt{\frac{R}{1k}} \times 4.06\ nV/\sqrt{Hz} \quad \text{for } 27°C \tag{35}$$

Diodes

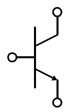
- Shot noise — white spectral density



- q is one electronic charge = 1.6×10^{-19} C
- I_D is the dc bias current through the diode

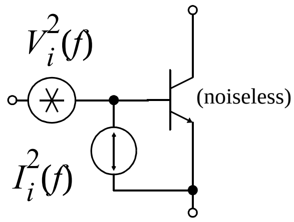
Bipolar Transistors

- Shot noise of collector and base currents
- Flicker noise due to base current
- Thermal noise due to base resistance



(active-region)

element



$$V_i^2(f) = 4kT\left(r_b + \frac{1}{2g_m}\right)$$

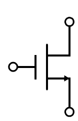
$$I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{|\beta(f)|^2}\right)$$

model

- $V_i(f)$ has base resistance thermal noise plus collector shot noise referred back
- $I_i(f)$ has base shot noise, base flicker noise plus collector shot noise referred back

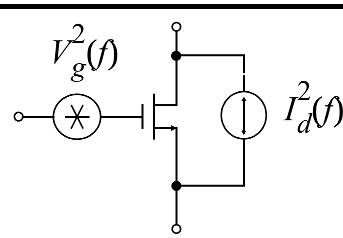
MOSFETS

- Flicker noise at gate
- Thermal noise in channel



(active-region)

element



$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$$

model

MOSFET Flicker (1/f) Noise

$$V_g^2(f) = \frac{K}{WLC_{ox}f} \qquad (36)$$

- K dependent on device characteristics, varies widely.
- W & L — Transistor's width and length
- C_{ox} — gate-capacitance/unit area

- ***Flicker noise is inversely proportional to the transistor area***, WL.
- 1/f noise is extremely important in MOSFET circuits as it can dominate at low-frequencies
- Typically p-channel transistors have less noise since holes are less likely to be trapped.

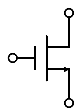
MOSFET Thermal Noise

- Due to resistive nature of channel
- In triode region, noise would be $I_d^2(f) = (4kT)/r_{ds}$ where r_{ds} is the channel resistance
- In active region, channel is not homogeneous and total noise is found by integration

$$I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m \qquad (37)$$

for the case $V_{DS} = V_{GS} - V_T$

Low-Moderate Frequency MOSFET Model



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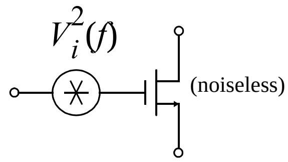
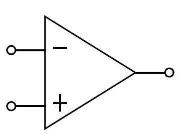


$$V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$$

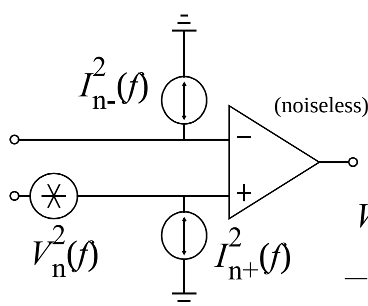
model

- Can lump thermal noise plus flicker noise as an input voltage noise source at low to moderate frequencies.
- At high frequencies, gate current can be appreciable due to capacitive coupling.

Opamps



element



$V_n(f)$, $I_{n-}(f)$, $I_{n+}(f)$

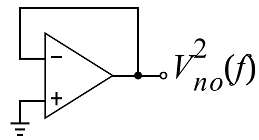
— values depend on opamp

— typically, all uncorrelated.

model

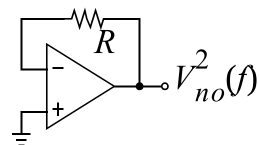
- Modelled as 3 uncorrelated input-referred noise sources.
- Current sources often ignored in MOSFET opamps

Why 3 Noise Sources?



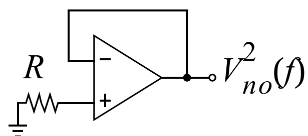
$V_n(f)$ ignored $\Rightarrow V_{no}^2 = 0$

Actual $V_{no}^2 = V_n^2$



$I_{n-}(f)$ ignored $\Rightarrow V_{no}^2 = V_n^2$

Actual $V_{no}^2 = V_n^2 + (I_{n-}R)^2$

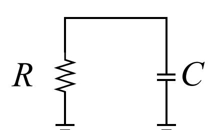


$I_{n+}(f)$ ignored $\Rightarrow V_{no}^2 = V_n^2$

Actual $V_{no}^2 = V_n^2 + (I_{n+}R)^2$

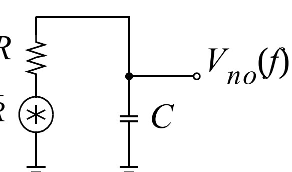
Capacitors

- Capacitors and inductors do not generate any noise but ... they accumulate noise.
- Capacitor noise mean-squared value equals kT/C when connected to an arbitrary resistor value.



$$f_0 = \frac{1}{2\pi RC}$$

$$V_R(f) = \sqrt{4kTR}$$



- Noise bandwidth equals $(\pi/2)f_o$

$$V_{no(rms)}^2 = V_R^2(f)\left(\frac{\pi}{2}\right)f_o = (4kTR)\left(\frac{\pi}{2}\right)\left(\frac{1}{2\pi RC}\right)$$

$$V_{no(rms)}^2 = \frac{kT}{C} \qquad (38)$$

Capacitor Noise Example

- At $300\ °K$, what capacitor size is needed to have 96dB dynamic range with $1\ V\text{rms}$ signal levels.
- Noise allowed:

$$V_{n(rms)} = \frac{1V}{10^{96/20}} = 15.8\ \mu V \text{ rms} \quad (39)$$

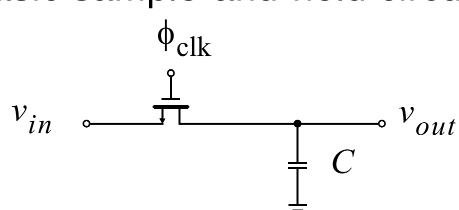
- Therefore

$$C = \frac{kT}{V^2_{n(\text{rms})}} = 16.6pF \quad (40)$$

- This min capacitor size determines max resistance size to achieve a given time-constant.

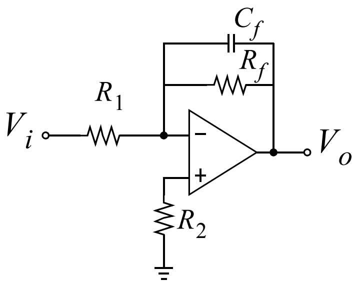
Sampled Signal Noise

- Consider basic sample-and-hold circuit

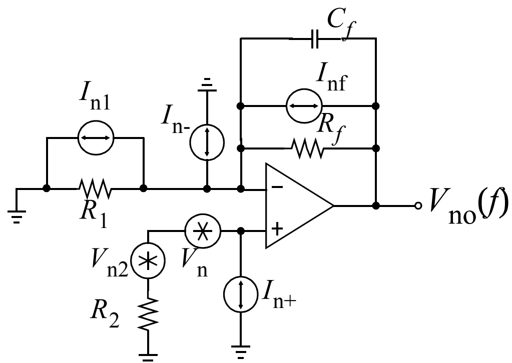


-
- When ϕ_{clk} goes low, noise as well as signal is held on C. — an rms noise voltage of $\sqrt{kT/C}$.
- Does not depend on sampling rate and is independent from sample to sample.
- Can use "oversampling" to reduce effective noise.
- Sample, say 1000 times, and average results.

Opamp Example



circuit



equivalent noise circuit

- Use superposition — noise sources uncorrelated
- Consider I_{n1}, I_{nf} and I_{n-} causing $V^2_{no1}(f)$

$$V^2_{no1}(f) = (I^2_{n1}(f) + I^2_{nf}(f) + I^2_{n-}(f))\left|\frac{R_f}{1 + j2\pi f R_f C_f}\right|^2 \qquad (41)$$

Opamp Example

- Consider I_{n+}, V_{n2} and V_n causing $V^2_{no2}(f)$

$$V^2_{no2}(f) = (I^2_{n+}(f)R^2_2 + V^2_{n2}(f) + V^2_n(f))\left|1 + \frac{R_f / R_1}{1 + j2\pi f C_f R_f}\right|^2 \qquad (42)$$

- If $R_f \ll R_1$ then gain $\cong 1$ for all freq and ideal opamp would result in infinite noise — practical opamp will lowpass filter noise at opamp f_t.
- If $R_f \gg R_1$, low freq $\text{gain} \cong R_f / R_1$ and $f_{3dB} = 1/(2\pi R_f C_f)$ similar to noise at negative input — however, gain falls to unity until opamp f_t.

$$\text{Total noise: } V^2_{no(rms)} = V^2_{no1(rms)} + V^2_{no2(rms)} \qquad (43)$$

Numerical Example

- Estimate total output noise rms value for a 10kHz lowpass filter when $C_f = 160pF$, $R_f = 100k$, $R_1 = 10k$, and $R_2 = 9.1k$.
- Assume $V_n(f) = 20\ nV/\sqrt{Hz}$, $I_n(f) = 0.6\ pA/\sqrt{Hz}$ opamp's $f_t = 5\ \mathrm{MHz}$.
- Assuming room temperature,

$$I_{nf} = 0.406\ pA/\sqrt{Hz} \tag{44}$$

$$I_{n1} = 1.28\ pA/\sqrt{Hz} \tag{45}$$

$$V_{n2} = 12.2\ nV/\sqrt{Hz} \tag{46}$$

Numerical Example

- The low freq value of $V_{no1}^2(f)$ is found by $f = 0$, in (41).

$$\begin{aligned} V_{no1}^2(0) &= (I_{n1}^2(0) + I_{nf}^2(0) + I_{n-}^2(0))R_f^2 \\ &= (0.406^2 + 1.28^2 + 0.6^2)(1 \times 10^9)^2(100k)^2 \\ &= (147\ nV/\sqrt{Hz})^2 \end{aligned} \tag{47}$$

- Since (41) indicates noise is first-order lowpass filtered,

$$\begin{aligned} V_{no1(rms)}^2 &= (147\ nV/\sqrt{Hz})^2 \times \frac{\pi/2}{2\pi(100k\Omega)(160pF)} \\ &= (18.4\ \mu V)^2 \end{aligned} \tag{48}$$

Numerical Example

- For $V_{no2}^2(0)$

$$V_{no2}^2(0) = (I_{n+}^2(f)R_2^2 + V_{n2}^2(f) + V_n^2(f))(1 + R_f/R_1)^2$$
$$= (24.1\ nV/\sqrt{Hz})^2 \times 11^2$$
$$= (265\ nV/\sqrt{Hz})^2 \qquad (49)$$

- Noise is lowpass filtered at f_o until $f_1 = (R_f/R_1)f_o$ where the noise gain reaches unity until $f_t = 5MHz$
- Breaking noise into two portions, we have

$$V_{no2(rms)}^2 = (265 \times 10^{-9})^2 \left(\frac{\pi/2}{2\pi R_f C_f}\right) + (24.1 \times 10^{-9})^2 \left(\frac{\pi}{2}\right)(f_t - f_1)$$
$$= (74.6\ \mu V)^2 \qquad (50)$$

Numerical Example

- Total output noise is estimated to be

$$V_{no(rms)} = \sqrt{V_{no1(rms)}^2 + V_{no2(rms)}^2} = 77\ \mu V\ \text{rms} \qquad (51)$$

- Note: major noise source is opamp's voltage noise.
- To reduce total output noise
  — use a lower speed opamp
  — choose an opamp with a lower voltage noise.
- Note: R_2 contributes to output noise with its thermal noise AND amplifying opamp's positive noise current.
- If dc offset can be tolerated, it should be eliminated in a low-noise circuit.