Integrated Circuits for Digital Communications

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Motivation

- Many exciting new applications for digital comm (previously, modems and satellites)
- xDSL (x=H,A, or V), ethernet, ATM, cable-modems, firewire, disk-drives, wireless, infrared, etc.
- Want to push up bit rate using same channels

Mainly Digital

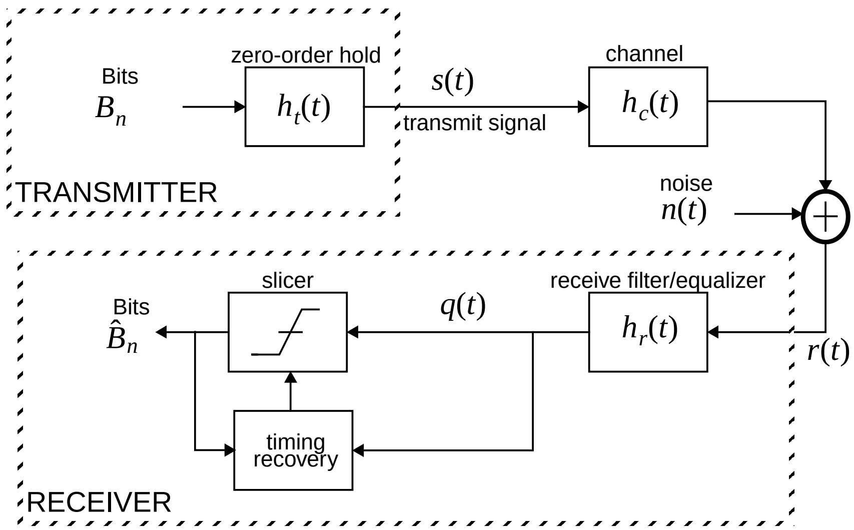
- Tolerant to large noise and channel variations
- Tolerant to analog IC freq and dc variations
- Easier to design and test

Mainly Analog

- Smaller size and power at high-speeds

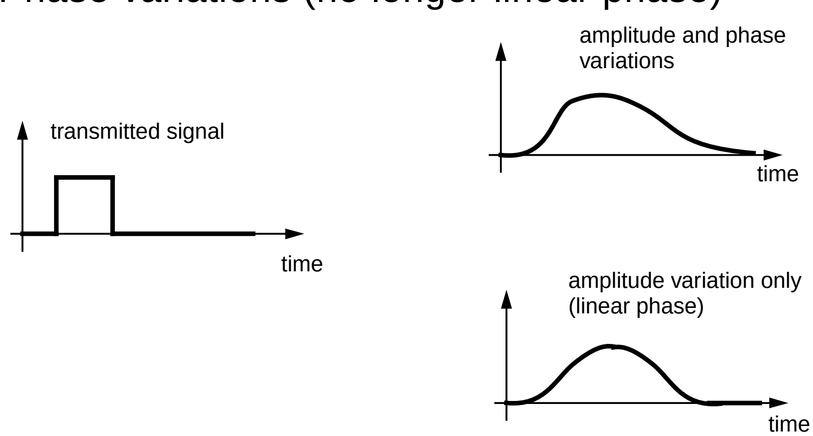
Basic Challenge

- Channel attenuates and phase changes signal, also noise added



Channel Amplitude and Phase Variations

- Amplitude variation (usually lowpass)
- Phase variations (no longer linear phase)

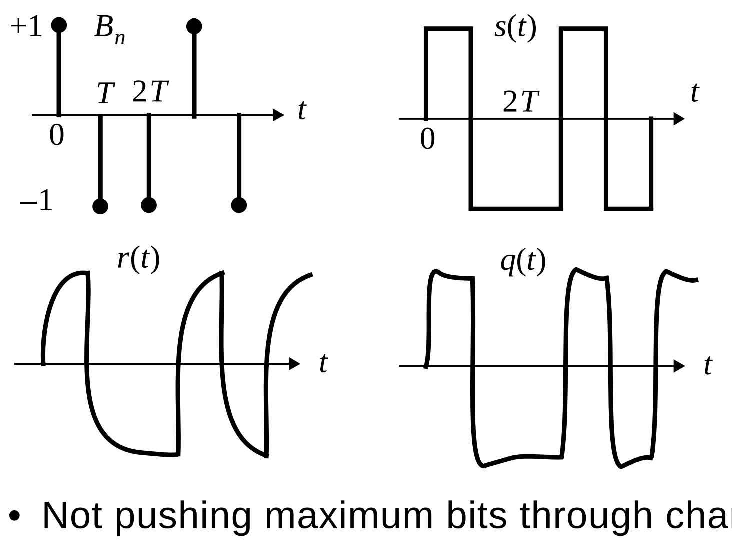


- Linear phase keeps symmetric pulses (intuitively better for less interference)

Channel Noise

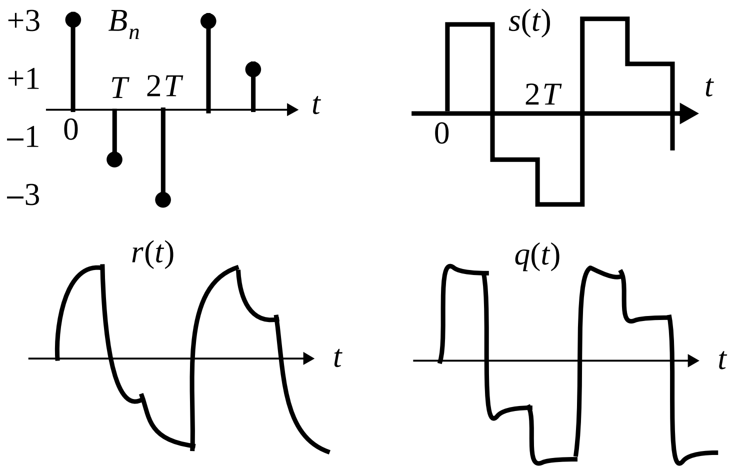
- Noise and bandwidth limits how much digital information can be sent through channel
- Usually model noise as being added after channel impairment
- Can use filtered noise if not white
- Usually assume Gaussian noise because of central limit theorem and mathematically tractable
- Note that receive equalizer might ***increase*** noise if it has boost (noise enhancement)

Low Noise - Large Bandwidth



- Not pushing maximum bits through channel
- Low noise — can use more levels for each symbol
- Large bandwidth — can increase symbol-rate

Multi-Level — Low-Noise, Large Bandwidth

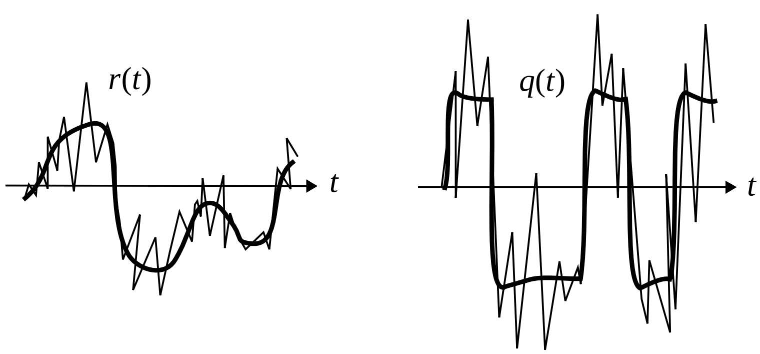


4-PAM

2 bits every symbol

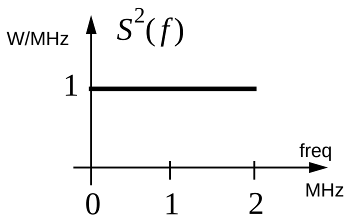
- Twice the bit information over same bandwidth!
- More prone to noise causing errors
- Commonly called PAM (here 2B1Q — 4-PAM)

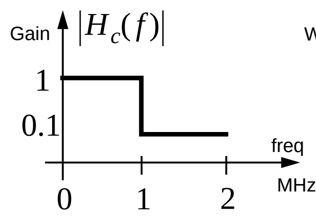
High Noise - Low Bandwidth

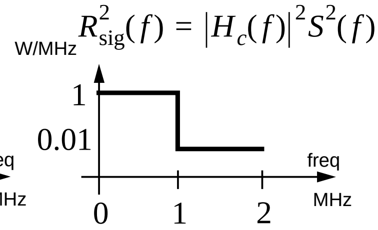


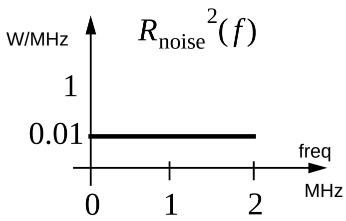
- Equalizing back to “square-wave” amplifies noise
- Particularly bad if most noise is at high frequencies
- Too many errors — need to equalize without much noise enhancement (or add coding)
- More errors with multi-level PAM (unless coded)

Noise Enhancement Example









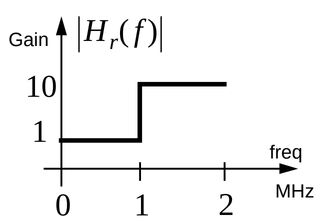
Before Equalization

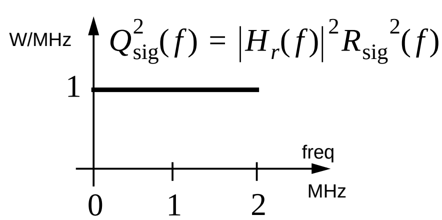
Signal power $= 1.01$ W

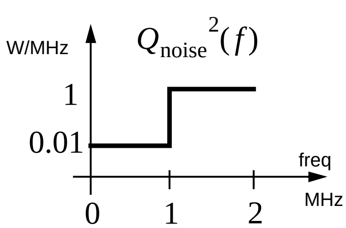
Noise power $= 0.02$ W

$$\mathrm{SNR} = 10\log\left(\frac{1.01}{0.02}\right) = 17\ \mathrm{dB}$$

Noise Enhancement Example







After Equalization

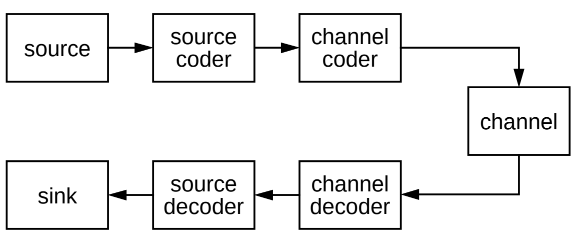
Signal power $= 2$ W

Noise power $= 1.01$ W

$$\mathrm{SNR} = 10\log\left(\frac{2}{1.01}\right) = 3\ \mathrm{dB}$$

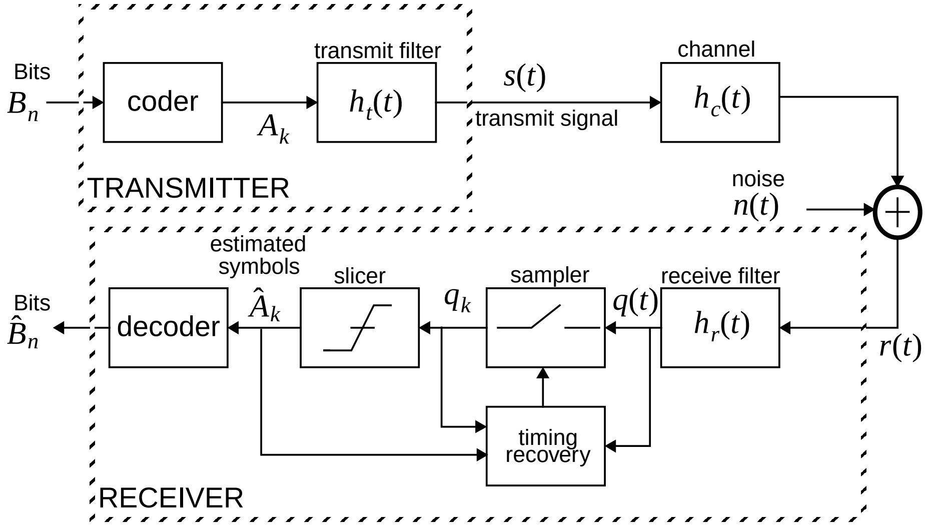
Basic Baseband PAM Concepts

General Data Communication System



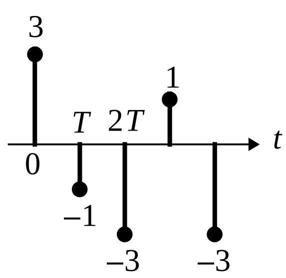
- Source coder removes redundancy from source (i.e. MPEG, ADPCM, text compression, etc.)
- Channel coder introduces redundancy to maximize information rate over channel. (i.e. error-correcting codes, trellis coding, etc.)
- Our interest is in channel coding/decoding and channel transmission/reception.

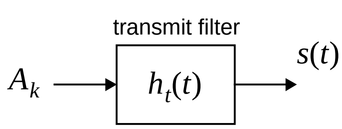
Basic Baseband System

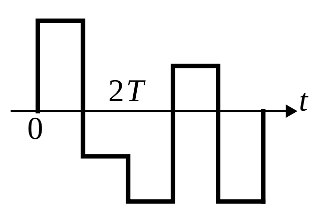


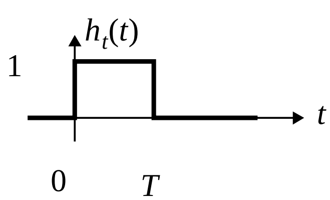
- In 2B1Q, coder maps pairs of bits to one of four levels
  — $A_k = \{-3, -1, 1, 3\}$

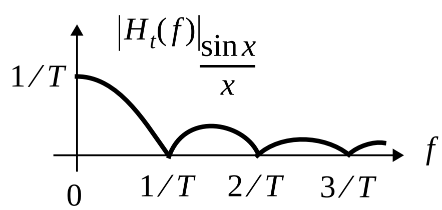
Rectangular Transmit Filter











- The spectrum of A_k is flat if random.
- The spectrum of $s(t)$ is same shape as $H_t(f)$

Nyquist Pulses

- $h(t)$ is the impulse response for transmit filter, channel and receive filter ($\otimes$ denotes convolution)

$$h(t) = h_t(t) \otimes h_c(t) \otimes h_r(t) \tag{1}$$

$$q(t) = \sum_{m=-\infty}^{\infty} A_m h(t - mT) + n(t) \otimes h_r(t) \tag{2}$$

- The received signal, $q(t)$, is sampled at kT.

$$q_k = \sum_{m=-\infty}^{\infty} A_m h(kT - mT) + u(kT) \quad , u(t) \equiv n(t) \otimes h_r(t) \tag{3}$$

- To have zero intersymbol interference (i.e. $q_k = A_k + u_k$)

$$h(kT) = \delta_k \qquad (\delta_k = 0, 1, 0, 0, 0, \ldots) \tag{4}$$

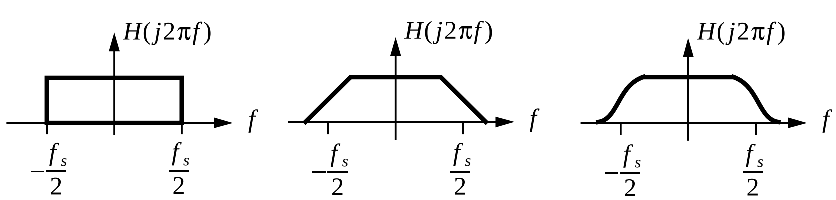
Nyquist Pulses

- For zero ISI, the same criteria in the frequency domain is: ($f_s = 1/T$)

$$\frac{1}{T} \sum_{m=-\infty}^{\infty} H(j2\pi f + jm2\pi f_s) = 1 \tag{5}$$

- Known as Nyquist Criterion

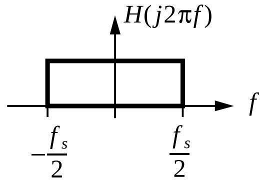
Example Nyquist Pulses (in freq domain)



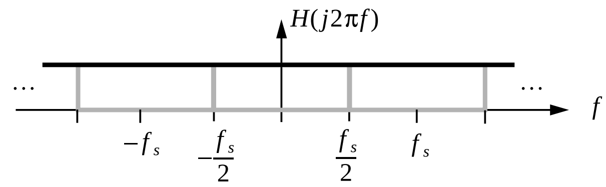
Sinc pulse

Raised-cosine pulse

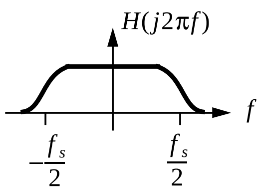
Nyquist Pulses



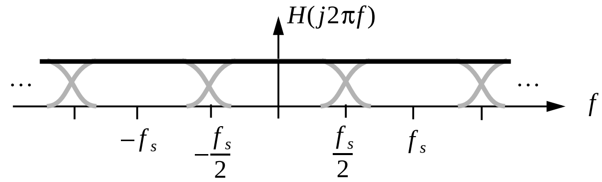
Sinc pulse



Sinc pulse

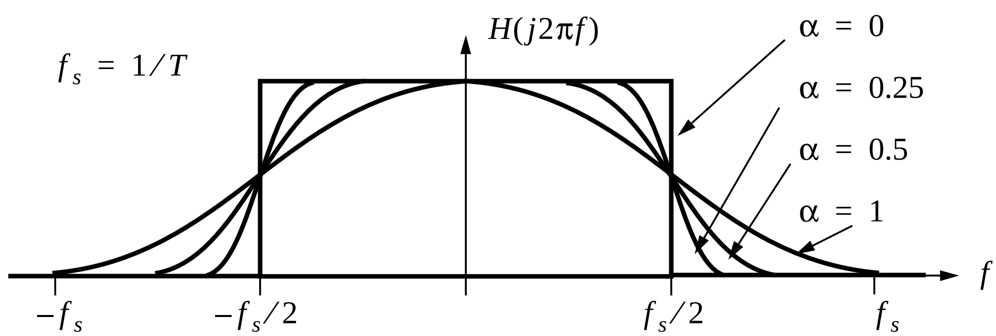


Raised-cosine pulse



Raised-cosine pulse

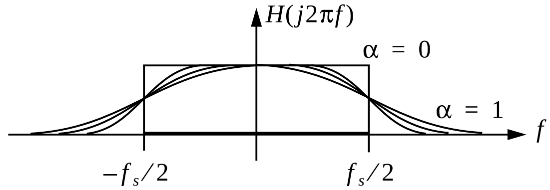
Raised-Cosine Pulse

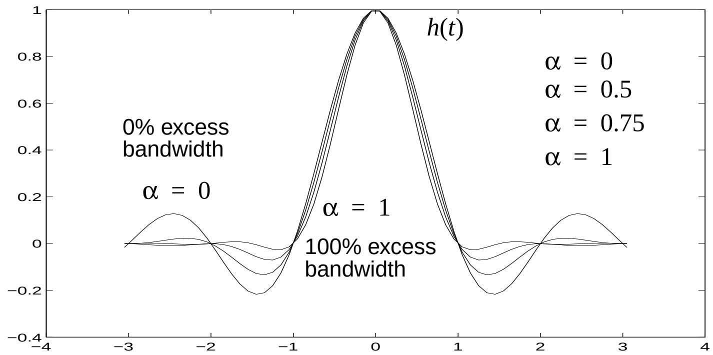


$$H(j2\pi f) = \begin{cases} T; & 0 \le |f| \le (1-\alpha)\left(\frac{f_s}{2}\right) \\ \frac{T}{2}\left[1 + \cos\left[\frac{\pi}{2\alpha}\left(\frac{|2f|}{f_s} - (1-\alpha)\right)\right]\right] & (1-\alpha)\left(\frac{f_s}{2}\right) \le |f| \le (1+\alpha)\left(\frac{f_s}{2}\right) \\ 0; & |f| > (1+\alpha)\left(\frac{f_s}{2}\right) \end{cases}$$

- α determines ***excess bandwidth***

Raised-Cosine Pulses





- More excess bandwidth — impulse decays faster.

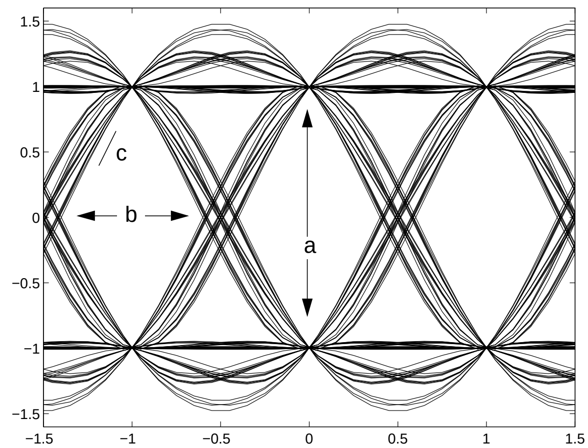
Raised-Cosine Pulse

- α determines amount of excess bandwidth past $f_s/2$
- Example: $\alpha = 0.25$ implies that bandwidth is 25 percent higher than $f_s/2$ while $\alpha = 1$ implies bandwidth extends up to f_s.
- Larger excess bandwidth — easier receiver
- Less excess bandwidth — more efficient channel use

Example

- Max symbol-rate if a 50% excess bandwidth is used and bandwidth is limited to 10kHz
- $1.5 \times (f_s/2) = 10\text{ kHz}$ implies $f_s = 13.333 \times 10^3\text{ symbols/s}$

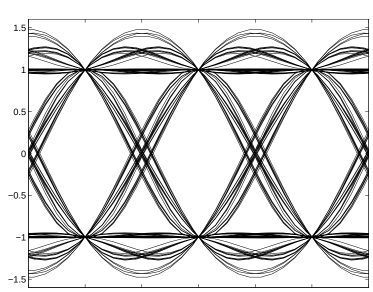
Eye Diagram



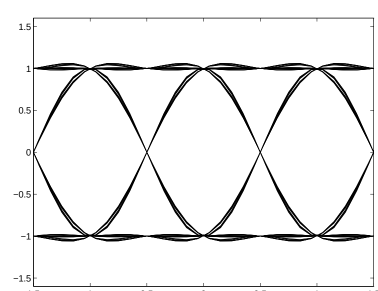
- “a” indicates immunity to noise
- “b” indicates immunity to errors in timing phase
- slope “c” indicates sensitivity to jitter in timing phase

Eye Diagram

- Zero crossing — NOT a good performance indicator
- 100% bandwidth has little zero crossing jitter
- 50% BW has alot of zero crossing jitter but it is using less bandwidth



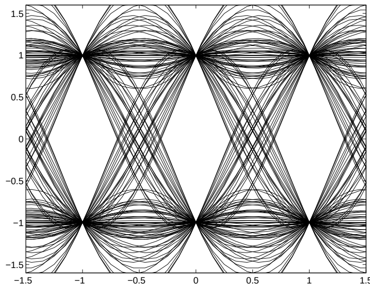
$\alpha = 0.5$ 50% excess bandwidth



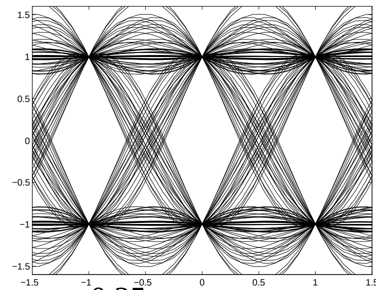
$\alpha = 1$ 100% excess bandwidth

- Less excess BW — more intolerant to timing phase

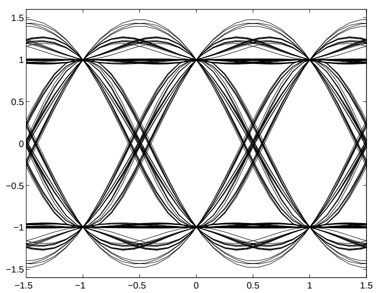
Example Eye Diagrams



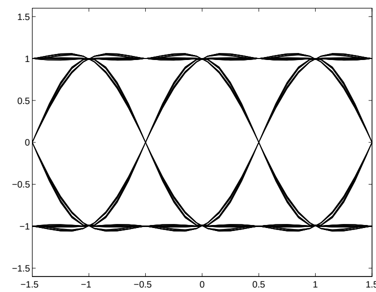
$\alpha = 0$ 0% excess bandwidth



$\alpha = 0.25$ 25% excess bandwidth

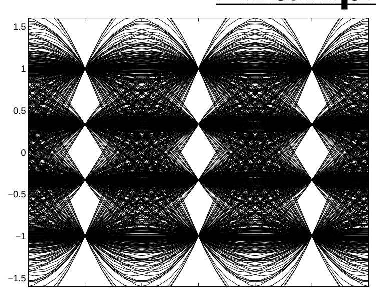


$\alpha = 0.5$ 50% excess bandwidth

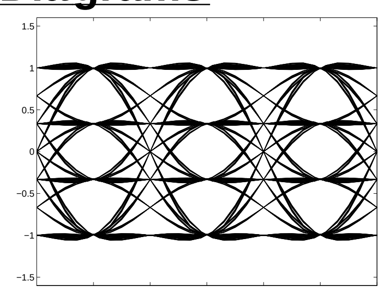


$\alpha = 1$ 100% excess bandwidth

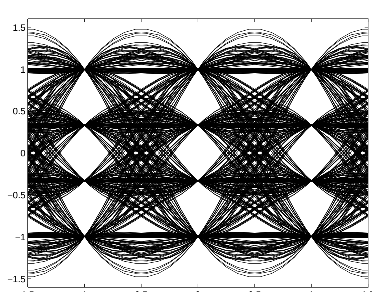
Example Eye Diagrams



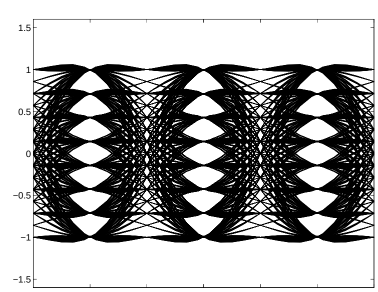
$\alpha = 0$ 0% excess bandwidth



$\alpha = 1$ 100% excess bandwidth

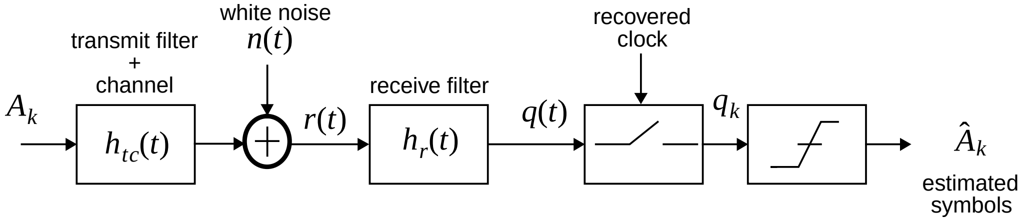


$\alpha = 0.5$ 50% excess bandwidth



$\alpha = 1$ 100% excess bandwidth

Matched-Filter



- For zero-ISI, $h_{tc}(t) \otimes h_r(t)$ satisfies Nyquist criterion.
- For optimum noise performance, $h_r(t)$ should be a ***matched-filter***.
- A matched-filter has an impulse response which is time-reversed of $h_{tc}(t)$

$$h_r(t) = K h_{tc}(-t) \tag{6}$$

where K is an arbitrary constant.

Matched-Filter (proof)

- Consider isolated pulse case (so no worry about ISI)

$$r(t) = A_0 h_{tc}(t) + n(t) \tag{7}$$

$$q_0 = \int_{-\infty}^{\infty} r(\tau) h_r(t - \tau) d\tau \Big|_{t=0} = \int_{-\infty}^{\infty} r(\tau) h_r(-\tau) d\tau \tag{8}$$

$$q_0 = A_0 \int_{-\infty}^{\infty} h_{tc}(\tau) h_r(-\tau) d\tau + \int_{-\infty}^{\infty} n(\tau) h_r(-\tau) d\tau \tag{9}$$

- Want to maximize signal term to noise term
- Variance of noise is

$$\sigma_n^2 = N_0 \int_{-\infty}^{\infty} h_r^{\,2}(-\tau) d\tau \tag{10}$$

Matched-Filter (proof)

- Assuming A_0 and $h_{tc}(t)$ fixed, want to maximize

$$\mathrm{SNR} = \frac{A_0^2\left[\int_{-\infty}^{\infty} h_{tc}(\tau)h_r(-\tau)d\tau\right]^2}{N_0^2\int_{-\infty}^{\infty} h_r^{\,2}(-\tau)d\tau} \qquad (11)$$

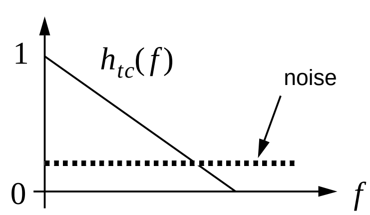
- Use Schwarz inequality

$$\left[\int_a^b f_1(x)f_2(x)dx\right]^2 \le \left[\int_a^b f_1^2(x)dx\right]\left[\int_a^b f_2^2(x)dx\right] \qquad (12)$$

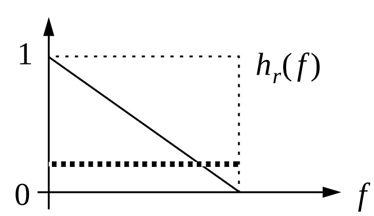
with equality if and only if $f_2(x) = Kf_1(x)$

- Maximizing (11) results in $h_r(t) = Kh_{tc}(-t)$ — QED

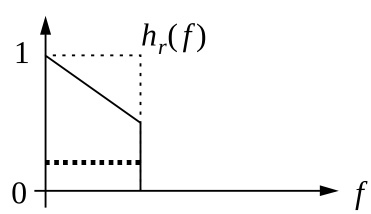
Matched-Filter — Why optimum?



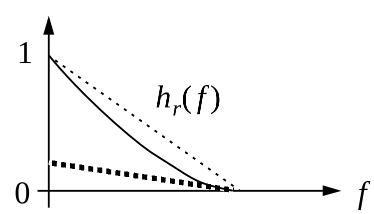
Transmit filter, channel and noise



Too much noise,
All of signal



Too little signal,
Less noise



Just right — max SNR

ISI and Noise

- In general, we need the output of a ***matched filter*** to obey Nyquist criterion
- Frequency response at output of matched filter is $|H_{tc}(j\omega)|^2$ leading to criterion

$$\frac{1}{T}\sum_{m=-\infty}^{\infty}|H_{tc}(j2\pi f + jm2\pi f_s)|^2 = 1 \qquad (13)$$

Example

- Assume a flat freq resp channel and raised-cosine pulse is desired at matched-filter output
- Transmit filter should be $\sqrt{\text{raised-cosine}}$
- Receive filter should be $\sqrt{\text{raised-cosine}}$

Gaussian Noise and SNR Requirement

Probability Distrib ution Function

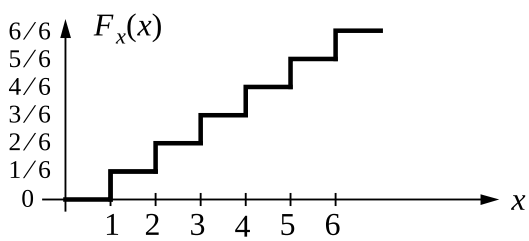
- Consider a random variable X
- Cumulative distribution function (c.d.f.) — $F_x(x)$

$$F_x(x) \equiv P_r(X \le x) \quad -\infty < x < \infty \tag{14}$$

$$1 \ge F_x(x) \ge 0 \tag{15}$$

Example

- Consider a fair die



Probability Density Function

- Derivative of $F_x(x)$ is p.d.f. defined as $f_x(x)$

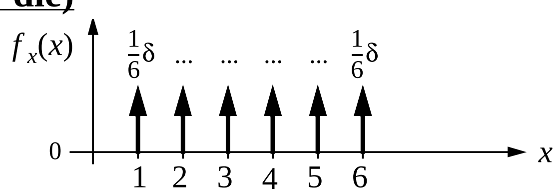
$$f_x(x) \equiv \frac{dF_x(x)}{dx} \qquad \text{or} \qquad F_x(x) = \int_{-\infty}^{\alpha} f_x(\alpha)\,d\alpha \tag{16}$$

- To find prob that X is between x_1 and x_2

$$P_r(x_1 < X \le x_2) = \int_{x_1}^{x_2} f_x(\alpha)\,d\alpha \tag{17}$$

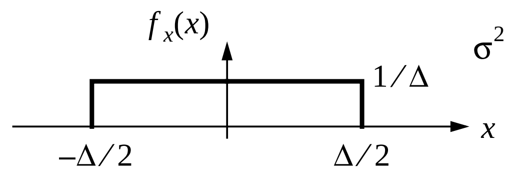
- It is the area under p.d.f. curve.

Example (fair die)



Uniform Distribution

- p.d.f. is a constant
- Variance is given by: $\sigma^2 = \frac{\Delta^2}{12}$ where Δ is range of random variables



$\sigma^2 = \Delta^2/12$

- Crest factor: $CF \equiv \frac{\max}{\sigma} = \frac{\Delta/2}{\Delta/\sqrt{12}} = \sqrt{3} = 1.732$

Example

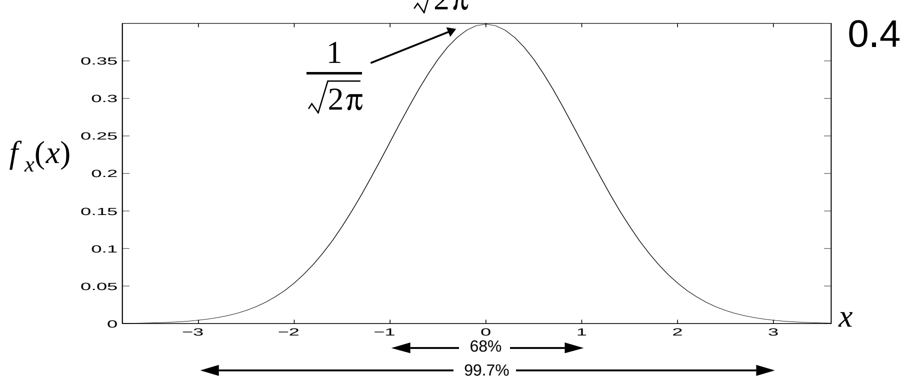
- A uniform random variable chosen between 0 and 1 has a mean, $\mu = 0.5$, and variance, $\sigma^2 = 1/12$

Gaussian Random Variables

Probability Density Function

- Assuming $\sigma^2 = 1$ (i.e. variance is unity) and $\mu = 0$ (i.e. mean is zero) then

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{18}$$

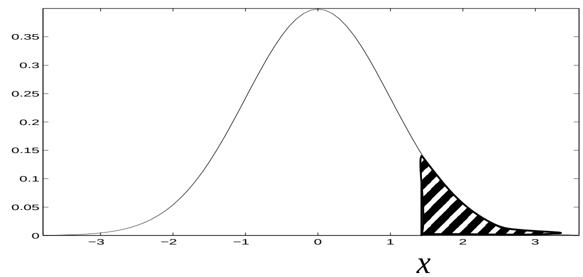


Gaussian Random Variables

- Often interested in how likely a random variable will be in tail of a Gaussian distribution

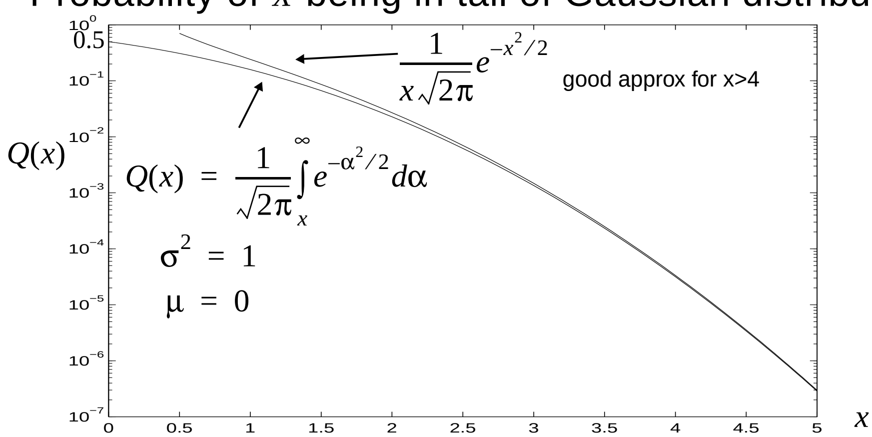
$$Q(x) \equiv P_r(X > x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\alpha^2/2} d\alpha \qquad (19)$$

$$Q(x) = \frac{1}{2} \mathrm{erfc}(x/\sqrt{2}) \qquad (20)$$



Gaussian Random Variables

- Probability of X being in tail of Gaussian distribution

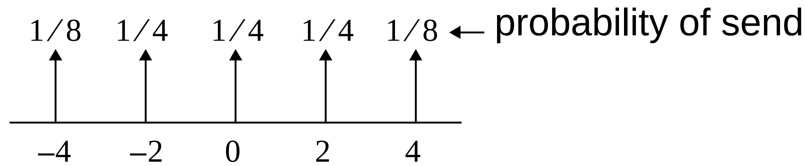


- If $\sigma^2 \neq 1$ or $\mu \neq 0$

$$P_r(X > x) = Q((x - \mu)/\sigma) \qquad (21)$$

Example SNR Calculation

- 100Base-T2 for fast-ethernet uses 5-PAM
- Want to calculate the receive SNR needed for a symbol-error-rate of 10^{-10} (assume rest is ideal).



- Signal power, P_s

$$P_s = \frac{1}{4}\times 0W + \frac{1}{2}\times 4W + \frac{1}{4}\times 16W = 6W \qquad (22)$$

- Using a reference of $1W$ as $0\mathrm{dB}$,

$$P_s = 10\log_{10}(6) = 7.78\mathrm{dB} \qquad (23)$$

Example SNR Calculation

- Assume Gaussian noise added to receive signal.
- Since symbols are distance 2 apart, a noise value greater than 1 will cause an error in receive symbol.
- Want to find σ of Gaussian distribution such that likelihood of random variable greater than 1 is 10^{-10}.
- Recall

$$Q(x/\sigma) = 0.5\mathrm{erfc}((x/\sigma)/\sqrt{2}) \qquad (24)$$

- Let $x = 1$ and set

$$2Q(1/\sigma) = 10^{-10} \qquad (25)$$

(2 value because variable might be >1 or <-1)

$$0.5\times 10^{-10} = Q(1/\sigma) = 0.5\mathrm{erfc}(1/(\sigma\sqrt{2})) \qquad (26)$$

Example SNR Calculation

- Trial and error gives $1/(\sigma\sqrt{2}) = 4.57$ implying that $\sigma = 0.1547 = 1/6.46$
- Noise with $\sigma = 0.1547$ has a power of (ref to $1W$)

$$P_n = 10\log_{10}(\sigma^2) = -16.2\mathrm{dB} \tag{27}$$

- Finally, SNR needed at receive signal is

$$\mathrm{SNR} = 7.78\mathrm{dB} - (-16.2\mathrm{dB}) = 24\mathrm{dB} \tag{28}$$

- Does not account that large positive noise on +4 signal will ***not*** cause symbol error (same on -4).
- It is slightly conservative
- BER approx same as symbol error rate if Gray coded

m-PAM

- For m bits/symbol $\Rightarrow$ 2^m levels
- Normalize distance between levels to 2 (so error of 1 causes a symbol error)
- $(m = 1) \Rightarrow \pm1$ $(m = 3) \Rightarrow \pm1, \pm3, \pm5, \pm7$ etc.
- Noise variance of $(\sigma = 0.1547) \Rightarrow \mathrm{BER} = 10^{-10}$
- Symbols spaced $\pm1, \pm3, \pm5, \ldots, \pm(2^m - 1)$
  — average power is: $S_m = (4^m - 1)/3$

$$\mathrm{SNR} = 10\log\left(\frac{S_m}{\sigma^2}\right) = 10\log\left(\frac{4^m - 1}{3\sigma^2}\right) \tag{29}$$

m-PAM

$$\text{SNR} = 10\log\left(\frac{S_m}{\sigma^2}\right) = 10\log\left(\frac{4^m - 1}{3\sigma^2}\right) \tag{30}$$

- equals 23.1 dB for $m = 2$, $\text{BER} = 10^{-10}$
- equals 28.2 dB for $m = 3$, $\text{BER} = 10^{-10}$ (approx +6dB)
- Can show $S_{m+1} = 4S_m + 1$
- Require 4 times more power to maintain same symbol error rate with same noise power (uncoded)
- In other words,
  — to send 1 more bit/symbol, need 6dB more SNR
  (but does not increase bandwidth)

Why Assume Gaussian Noise?

Central-Limit Theorem

- Justification for modelling many random signals as having a Gaussian distribution

Sum of independent random variables approaches Gaussian as sum increases

- Assumes random variables have identical distributions.
- No restrictions on original distribution (except finite mean and variance).
- Sum of Gaussian random variables is also Gaussian.

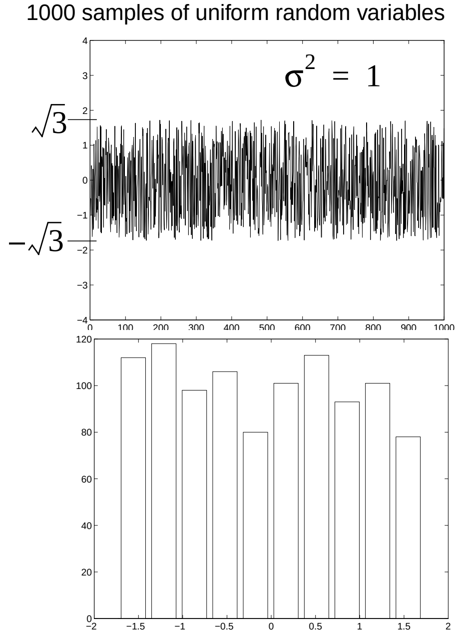
Central Limit Theorem Example

- Consider averaging 10 uniformly random variables.

Sample	1	2	3	4	5	6	7	8	9	10
Uniform Distribution between -100, 100	14	68	42	-22	-18	26	80	-27	-57	-8
	60	-68	88	0	-72	-75	-15	-49	36	90
	-93	-57	-52	-70	13	30	-72	-73	82	27
	7	43	-64	17	-50	24	89	57	-50	-12
	0	-74	-36	69	-2	61	-18	-9	72	65
	91	-82	77	18	-7	-50	-74	-30	-6	38
	50	-45	30	91	92	-5	77	-10	1	40
	11	-99	-70	11	-75	-22	-82	62	20	97
	78	-17	36	-70	-60	-59	-68	86	64	91
	25	-95	-23	97	-36	-94	-86	30	51	70
Average	24	-43	3	14	-22	-16	-17	4	21	50

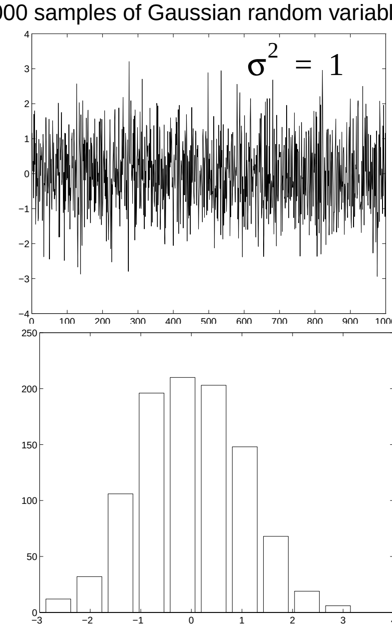
Uniform and Gaussian Signals

1000 samples of uniform random variables



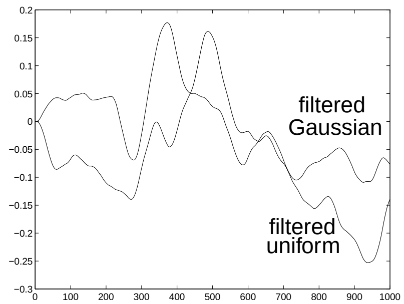
histogram of uniform variable

1000 samples of Gaussian random variables



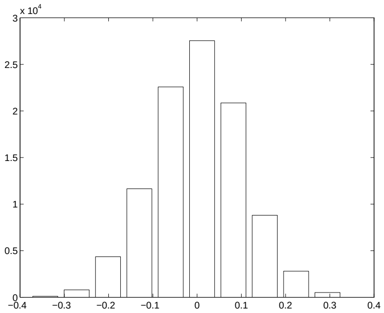
histogram of Gaussian variable

Filtered Random Signals

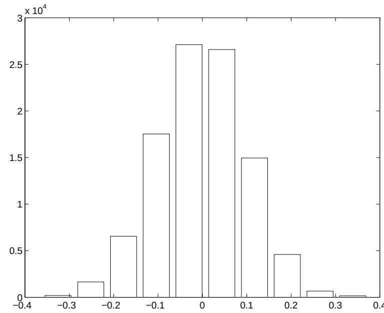


Filtered with 3'rd order Butterworth lowpass with cutoff $f_s / 200$

No longer independent from sample to sample



histogram of filtered uniform
(100,000 samples)

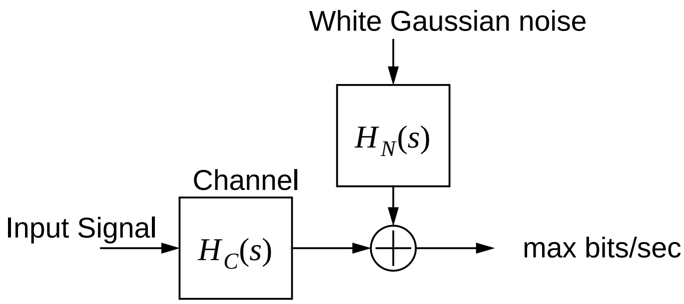


histogram of filtered Gaussian
(100,000 samples)

Channel Capacity

Limits of Communications

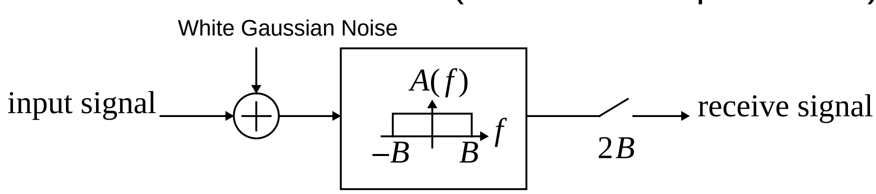
- Want to find max bits/sec for a given channel with arbitrary noise spectrum.



- Limited input signal power
- First find capacity for flat channels with white noise and then integrate to find overall capacity

Channel Capacity of a Bandlimited Channel

- Channel with bandwidth B (baseband or passband).



- Highest rate is $2B$ samples/s to maintain independent noise samples and independent signal samples.
- Receive signal is $2B$ symbols/s
- Thus, the capacity in bits per second is given by

$$C = 2BC_s \tag{31}$$

where C_s is the capacity in bits/symbol

Capacity, C_S — bits/symbol

- C_s is the max number of bits/symbol that can be transmitted through a channel
- In our case, each symbol has additive independent noise samples
- Need to define ***entropy*** and ***mutual information***
- ***Entropy***
  - Amount of information required on average to describe a random variable.
  - Also called average information
- ***Mutual Information***
  - Measure of average information passed through a channel

Entropy (Average Information)

- Information in rare event higher than common event.
- Consider a discrete-valued random process $\{X_k\}$ with outcomes $\{a_1, a_2, \ldots, a_K\}$
- Entropy of X is defined to be

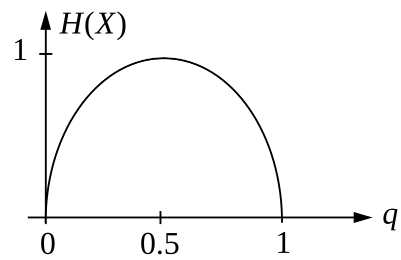
$$H(X) = E[-\log_2(p_x(x))] = -\sum p_x(x)\log_2(p_x(x)) \qquad (32)$$

- Because of $\log_2$ — information measured in "bits"
- Note that actual values are unimportant — only probabilities.
- ***Source coding theorem*** — if independent trials of X occur at r trials/s then can encode source by a bit stream of rate $rH(X) + \varepsilon$ for any $\varepsilon > 0$.

Entropy Example 1

- Consider a binary random variable with alphabet $\{2, 7\}$. If $q = p_x(7)$ then

$$H(X) = -q\log_2(q) - (1-q)\log_2(1-q) \quad (33)$$



- Peaks at 1 bit and no information when $q = 0$ or $q = 1$.
- When $q = 0.5$, source coder is to just transmit 0 for 2 and 1 for 7.

Entropy Example 1

- When $q = 0.1$ in example above, then

$$H(X) = -0.1\log_2(0.1) - 0.9\log_2(0.9) = 0.47 \quad (34)$$

- Tough to achieve optimum but one coding scheme is

2 trials	bits	likelihood
2,2	0	0.81
2,7	10	0.09
7,2	110	0.09
7,7	111	0.01

- Average number of bits is $\text{Rate} = 0.645 \text{ bits/trial}$

$$\text{Rate} = 0.5 \times (0.81 \times 1 + 0.09 \times 2 + 0.09 \times 3 + 0.01 \times 3) \quad (35)$$

- 0.5 because 2 trials are determined at once.

Entropy Example 2

- Consider a 8 horse race with each horse having a probability of winning as:

$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}\right\} \tag{36}$$

$$H(X) = -\frac{1}{2}\log_2(2^{-1}) - \frac{1}{4}\log_2(2^{-2}) - \frac{1}{8}\log_2(2^{-3}) - \frac{1}{16}\log_2(2^{-4}) - \frac{4}{16}\log_2(2^{-6})$$
$$H(X) = \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{4}{16} + \frac{24}{64} = 2 \text{ bits} \tag{37}$$

- Want to send info of winning horse to bookie
- Code as: $[0, 10, 110, 1110, 111100, 111101, 111110, 111111]$
- Average bit length is:

$$\frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4 + 4\times\frac{1}{64}\times 6 = 2 \text{ bits} \tag{38}$$

Min and Max Entropy

- $0 \le H(X)$

Discrete Valued

- If K is size of alphabet, then

$$0 \le H(X) \le \log_2(K) \tag{39}$$

- If and only if all outcomes equally likely,

$$H(X) = \log_2(K) \tag{40}$$

Example:

- If eight horses running all have an equal chance of winning, 3 bits needed to send info to bookie.

Differential Entropy

- All continuous-valued random variables have infinite entropy (try sending π to someone)
- Use differential entropy for cont-valued variables.
- It is a measure on how fast the entropy goes to ∞

Min and Max

- If Y has zero mean and variance σ^2

$$0 \le h(Y) \le 0.5\log_2(2\pi e\sigma^2) \qquad (41)$$

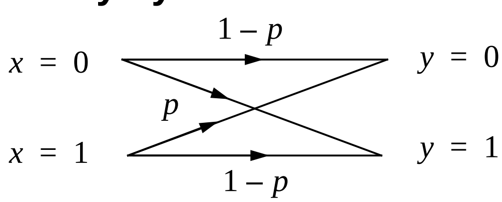
- If and only if Y is Gaussian, then

$$h(Y) = 0.5\log_2(2\pi e\sigma^2) \qquad (42)$$

Conditional Entropy

- ***Conditional entropy***, $H(X|Y)$ — average info needed to represent X after observing Y
- If $Y = X$, then $H(X|Y) = 0$
- If knowing Y gives no info for X then $H(X|Y) = H(X)$

Example: Binary symmetric channel



- $p = 0 \text{ or } 1$ implies $H(X|Y) = 0$
- $p = 0.5$ implies $H(X|Y) = H(X)$

Mutual Information

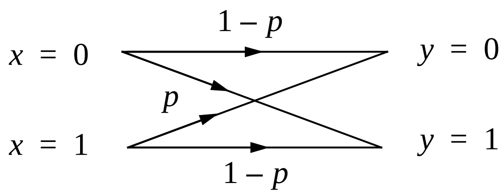
- Average ***mutual information***: $I(X, Y)$

$$I(X, Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{43}$$

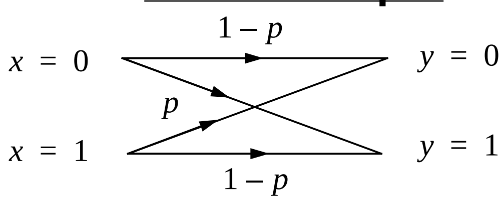
- It is always positive and is a measure of the average information passed through channel.

Example:

- Binary symmetric channel



BSC Example



- Assume $X = \{0.5, 0.5\}$ (i.e. 0 or 1 equally likely)

Y \ X	0	1
0	0.5(1-p)	0.5(p)
1	0.5(p)	0.5(1-p)

$$X = \{0.5, 0.5\} \qquad Y = \{0.5, 0.5\}$$

$$H(X) = 1 \qquad H(Y) = 1$$

BSC Example

$$H(X|Y) = \sum_{i=1}^{2} P(Y=y_i)H(X|Y=y_i)$$
$$= 0.5H(p, 1-p) + 0.5H(p, 1-p)$$
$$= H(p, 1-p) \quad (44)$$

which is the entropy of 1 random variable with 2 levels having probabilities of p and $1-p$.

$$H(Y|X) = H(p, 1-p) \quad (45)$$

$$I(X, Y) = H(X) - H(X|Y) = 1 - H(p, 1-p) \quad (46)$$

- If $p = 0.5$, $H(0.5, 0.5) = 1$ and $I(X, Y) = 0$: no transmission
- If $p = 1$, $H(1, 0) = 0$ and $I(X, Y) = 1$: full transmission

Channel Capacity, C_s

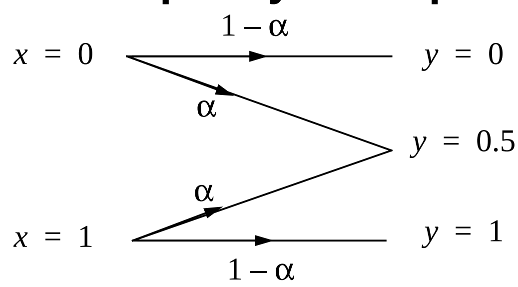
- Channel capacity per symbol: C_s

$$C_s = \max_{p_x(x)} I(X, Y) \quad (47)$$

- C_s is in units of bits/symbol
- ***It is the maximum*** $I(X, Y)$ ***over all possible input probability distributions.***
- If channel is used s times per second, then capacity in bits/second: C

$$C = sC_s \quad (48)$$

Capacity Example



- Above is a binary erasure channel
- Receiver knows when a bit was erased
- However, sent bit is unknown
- What should the pdf of the input signal be for capacity to be reached in channel?

Capacity Example

- Can show that capacity is achieved when X is equally likely to be a 0 or a 1
- Capacity is then equal to

$$C_s = 1 - \alpha \tag{49}$$

- With feedback from Y to X, can easily build a channel to achieve capacity (ask for a re-transmit)
- But, ***capacity can be achieved without feedback!***

- In general, feedback does not increase capacity of channels (though it often can make the design much simpler).

Channel Capacity with additive Gaussian

- Assume a channel with $Y = X + N$
- N is an independent zero-mean Gaussian random variable with variance σ_n^2
- From (42), $h(N) = 0.5\log_2(2\pi e\sigma_n^2)$
- Assume the variance of X is σ_x^2
- From (47), we want to maximize

$$I(X, Y) = h(Y) - h(Y|X) \tag{50}$$

Channel Capacity with additive Gaussian

- However, $h(Y|X) = h(N)$ since once X is known, information left in Y is the noise and this is fixed.

$$h(Y|X) = 0.5\log_2(2\pi e\sigma_n^2) \tag{51}$$

- Need to maximize $h(Y)$ and this is done if Y is Gaussian which is possible if X is Gaussian.
- Maximum of $h(Y)$ given by

$$h(Y) = 0.5\log_2(2\pi e(\sigma_x^2 + \sigma_n^2)) \tag{52}$$

Channel Capacity with additive Gaussian

- Thus, channel capacity is achieved with a Gaussian input resulting in

$$C_s = 0.5\log_2(2\pi e(\sigma_x^2 + \sigma_n^2)) - 0.5\log_2(2\pi e\sigma_n^2) \quad (53)$$

$$\boxed{C_s = 0.5\log_2(1 + \sigma_x^2/\sigma_n^2)} \quad (54)$$

in bits/symbol

- This assumes a real-valued random signal.
- σ_x^2/σ_n^2 is the signal-to-noise ratio of the received signal

m-PAM and Channel Capacity

- For $(\text{BER} = 10^{-10}) \Rightarrow \sigma_n{}^2 = 0.1547^2 = 1/41.8$
- At m-bits per symbol, channel capacity is actually

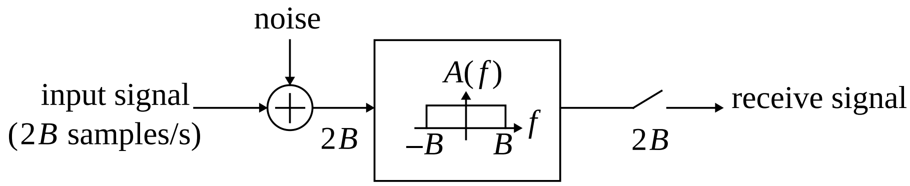
$$\begin{aligned} C_s &= 0.5\log_2(1 + S_m/\sigma_n^2) \\ &= 0.5\log_2(1 + (4^m - 1)/3\sigma_n^2) \\ &\cong 0.5\log_2((4^m)/(3\sigma_n^2)) \end{aligned} \quad (55)$$

- which for $\sigma_n{}^2 = 1/41.8$

$$\begin{aligned} C_s &= \log_2(2^m) + 0.5\log_2(13.9) \\ &= m + 1.9 \end{aligned} \quad (56)$$

- So channel capacity is 1.9 more bits/symbol — almost 12 dB power savings (for m large)

Channel Capacity of a Bandlimited Channel



- Recall

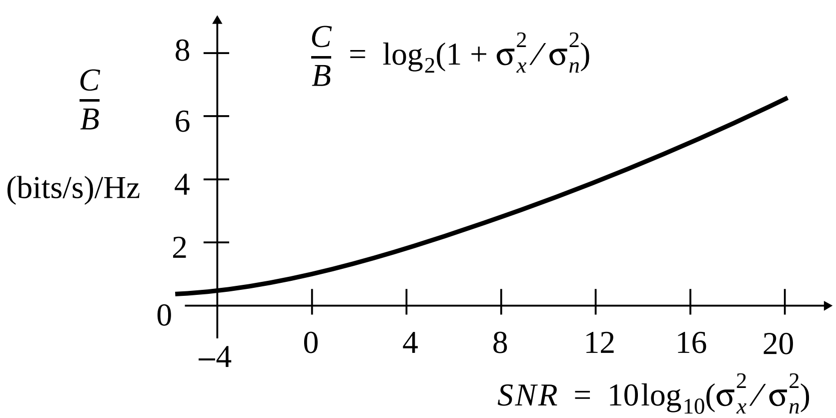
$$C = 2BC_s \tag{57}$$

- Thus, the capacity in bits per second is given by

$$\boxed{C = B\log_2(1 + \sigma_x^2/\sigma_n^2)} \tag{58}$$

- σ_x^2/σ_n^2 is the signal-to-noise ratio of the received signal

Normalized Channel Capacity



- For example, a SNR of 20dB has a capacity of 6.66 (bits/s)/Hz.

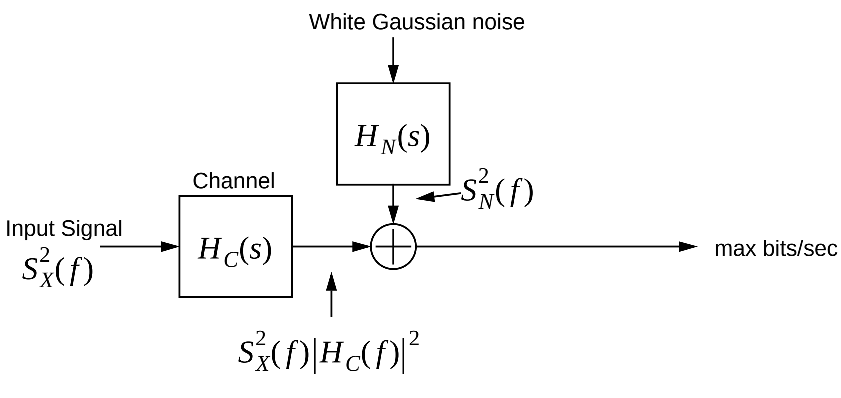
Capacity of a Telephone Channel

- The dominant noise source is PCM quantization error
- SNR approximately 35dB
- Bandwidth approximately 3kHz

$$C = 3000 \times \log_2(1 + 10^{3.5}) = 3000 \times 11.63 \approx 34 \text{ kbits/s} \qquad (59)$$

- To achieve this rate, one needs $11.63\ \text{(bits/s)/Hz}$.
- X56 modems achieve higher rates since the quantization error is not random but known
- 8-bit A/D converters used at 8kS/s - potential for 64kb/s

Capacity for Non-Flat Channels and Noise



- Break the channel into small Δf regions and integrate.

$$C = \int_0^{\infty} \log_2\left(1 + \frac{S_X^2(f)|H_C(f)|^2}{S_N^2(f)}\right) df \qquad (60)$$

Capacity for Non-Flat Channels

- Maximize capacity under fixed input power constraint

$$P_s = \int_{-\infty}^{\infty} S_X^2(f)df \qquad (61)$$

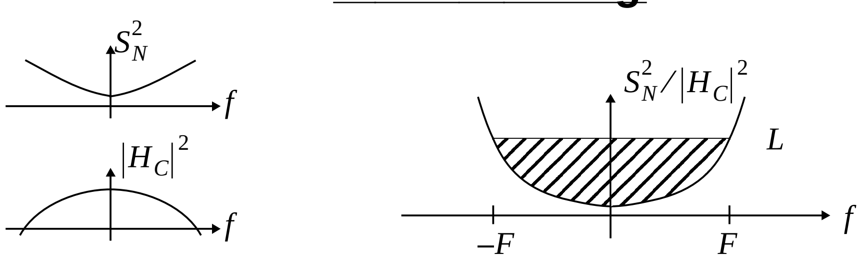
- Can show that $S_X^2(f)$ should be chosen such that

$$S_X^2(f) = \begin{cases} L - \dfrac{S_N^2(f)}{|H_C(f)|^2}, & 0 \le f \le F \\ 0, & \text{otherwise} \end{cases} \qquad (62)$$

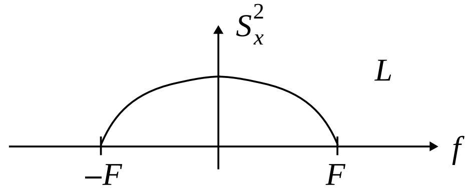
where L is chosen to meet the power constraint and F is the set of freq where $L > S_N^2 / |H_C|^2$

- Above known as the ***water-pouring spectrum.***

Water-Pouring



- The area equals the transmit power, P_s.
- Note that the spectrum shape of the transmit signal should be large where the SNR is large.



Adaptive Equalization

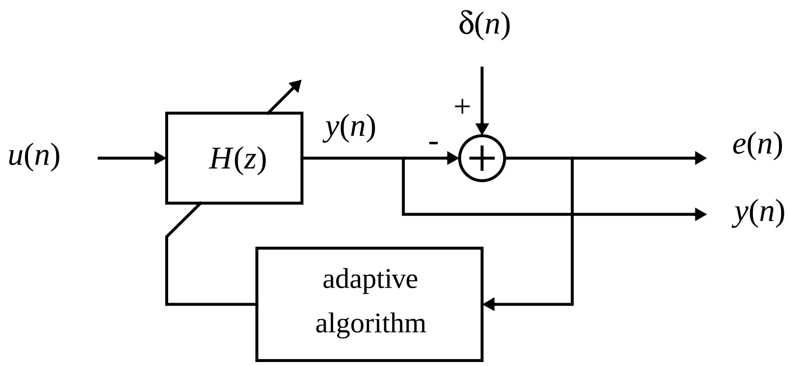
Adaptive Filter Introduction

- Adaptive filters are used in:
  - Noise cancellation
  - Echo cancellation
  - Sinusoidal enhancement (or rejection)
  - Beamforming
  - Equalization

- Adaptive equalization for data communications proposed by R.W. Lucky at Bell Labs in 1965.
- LMS algorithm developed by Widrow and Hoff in 60s for neural network adaptation

Adaptive Filter Introduction

- A typical adaptive system consists of the following two-input, two output system



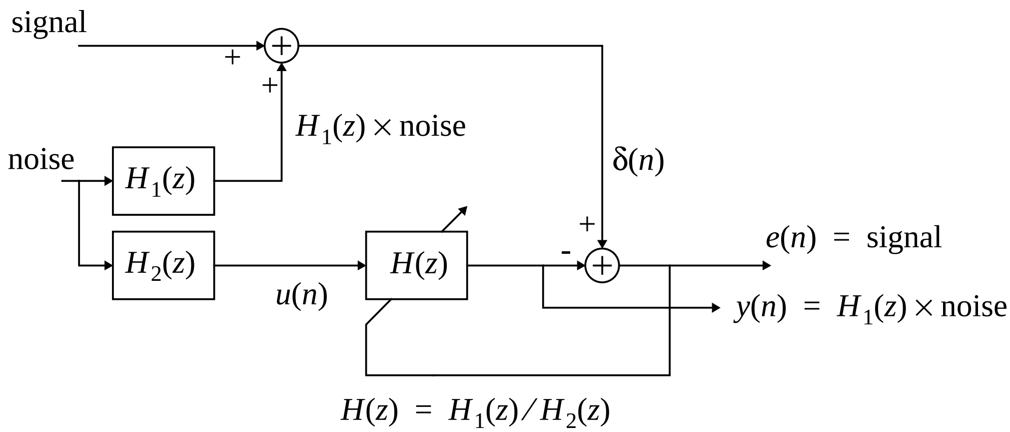
- $u(n)$ and $y(n)$ are the filter's input and output
- $\delta(n)$ and $e(n)$ are the reference and error signals

Adaptive Filter Goal

- Find a set of filter coefficients to minimize the power of the error signal, $e(n)$.
- Normally assume the time-constant of the adaptive algorithm is ***much slower*** than those of the filter, $H(z)$.
- If it were instantaneous, it could always set $y(n)$ equal to $\delta(n)$ and the error would be zero (this is useless)

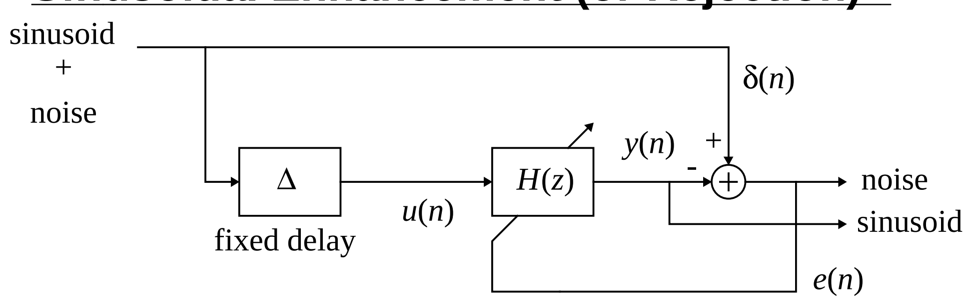
- Think of adaptive algorithm as an optimizer which finds the best set of ***fixed*** filter coefficients that minimizes the power of the error signal.

Noise (and Echo) Cancellation



- Useful in cockpit noise cancelling, fetal heart monitoring, acoustic noise cancelling, echo cancelling, etc.

Sinusoidal Enhancement (or Rejection)



- The sinusoid's frequency and amplitude are unknown.
- If $H(z)$ is adjusted such that its phase plus the delay equals 360 degrees at the sinusoid's frequency, the sinusoid is cancelled while the noise is passed.
- The "noise" might be a broadband signal which should be recovered.

Adaptation Algorithm

- Optimization might be performed by:
  - perturb some coefficient in $H(z)$ and check whether the power of the error signal increased or decreased.
  - If it decreased, go on to the next coefficient.
  - If it increased, switch the sign of the coefficient change and go on to the next coefficient.
  - Repeat this procedure until the error signal is minimized.
- This approach is a steepest-descent algorithm but is slow and not very accurate.
- The LMS (Least-Mean-Square) algorithm is also a steepest-descent algorithm but is more accurate and simpler to realize

Steepest-Descent Algorithm

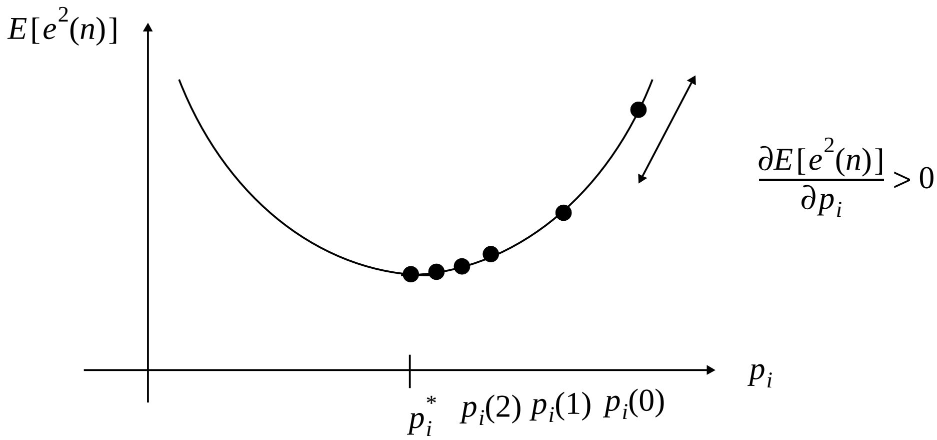
- Minimize the power of the error signal, $E[e^2(n)]$
- General steepest-descent for filter coefficient $p_i(n)$:

$$p_i(n+1) = p_i(n) - \mu\left(\frac{\partial E[e^2(n)]}{\partial p_i}\right)$$

- Here $\mu > 0$ and controls the adaptation rate

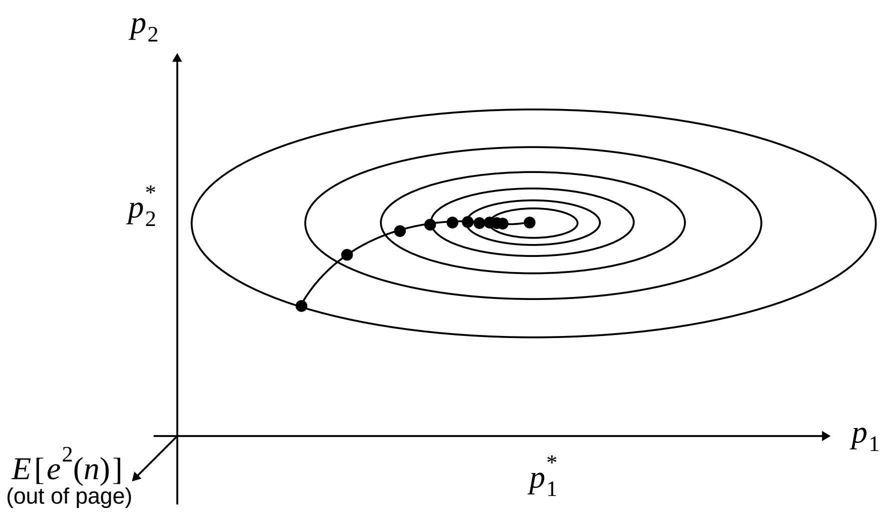
Steepest Descent Algorithm

- In the one-dimensional case



Steepest-Descent Algorithm

- In the two-dimensional case



- Steepest-descent path follows perpendicular to tangents of the contour lines.

LMS Algorithm

- Replace expected error squared with instantaneous error squared. Let adaptation time smooth out result.

$$p_i(n+1) = p_i(n) - \mu\left(\frac{\partial e^2(n)}{\partial p_i}\right)$$

$$p_i(n+1) = p_i(n) - 2\mu e(n)\left(\frac{\partial e(n)}{\partial p_i}\right)$$

- and since $e(n) = \delta(n) - y(n)$, we have

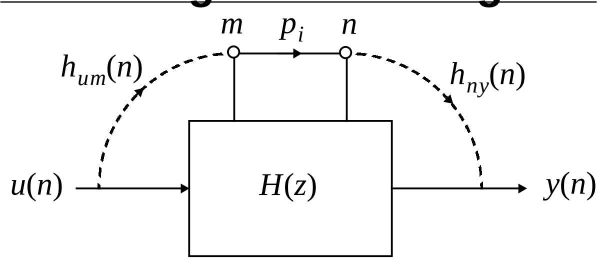
$$p_i(n+1) = p_i(n) + 2\mu e(n)\phi_i(n) \quad \text{where } \phi_i = \partial y(n) / \partial p_i$$

- $e(n)$ and $\phi_i(n)$ are uncorrelated after convergence.

Variants of the LMS Algorithm

- To reduce implementation complexity, variants are taking the sign of $e(n)$ and/or $\phi_i(n)$.
- ***LMS*** — $p_i(n+1) = p_i(n) + 2\mu e(n) \times \phi_i(n)$
- ***Sign-data LMS*** — $p_i(n+1) = p_i(n) + 2\mu e(n) \times \operatorname{sgn}(\phi_i(n))$
- ***Sign-error LMS*** — $p_i(n+1) = p_i(n) + 2\mu \operatorname{sgn}(e(n)) \times \phi_i(n)$
- ***Sign-sign LMS*** — $_i(n+1) = p_i(n) + 2\mu \operatorname{sgn}(e(n)) \times \operatorname{sgn}(\phi_i(n)$
- However, the sign-data and sign-sign algorithms have gradient misadjustment — ***may not converge!***
- These LMS algorithms have different dc offset implications in analog realizations.

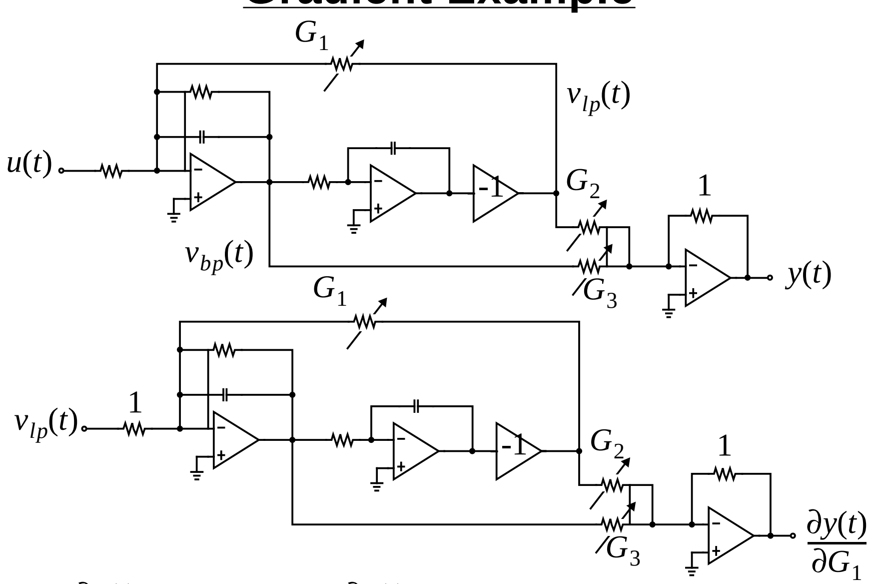
Obtaining Gradient Signals



$$\phi_i(n) = \frac{\partial y(n)}{\partial p_i} = h_{ny}(n) \otimes h_{um}(n) \otimes u(n)$$

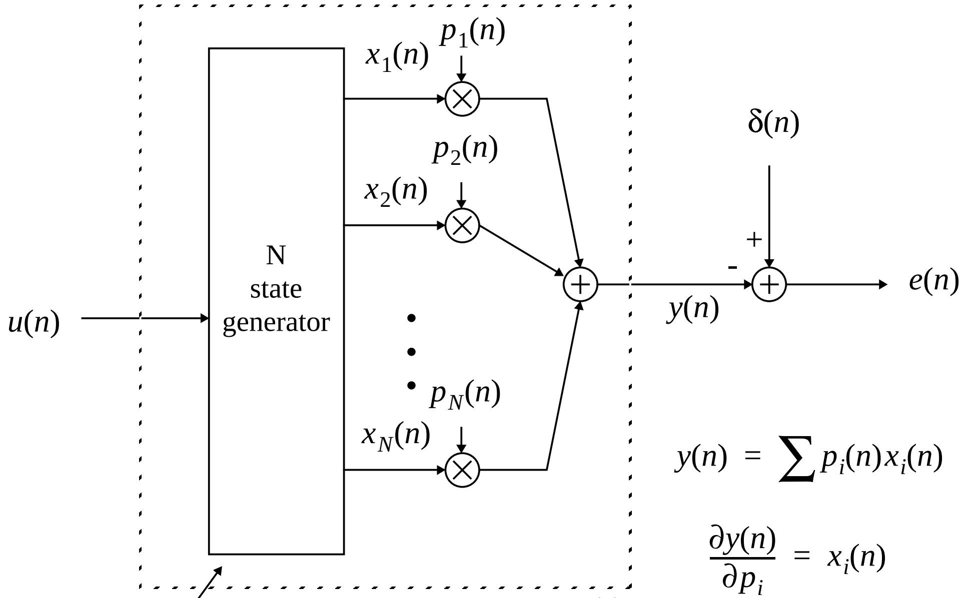
- $H(z)$ is a LTI system where the signal-flow-graph arm corresponding to coefficient p_i is shown explicitly.
- $h_{um}(n)$ is the impulse response of from u to m
- The gradient signal with respect to element p_i is the convolution of $u(n)$ with $h_{um}(n)$ convolved with $h_{ny}(n)$.

Gradient Example



$$\frac{\partial y(t)}{\partial G_2} = -v_{lp}(t) \qquad \frac{\partial y(t)}{\partial G_3} = -v_{bp}(t)$$

Adaptive Linear Combiner



$$y(n) = \sum p_i(n) x_i(n)$$

$$\frac{\partial y(n)}{\partial p_i} = x_i(n)$$

often, a tapped delay line

$$H(z) = \frac{Y(z)}{U(z)}$$

Adaptive Linear Combiner

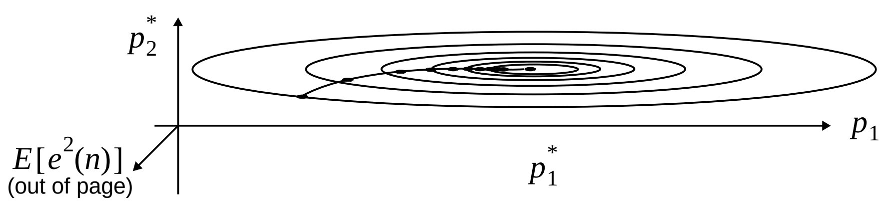
- The gradient signals are simply the state signals

$$p_i(n+1) = p_i(n) + 2\mu e(n) x_i(n) \quad (63)$$

- Only the zeros of the filter are being adjusted.
- There is no need to check that for filter stability (though the adaptive algorithm could go unstable if μ is too large).
- The ***performance surface is guaranteed unimodal*** (i.e. there is only one minimum so no need to worry about being stuck in a local minimum).
- The performance surface becomes ill-conditioned as the state-signals become correlated (or have large power variations).

Performance Surface

- Correlation of two states is determined by multiplying the two signals together and averaging the output.
- Uncorrelated (and equal power) states result in a “hyper-paraboloid” performance surface — good adaptation rate.
- Highly-correlated states imply an ill-conditioned performance surface — more residual mean-square error and longer adaptation time.



Adaptation Rate

- Quantify performance surface — state-correlation matrix

$$R \equiv \begin{bmatrix} E[x_1x_1] & E[x_1x_2] & E[x_1x_3] \\ E[x_2x_1] & E[x_2x_2] & E[x_2x_3] \\ E[x_3x_1] & E[x_3x_2] & E[x_3x_3] \end{bmatrix}$$

- Eigenvalues, λ_i, of R are all positive real — indicate curvature along the principle axes.
- For adaptation stability, $0 < \mu < \frac{1}{\lambda_{max}}$ but adaptation rate is determined by least steepest curvature, λ_{min}.
- Eigenvalue spread indicates performance surface conditioning.

Adaptation Rate

- Adaptation rate might be 100 to 1000 times slower than time-constants in programmable filter.
- Typically use same μ for all coefficient parameters since orientation of performance surface not usually known.
- A large value of μ results in a larger coefficient "bounce".
- A small value of μ results in slow adaptation
- Often "gear-shift" μ — use a large value at start-up then switch to a smaller value during steady-state.
- Might need to detect if one should "gear-shift" again.

Adaptive IIR Filtering

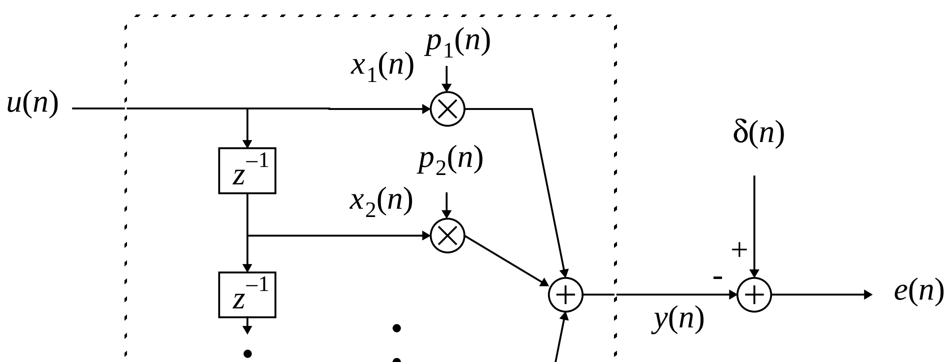
- The poles (and often the zeros) are adjusted — useful in applications with long impulse responses.
- A stability check might be needed for the adaptive filter itself to ensure the poles do not go outside the unit circle for too long a time (or perhaps at all).
- In general, a multi-modal performance surface occurs. Thus, one should try not be stuck in a local minimum too far from the global minimum
- However, if the order of the adaptive filter is greater than the order of the system being matched (and all poles and zeros are being adapted) — the performance surface is unimodal.
- To obtain the gradient signals for poles, extra filters are generally required.

Adaptive IIR Filtering

- Direct-form structure needs only one additional filter to obtain all the gradient signals.
- However, choice of structure for programmable filter is VERY important — sensitive structures tend to have ill-conditioned performance surfaces.
- Equation error structure has unimodal performance surface but has a bias.
- SHARF (simplified hyperstable adaptive recursive filter) — the error signal is filtered to guarantee adaptation — needs to meet a strictly-positive-real condition
- There are few commercial use of adaptive IIR filters

Digital Adaptive Filters

- FIR tapped delay line is the most common



$$y(n) = \sum p_i(n) x_i(n)$$

$$\frac{\partial y(n)}{\partial p_i} = x_i(n)$$

FIR Adaptive Filters

- All poles at $z = 0$ and zeros only adapted.
- Special case of an adaptive linear combiner
- Unimodal performance surface
- States are uncorrelated and equal power if input signal is white — hyper-paraboloid
- If not sure about correlation matrix, can guarantee adaptation stability by choosing

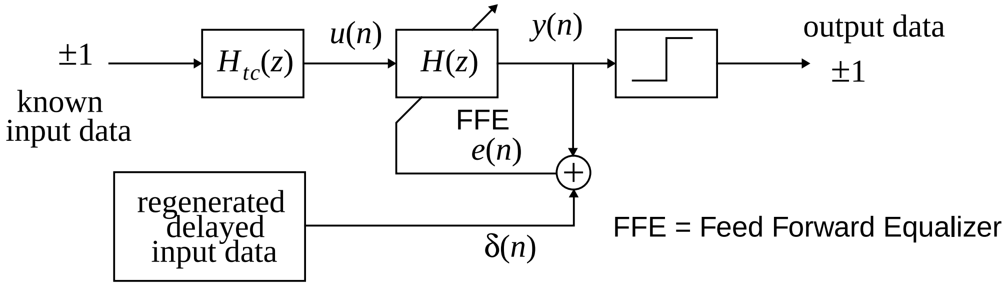
$$0 < \mu < \frac{1}{(\#\text{ of taps})(\text{input signal power})}$$

- Usually need an AGC so signal power is known.

FIR Adaptive Filter

- Coefficient word length typically $2 + 0.5\log_2(\#\text{ of taps})$ bits longer than “bit-equivalent” dynamic range
- Example: 6-bit input with 8-tap FIR might have 10-bit coefficient word lengths.
- Example: 12-bit input with 128-tap FIR might have 18-bit coefficient word lengths for 72 dB output SNR.
- Requires multiplies in filter and adaptation algorithm (unless an LMS variant used or slow adaptation rate) — twice the complexity of FIR fixed filter.
- **POWER:** typically 1 (mW/MHz)/tap
- Example [Abbot, ISSCC, 94]: 9 tap FIR filter with 6-bit input and 10b coeff — 500 mW at 72 MHz.

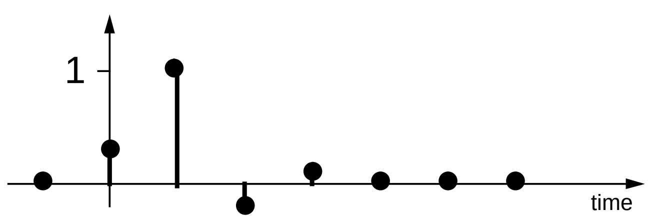
Equalization — Training Sequence



- The reference signal, $\delta(n)$ is equal to a delayed version of the transmitted data
- The training pattern should be chosen so as to ease adaptation — pseudorandom is common.
- Above is a feedforward equalizer (FFE) since $y(n)$ is not directly created using derived output data

FFE Example

- Suppose channel, $H_{tc}(z)$, has impulse response 0.3, 1.0, -0.2, 0.1, 0.0, 0.0



- If FFE is a 3-tap FIR filter with

$$y(n) = p_1 u(n) + p_2 u(n-1) + p_3 u(n-2) \tag{64}$$

- Want to force $y(1) = 0$, $y(2) = 1$, $y(3) = 0$
- Not possible to force all other $y(n) = 0$

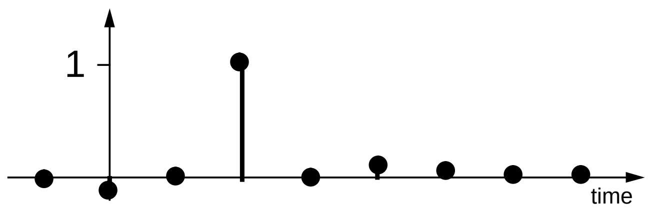
FFE Example

$$y(1) = 0 = 1.0p_1 + 0.3p_2 + 0.0p_3$$

$$y(2) = 1 = -0.2p_1 + 1.0p_2 + 0.3p_3$$

$$y(3) = 0 = 0.1p_1 + (-0.2)p_2 + 1.0p_3 \quad (65)$$

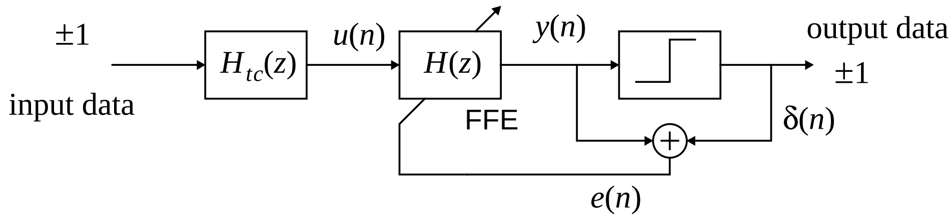
- Solving results in $p_1 = -0.266$, $p_2 = 0.886$, $p_3 = 0.204$
- Now the impulse response through both channel and equalizer is: 0.0, -0.08, 0.0, 1.0, 0.0, 0.05, 0.02, ...



FFE Example

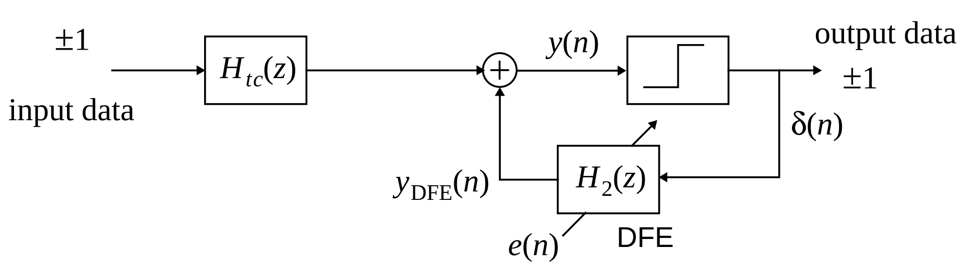
- Although ISI reduced around peak, introduction of slight ISI at other points (better overall)
- Above is a “zero-forcing” equalizer — usually boosts noise too much
- An LMS adaptive equalizer minimizes the mean squared error signal (i.e. find low ISI and low noise)
- In other words, do not boost noise at expense of leaving some residual ISI

Equalization — Decision-Directed



- After training, the channel might change during data transmission so adaptation should be continued.
- The reference signal is equal to the recovered output data.
- As much as 10% of decisions might be in error but correct adaptation will occur

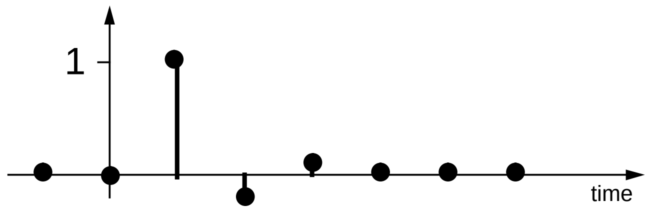
Equalization — Decision-Feedback



- ***Decision-feedback equalizers*** make use of $\delta(n)$ in directly creating $y(n)$.
- They enhance noise less as the ***derived*** input data is used to cancel ISI
- The error signal can be obtained from either a training sequence or decision-directed.

DFE Example

- Assume signals 0 and 1 (rather than -1 and +1) (makes examples easier to explain)
- Suppose channel, $H_{tc}(z)$, has impulse response 0.0, 1.0, -0.2, 0.1, 0.0, 0.0

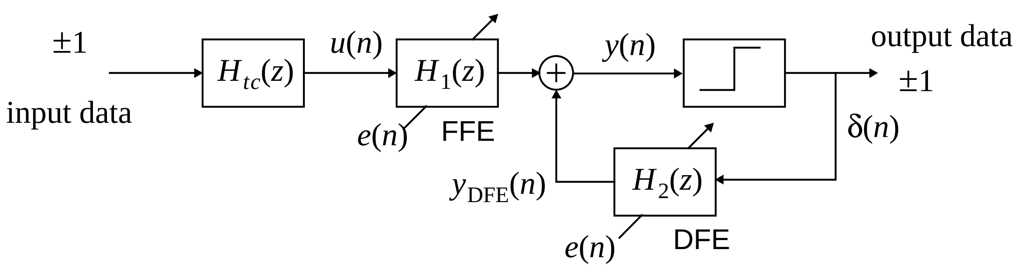


- If DFE is a 2-tap FIR filter with

$$y_{\mathrm{DFE}}(n) = 0.2\delta(n-1) + (-0.1)\delta(n-2) \quad (66)$$

- Input to slicer is now 0.0, 1.0, 0.0, 0.0 0.0 0.0

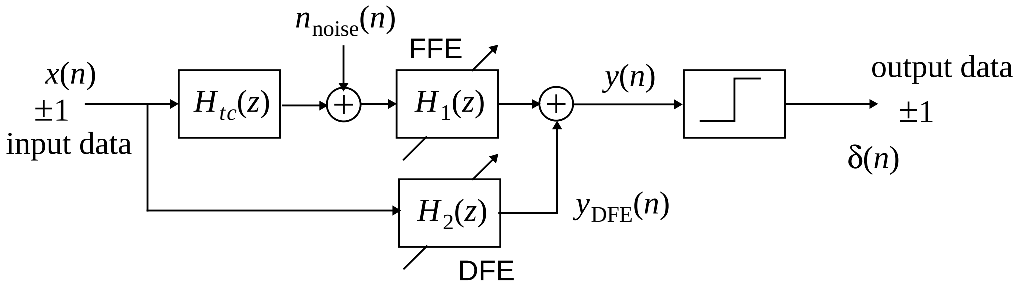
FFE and DFE Combined



- Assuming correct operation, output data = input data
- $e(n)$ same for both FFE and DFE
- $e(n)$ can be either training or decision directed

FFE and DFE Combined

Model as:

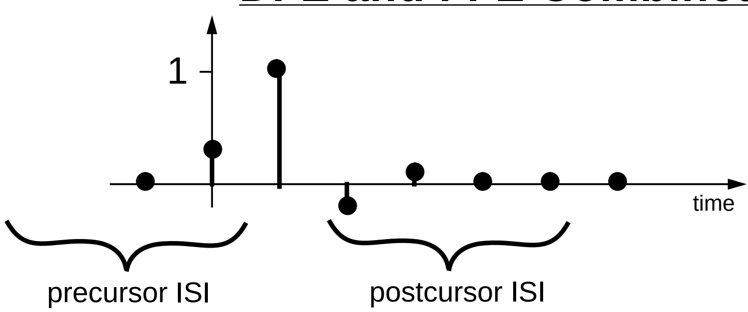


$$\frac{Y}{N} = H_1 \tag{67}$$

$$\frac{Y}{X} = H_{tc}H_1 + H_2 \tag{68}$$

- When H_{tc} small, make $H_2 = 1$ (rather than $H_1 \rightarrow \infty$)

DFE and FFE Combined



- FFE can deal with precursor ISI and postcursor ISI
- DFE can only deal with postcursor ISI
- However, FFE enhances noise while DFE does not

When both adapt

- FFE trys to add little boost by pushing precursor into postcursor ISI (allpass)

Equalization — Decision-Feedback

- The multipliers in the decision feedback equalizer can be simple since received data is small number of levels (i.e. +1, 0, -1) — can use more taps if needed.
- An error in the decision will propagate in the ISI cancellation — error propagation
- More difficult if Viterbi detection used since output not known until about 16 sample periods later (need early estimates).
- Performance surface might be multi-modal with local minimum if changing DFE affects output data

Fractionally-Spaced FFE

- Feed forward filter is often a FFE sampled at 2 or 3 times symbol-rate — fractionally-spaced (i.e. sampled at $T/2$ or at $T/3$)
- Advantages:

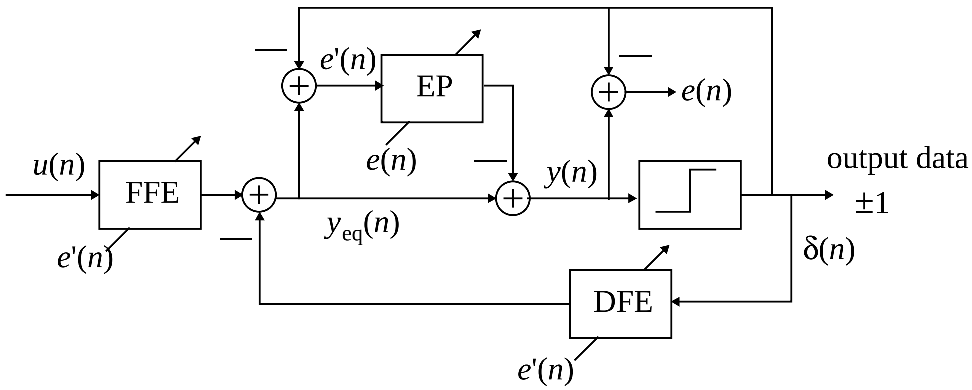
  — Allows the matched filter to be realized digitally and also adapt for channel variations (not possible in symbol-rate sampling)

  — Also allows for simpler timing recovery schemes (FFE can take care of phase recovery)
- Disadvantage

  Costly to implement — full and higher speed multiplies, also higher speed A/D needed.

Equalization (Error Prediction)

- Error prediction filters also can be used to whiten error and thereby improve adaptation.



- Decorrelates the error at slicer due to FFE noise enhancement to make up for channel attenuation.

Wired Digital Communications

Wired Digital Transmission

Long Twisted-Pair Applications (1km - 6km)

- T1/E1 — 1.5/2Mb/s (2km)
- ISDN — Integrated Services Digital Network
- HDSL — High data-rate Digital Subscriber Line
- ADSL — Asymmetric DSL
- VDSL — Very high data-rate DSL

Short Twisted-Pair Applications (20m - 100m)

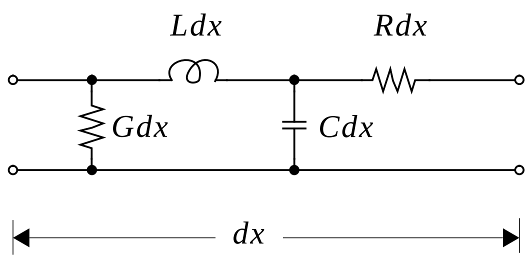
- 100Mb/s Fast-Ethernet — TX, T4, T2
- Gigabit Ethernet — Short haul, Long haul
- ATM — 155Mb/s

Short Coax (300m)

- Digital video delivery — 300Mb/s

Cable Modelling

- Modelled as a transmission line.



Twisted-Pair Typical Parameters:

- $R(f) = (1 + j)\sqrt{f/4}\ \Omega/\mathrm{km}$ due to the skin effect
- $L = 0.6\ \mathrm{mH/km}$ (relatively constant above 100kHz)
- $C = 0.05\ \mu\mathrm{F/km}$ (relatively constant above 100kHz)
- $G = 0$

Skin Effect

- "Resistance" is not constant with frequency and is complex valued.
- Can be modelled as:

$$R(\omega) = k_R(1 + j)\sqrt{\omega} \quad (69)$$

where k_R is a constant given by

$$k_R = \frac{1}{\pi d_c}\left(\frac{\mu}{2\sigma}\right)^{1/2} \quad (70)$$

- d_c is conductor diameter, μ is permeability, σ is conductivity
- Note resistance is inversely proportional to d_c.

• Jordan and Balmain, "Electromagnetic Waves and Radiating Systems", pg. 563, Prentice-Hall, 1968.

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (71)$$

- Making use of (69) and assuming $G = 0$

$$Z_0 = \left(\frac{k_R\sqrt{\omega}(1 + j) + j\omega L}{j\omega C}\right)^{1/2} \quad (72)$$

$$Z_0 = \sqrt{\frac{L}{C}}\left(1 + \frac{k_R}{L\sqrt{\omega}}(1 - j)\right)^{1/2} \quad (73)$$

Now using approx $(1 + x)^{1/2} \approx 1 + x/2$ for $x \ll 1$

$$Z_0 \approx \sqrt{\frac{L}{C}} + \frac{k_R}{2\sqrt{\omega LC}}(1 - j) \quad (74)$$

- At high freq, Z_0 appears as constant value $\sqrt{L/C}$

Characteristic Impedance

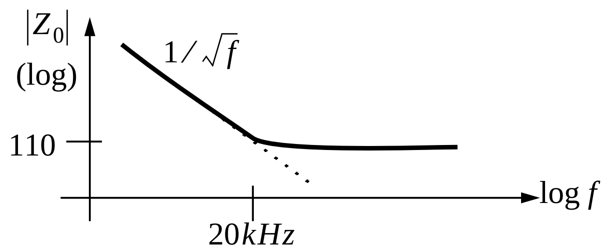
- From (71), when $\omega L \gg R$ (typically $\omega \gg 2\pi \times 16kHz$)

$$Z_{0h} = \sqrt{\frac{L}{C}} \tag{75}$$

resulting in

$$Z_{0h} \approx 110\ \Omega \tag{76}$$

- Thus, when terminating a line, a resistance value around $110\ \Omega$ should be used.



Cable Transfer-Function

- When properly terminated, a cable of length d has a transfer-function of

$$H(d, \omega) = e^{-d\gamma(\omega)} \tag{77}$$

where $\gamma(\omega)$ is given by

$$\gamma(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{78}$$

- Breaking $\gamma(\omega)$ into real and imaginary parts,

$$\gamma(\omega) \equiv \alpha(\omega) + j\beta(\omega) \tag{79}$$

$$H(d, \omega) = e^{-d\alpha(\omega)} e^{-jd\beta(\omega)} \tag{80}$$

- $\alpha(\omega)$ determines ***attenuation***.
- $\beta(\omega)$ determines ***phase***.

Cable Transfer-Function

- Assuming $G = 0$, then from (78)

$$\gamma = (j\omega CR - \omega^2 LC)^{1/2} \tag{81}$$

- Substituting in (69)

$$\gamma = (j\omega^{1.5} k_R C(1+j) - \omega^2 LC)^{1/2} \tag{82}$$

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{k_R}{L\sqrt{\omega}}(1-j)\right)^{1/2} \tag{83}$$

Now using approx $(1+x)^{1/2} \approx 1 + x/2$ for $x \ll 1$

$$\gamma \approx \frac{k_R}{2}\sqrt{\frac{\omega C}{L}} + j\left(\omega\sqrt{LC} + \frac{k_R}{2}\sqrt{\frac{\omega C}{L}}\right) \tag{84}$$

Cable Attenuation

- Equating (79) and (84)

$$\alpha(\omega) \approx \frac{k_R}{2}\sqrt{\frac{C}{L}} \times \sqrt{\omega} \tag{85}$$

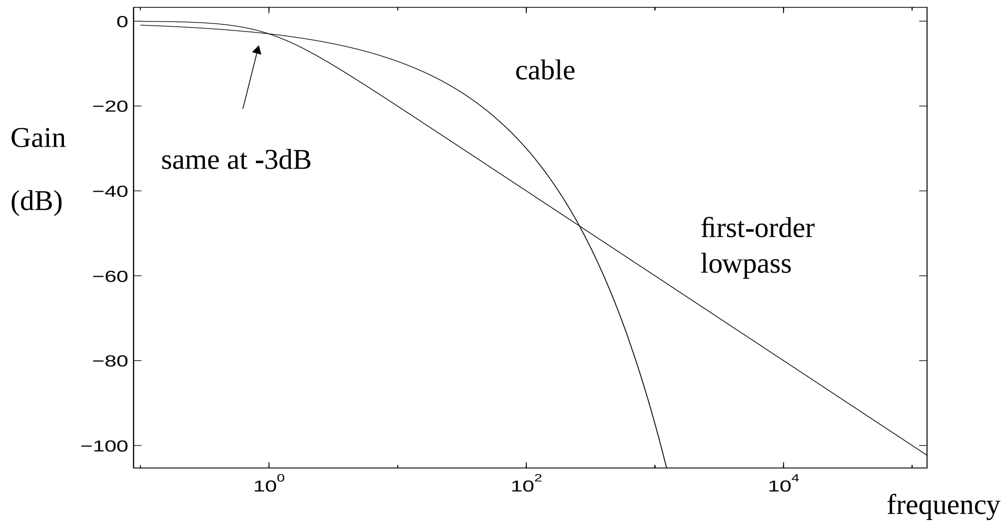
- Therefore gain in dB is

$$\boxed{H_{dB}(d, \omega) \approx -8.68 d \times \frac{k_R}{2}\sqrt{\frac{C}{L}} \times \sqrt{\omega}} \tag{86}$$

- Note that attenuation in dB is proportional to cable length (i.e. 2x distance doubles attenuation in dB)
- Can reduce attenuation by using a larger diameter cable
- Attenuation proportional to root-frequency

Cable Attenuation

- Gain in dB is proportional to $\sqrt{f}$ due to skin effect.



- Do not confuse with 1/f noise slow frequency roll-off.

Cable Phase

- Equating (79) and (84)

$$\beta(\omega) \approx \omega\sqrt{LC} + \frac{k_R}{2}\sqrt{\frac{C}{L}} \times \sqrt{\omega} \qquad (87)$$

- The linear term usually dominates
- The linear term implies a constant group delay.
- In other words, the linear term simply accounts for the delay through the cable.
- Ignoring linear phase portion, remaining phase is proportional to $\sqrt{f}$.
- Note it has the same multiplying term as attenuation.

IIR Filter Cable Match using Matlab

```
% this program calculates an iir num/den transfer-function
% approx for a transmission line with exp(sqrt(s)) type response.
clear;

% Order of IIR filter to match to cable
% nz is numerator order and np is denominator order
nz = 9;
np = 10;

% important parameters of cable
c = 0.05e-6 % capacitance per unit length in farads/km
l = 0.6e-3 % inductance per unit length in henries/km
kr = 0.25 % resistance per unit length in ohms/km (times (1+j)*sqrt(omega))
d = 0.1 % cable length in km
% above values adjusted to obtain -20dB atten for 100m at 125MHz
k_cable = (kr/2)*sqrt(c/l);

% the frequency range for finding tf of cable
fmin=1;
fmax=1e9;
```



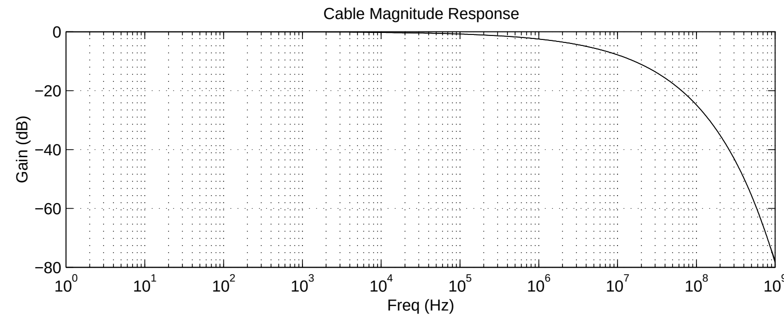
```
% specify frequency points to deal with
nmax=1000;
f=logspace(log10(fmin), log10(fmax), nmax);
w=2*pi*f;
s=j*w;

% 'cable' is desired outcome in exponential form
cable = exp(-d*k_cable*sqrt(2)*sqrt(s));

% Perform IIR approximate transfer-function match
% Since invfreqs miminizes (num-cable*den)
% first need an approximate den so that it can be used
% as a freq weighting to minimize (num/den - cable)
[num,den]=invfreqs(cable,w,nz,np, 1./w);
[denor]=freqs(den,1,w);
% re-iterate process with weighting for the denominator
% which now minimizes (num/den - cable)
[num,den]=invfreqs(cable,w,nz,np, (1./denor).^2);
[denor]=freqs(den,1,w);
[num,den]=invfreqs(cable,w,nz,np, (1./denor).^2);

% find approximate transfer function 'cable_approx' to 'cable'
[cable_approx]=freqs(num,den,w);
```

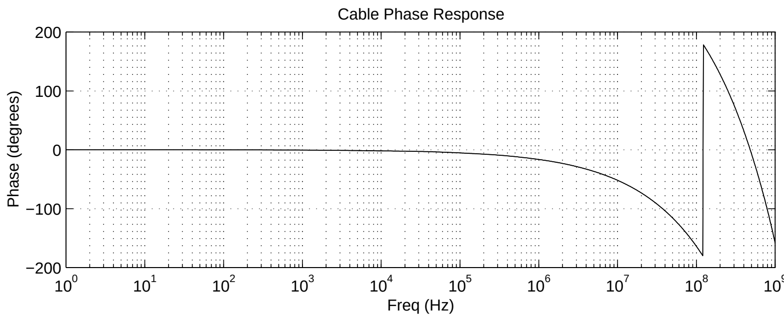
```
% also find pole-zero model
[Z,p,k]=tf2zp(num,den);


% PLOT RESULTS
clf;
figure(1);
subplot(211);
semilogx(f,20*log10(abs(cable)),'r');
hold on;
semilogx(f,20*log10(abs(cable_approx)),'b');
title('Cable Magnitude Response');
xlabel('Freq (Hz)');
ylabel('Gain (dB)');
grid;

hold off;
subplot(212);
semilogx(f,angle(cable)*180/pi,'r');
hold on;
semilogx(f,angle(cable_approx)*180/pi,'b');
title('Cable Phase Response');
xlabel('Freq (Hz)');
```



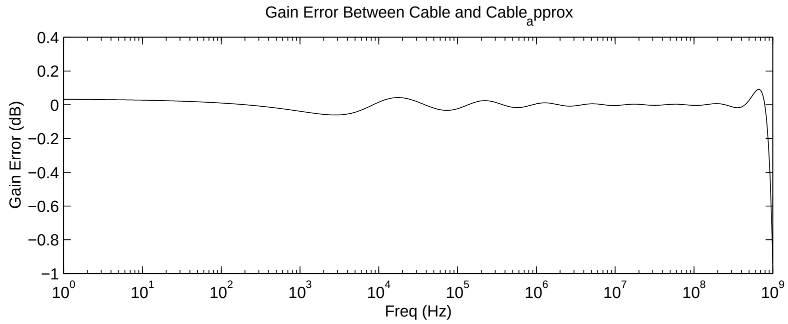
```
ylabel('Phase (degrees)');
grid;

hold off;
figure(2);
subplot(211);
semilogx(f,20*log10(abs(cable)./abs(cable_approx)));
title('Gain Error Between Cable and Cable_approx');
xlabel('Freq (Hz)');
ylabel('Gain Error (dB)');
subplot(212);
semilogx(f,(angle(cable)-angle(cable_approx))*180/pi);
title('Phase Error Between Cable and Cable_approx');
xlabel('Freq (Hz)');
ylabel('Phase Error (degrees)');
grid;
```

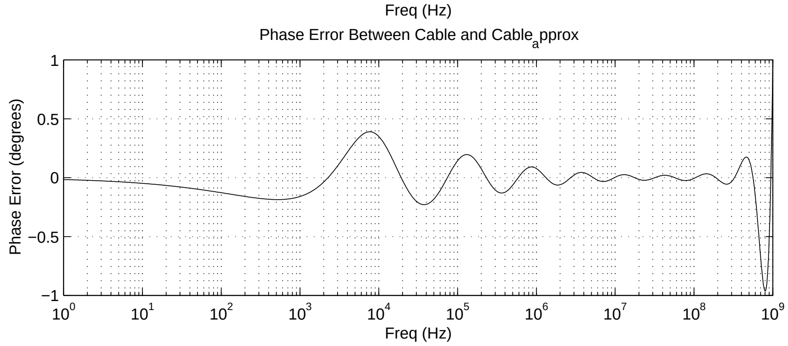
Cable Response



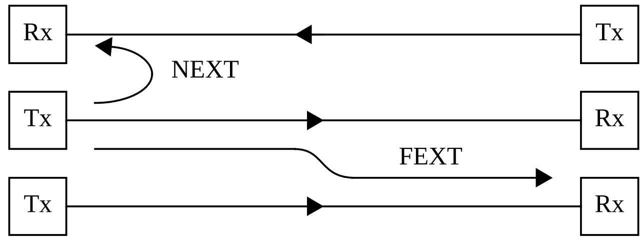


IIR Matching Results





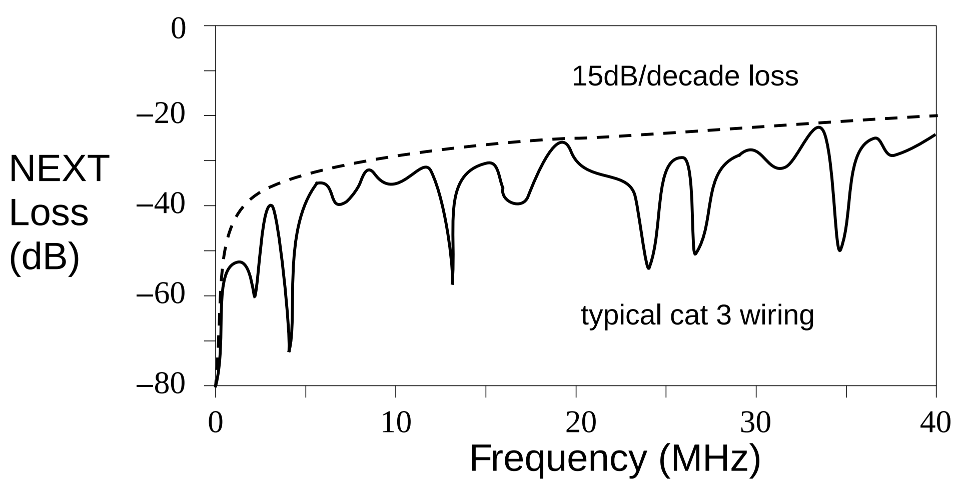
Near and Far End Crosstalk



- In FEXT, interferer and signal both attenuated by cable
- In NEXT, signal attenuated but interferer is coupled directly in.
- When present, NEXT almost always dominates.
- Can cancel NEXT if nearby interferer is known.
- Envelope of squared gain of NEXT increases with $f^{1.5}$

Twisted-Pair Crosstalk

- Crosstalk depends on turns/unit length, insulator, etc.
- Twisted-pairs should have different turns/unit length within same bundle



Transformer Coupling

- Almost all long wired channels (>10m) are AC coupled systems
- AC coupling introduces ***baseline wander*** if random PAM sent
- A long string of like symbols (for example, +1) will decay towards zero degrading performance
- Requires baseline wander correction (non-trival)
- Can use passband modulation schemes (CAP, QAM, DMT)
- ***Why AC couple long wired channels??***

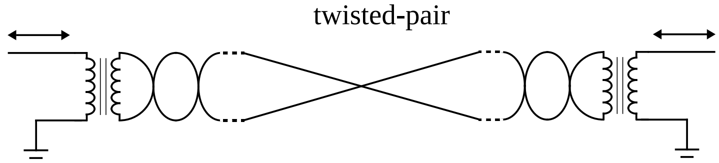
Transformer Coupling

Eliminates need for similar grounds

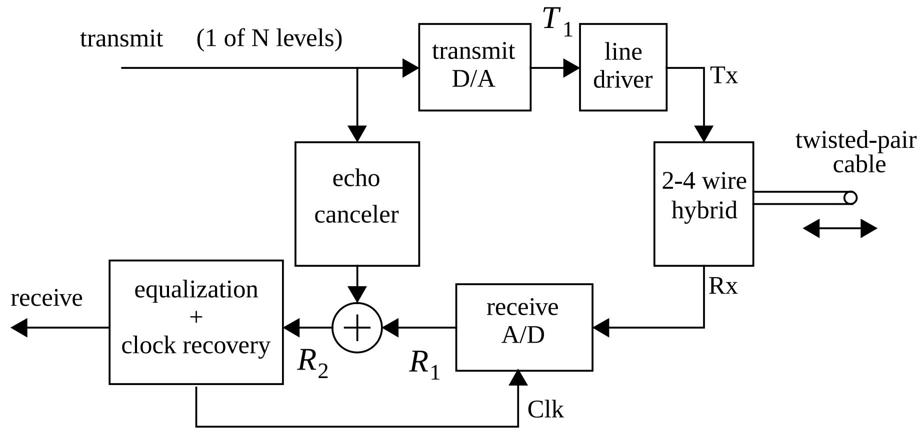
- If ground potentials not same — large ground currents

Rejects common-mode signals

- Transformer output only responds to differential signal current
- Insensitive to common-mode signal on both wires



Generic Wired PAM Transceiver



- Look at approaches for each block

HDSL Application

- 1.544Mb/s over 4.0km of existing telephone cables.
- Presently 4-level PAM code (2B1Q) over 2 pairs (a CAP implementation also exists).
- Symbol-rate is 386 ksymbols/s

Possible Bridged-Taps

- Can have unterminated taps on line
- Modelling becomes more complicated but DFE equalizes effectively
- Also causes a wide variation in input line impedance to which echo canceller must adapt — difficult to get much analog echo cancellation

HDSL Application

- Symbol-rate is 386 ksymbols/s

Received Signal

- For $d = 4km$, a 200kHz signal is attenuated by $40dB$.
- Thus, high-freq portion of a 5Vpp signal is received as a 50mVpp signal — ***Need effective echo cancellation***

Transmit Path

- Due to large load variations, echo cancellation of analog hybrid is only 6dB
- To maintain 40dB SNR receive signal, linearity and noise of transmit path should be better than 74dB.

ISDN Application

- Similar difficulty to HDSL but lower frequency
- 160kb/s over 6km of 1 pair existing telephone cables
- 4-level PAM coding — 2B1Q
- Receive signal at $40\mathrm{kHz}$ atten by $40\mathrm{dB}$
- Requires highly linear line-drivers + A/D converters for echo cancellation (similar to HDSL)

Fast-Ethernet Application

CAT3	CAT5
$H_{dB}(f) = 2.32\sqrt{f} + 0.238f$	$H_{dB}(f) = 1.967\sqrt{f} + 0.023f + 0.05/\sqrt{f}$
$12.5MHz \leftrightarrow 11dB$	$12.5MHz \leftrightarrow 7dB$
crosstalk worse	crosstalk better

100Base-T4

- 4 pair CAT3 — 3 pair each way, 25MS/s with coding

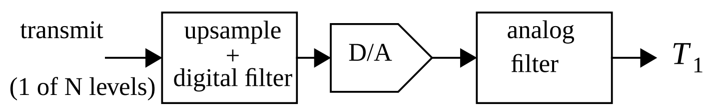
100Base-TX

- 2 pair CAT5 — 3 level PAM to reduce radiation

100Base-T2

- 2 pair CAT3 — 5x5 code, 25MS/s on each pair

Typical Transmit D/A Bloc k



- Polyphase filter to perform upsampling+filtering

HDSL

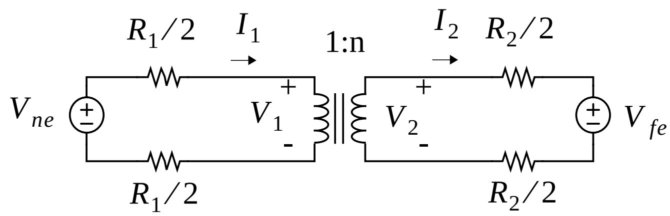
- D/A and filter needs better than 12-bit linearity
- Might be an oversampled 1-bit DAC
- One example: $\uparrow 16$; 48 tap FIR; $\uparrow 4$; $\Delta\Sigma$ DAC

Fast-Ethernet

- Typically around 35 dB linearity + noise requirement
- 100Base-T2 example: $\uparrow 3$; simple FIR; 75MHz 4-bit DAC; 3’rd-order LP cont-time filter

Line Drivers

- Line driver supplies drive current to cable.
- Commonly realized as voltage buffers.
- Often the most challenging part of analog design.
- Turns ratio of transformer determines equivalent line impedance.



$$V_{ne} = \frac{2}{n}V_2$$

$$V_1 = V_2/n$$
$$I_1 = nI_2$$
$$R_1 = R_2/n^2$$

Typical Values

$R_2 = 100\Omega$

$V_2 = \pm 2.5\text{V}$

$I_2 = \pm 25\text{mA}$

Line Driver Efficiency

- Efficiency improves as power supply increased

Example (assume can drive within 1V of supplies)

- From typical values, max power delivered by line driver is $P_{\text{line+R}} = 2 \times 2.5 \times 25\text{mA} = 125\text{mW}$

12V Case

- Consider 12V supply — use $n = 0.5$, $V_{ne,\max} = 10V$, $I_{1,\max} = 12.5\text{mA}$ leading to $P = 12 \times 12.5\text{mA} = 150\text{mW}$ (and drive an 800 ohm load)

3V Case

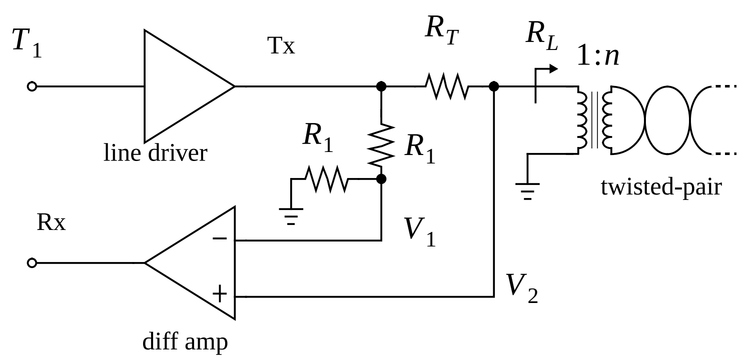
- Consider 3V supply — use $n = 5$, $V_{ne,\max} = 1V$, $I_{1,\max} = 125\text{mA}$ leading to $P = 3 \times 125\text{mA} = 375\text{mW}$ (and drive an 8 ohm load!!!)

Line Driver

- In CMOS, W/L of output stage might have transistors on the order of 10,000!
- Large sizes needed to ensure some gain in final stage so that feedback can improve linearity — might be driving a 30 ohm load
- When designing, ensure that enough phase margin is used for the wide variation of bias currents
- Nested Miller compensation has been successfully used in HDSL application with class AB output stage
- Design difficulties will increase as power supplies decreased

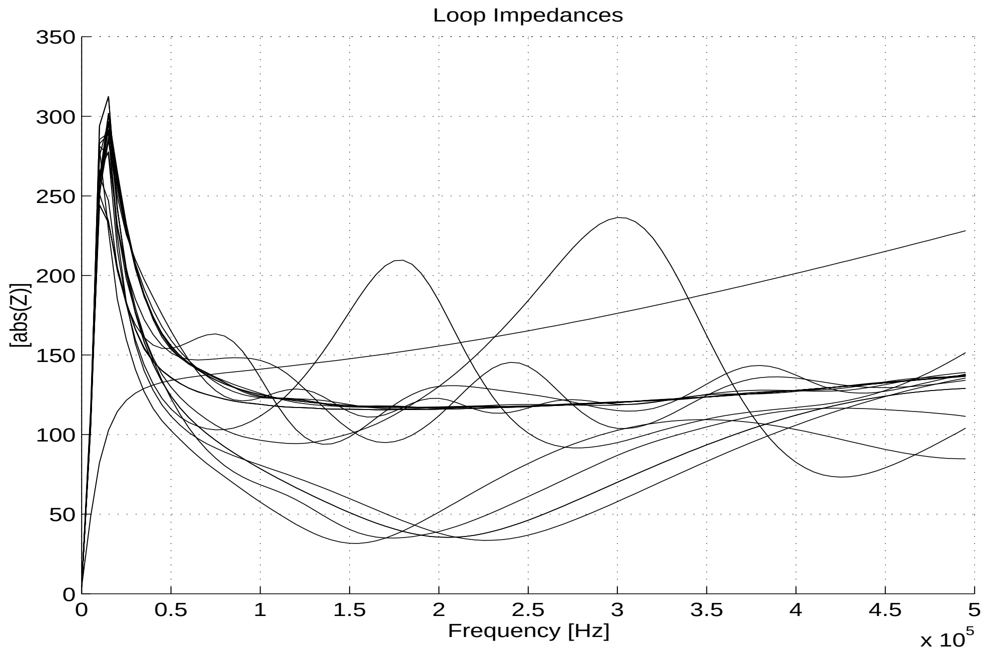
2-4 Wire Hybrids

- Dual-duplex often used to reduce emission.
- However, dual-duplex requires hybrids and echo cancellation.



- If $R_L = R_T$, no echo through hybrid
- Can be large impedance variation.

Typical HDSL Line Impedances



Hybrid Issues

- Note zero at dc and pole at 10kHz.
- Low frequency pole causes long echo tail (HDSL requires 120 tap FIR filter)

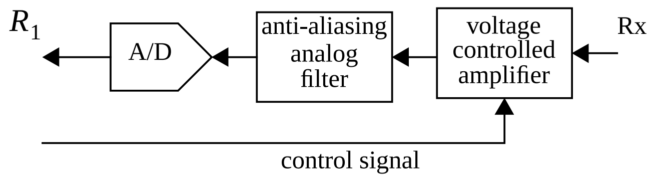
Alternatives

- Could eliminate R_1 circuit and rely on digital echo cancellation but more bits in A/D required.

OR

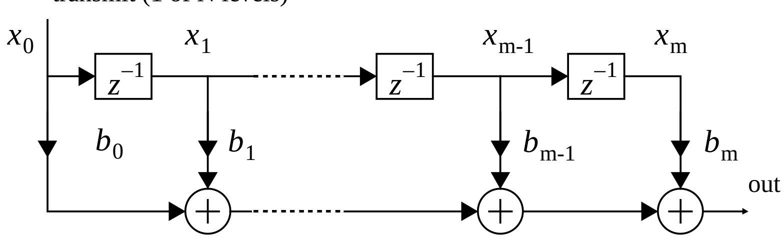
- Can make R_1 circuit more complex to ease A/D specs.
- Less echo return eases transmit linearity spec.
- Might be a trend towards active hybrids with or without extra A/D and D/A converters (particularly for higher speeds).

Typical Receive A/D



- Often, VGA is controlled from digital signal.
- Anti-aliasing can be simple in oversampled systems.
- Continuous-time filters are likely for fast-ethernet
- Example: 100Base-T2 suggests a 5'th order cont-time filter at 20MHz with a 6-bit A/D at 75MHz.
- Challenge here is to keep size and power of A/D small.

Echo Cancellation



$$b_i(k+1) = b_i(k) + \mu e(k) x_i(k) \quad \text{LMS}$$

- Typically realized as an adaptive FIR filter.
- Note input is transmit signal so delay lines and multiplies are trivial.
- HDSL uses about a 120 tap FIR filter
- Coefficient accuracy might be around 20 bits for dynamic range of 13 bits.

Echo Cancellation

- Fast-ethernet might be around 30 taps and smaller coefficient accuracy
- Can also perform some NEXT cancellation if signal of nearby transmitter is available (likely in 100Base-T2 and gigabit ethernet)

Alternatives

- Higher data rates may have longer echo tails.
- Might go to FIR/IIR hybrid to reduce complexity.
- Non-linear echo cancellation would be VERY useful in reducing transmit linearity spec.
- However, these non-linearities have memory and thus Volterra series expansions needed.

Equalization

HDSL

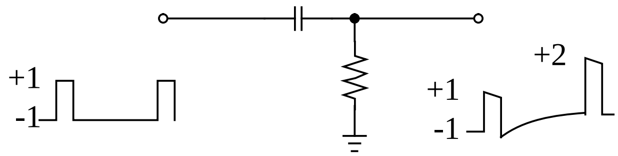
- Echo canceller required ***before*** equalization so fractional spaced equalizer not practical
- Typically 9 tap FFE and 120 tap DFE
- Long DFE also performs dc recovery (baseline wander)

Fast Ethernet

- Often fractional-spaced EQ - 30 taps
- DFE — 20 taps (dc recovery)

dc Recovery (Baseline Wander)

- Wired channels often ac coupled
- Reduces dynamic range of front-end circuitry and also requires some correction if not accounted for in transmission line-code



- Front end may have to be able to accomodate twice the input range!
- DFE can restore baseline wander - lower frequency pole implies longer DFE
- Can use line codes with no dc content

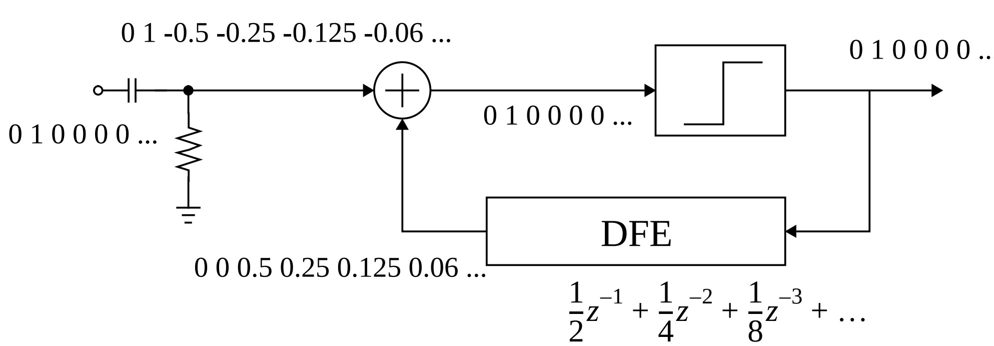
Baseline Wander Correction #1

DFE Based

- Treat baseline wander as postcursor interference
- May require a long DFE

$$\frac{z-1}{z-0.5} = 1 - \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3} - \dots$$

IMPULSE INPUT

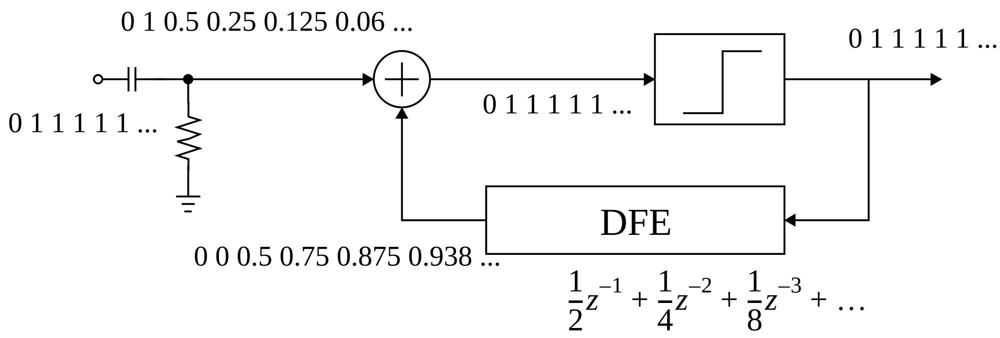


Baseline Wander Correction #1

DFE Based

$$\frac{z-1}{z-0.5} = 1 - \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3} - \dots$$

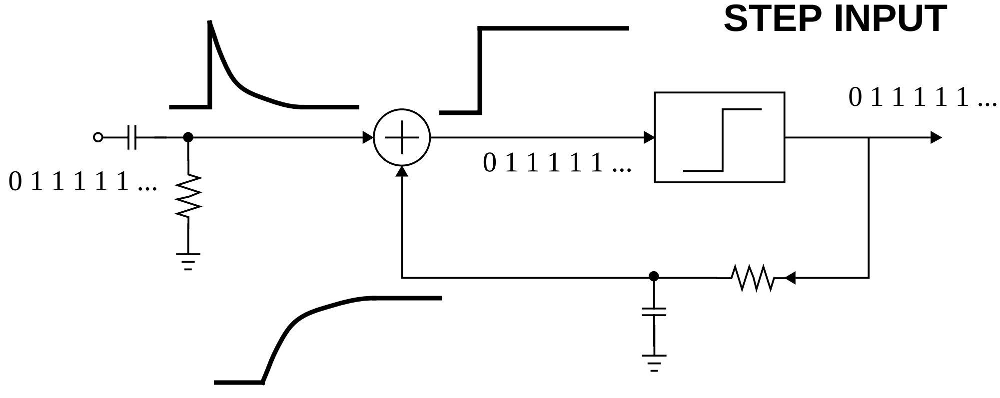
STEP INPUT



Baseline Wander Correction #2

Analog dc restore

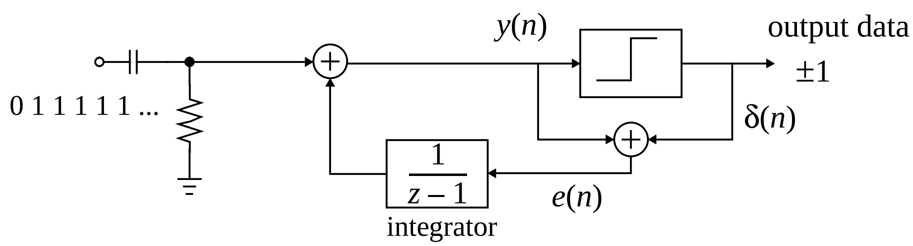
STEP INPUT



- Equivalent to an analog DFE
- Needs to match RC time constants

Baseline Wander Correction #3

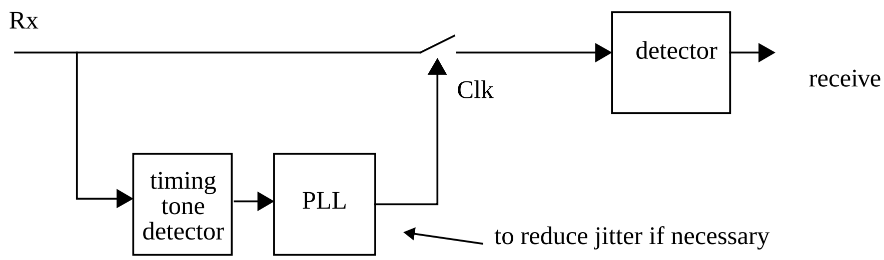
Error Feedback



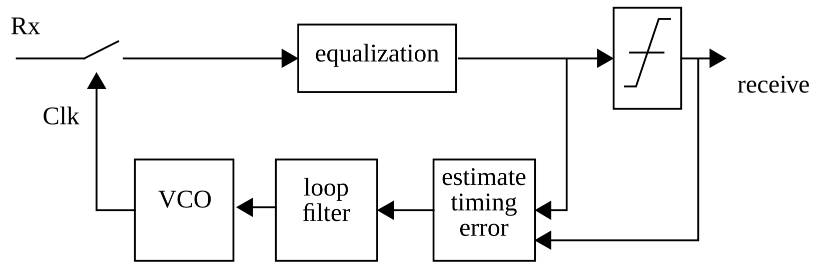
- Integrator time-constant should be faster than ac coupling time-constant
- Effectively forces error to zero with feedback
- May be difficult to stablilize if too much in loop (i.e. AGC, A/D, FFE, etc)

Timing Recovery

Timing Recovery (two types)



deductive



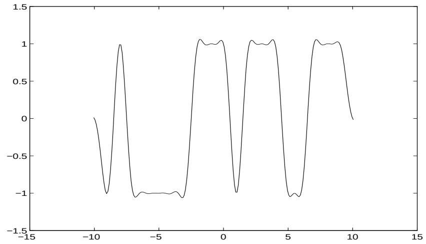
inductive

- Timing more difficult with less excess bandwidth.

Deductive Timing Recovery

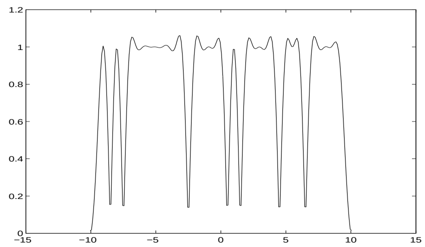
- Non-linear spectral line method most popular (linear spectral line method used if f_s tone present).
- Apply a non-linearity to receive signal and bandpass filter to recover f_s tone (usually with PLL).
- Works because receive signal is cyclostationary (i.e. its moments vary in time and are periodic).
- Common non-linearities used are squaring and absolute circuit (rectifier) (for low excess BW)
- ***Ensemble average*** of non-linear circuit output is periodic in T
- Thus, a f_s component exists (scrambled data)

Example (100% excess BW)

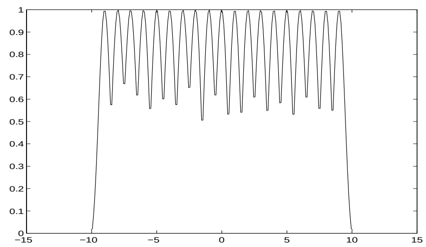


receive signal

average NOT in time but
over transmit sequences
(100 sequences in this case)

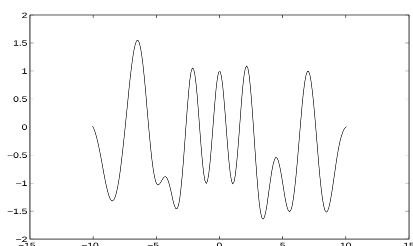


abs(receive signal)



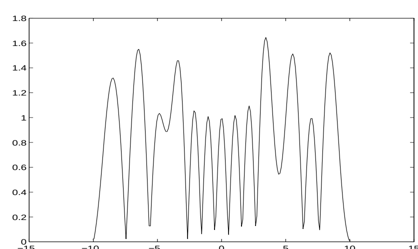
average(abs(receive signal))

Example (20% excess BW)

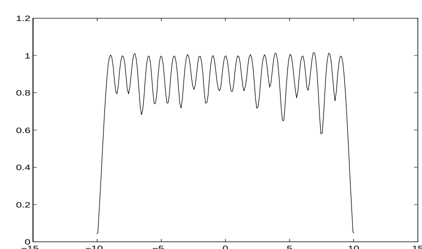


receive signal

average NOT in time but
over transmit sequences
(100 sequences in this case)



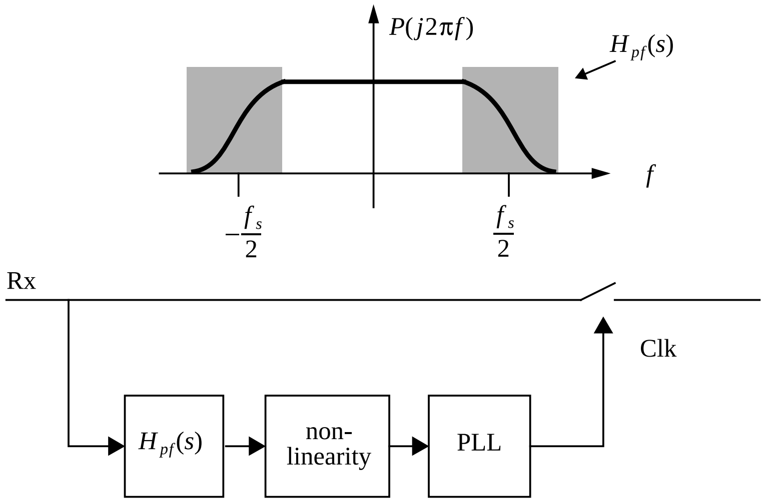
abs(receive signal)



average(abs(receive signal))

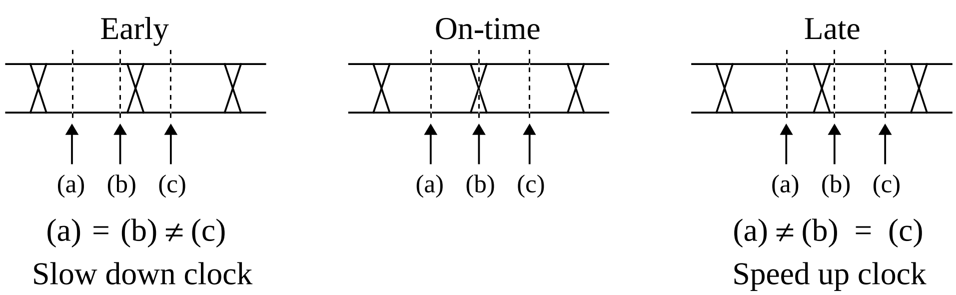
Deductive Timing

- Can pre-filter receive signal to only non-flat portion to reduce jitter — eliminate portion that does not contribute to timing tone.



Inductive Timing — Early Late

- Can sample at 2X and determine if clock is early or late when a transition occurs.

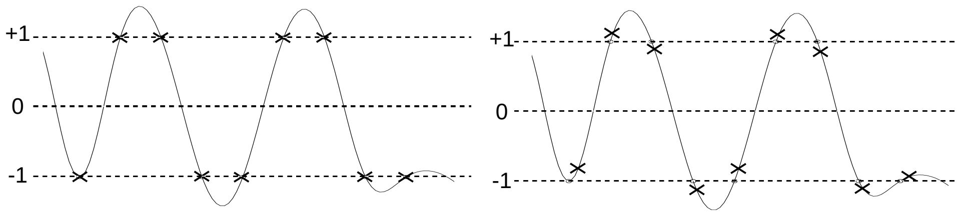


- If (a) = (b) = (c), do nothing
- However, (b) sample does not indicate how far away from zero crossing — can add dither to (b) to aid estimate.

Inductive Timing (MMSE)

- Commonly realized as minimum mean-square error (i.e. MMSE timing)
- Also called LMS timing.
- Assume sample times are $kT + \tau_k$

$$A_k = \pm 1$$



Correct sampling phase

Late sampling phase $\tau_k > 0$

Inductive Timing (MMSE)

- MMSE adjusts τ_k to minimize

$$E[E_k^2(\tau_k)] = E[(Q_k(\tau_k) - A_k)^2] \tag{88}$$

where $E[\bullet]$ denotes expectation, $Q_k(\tau_k)$ is the sampled signal (it is a function of τ_k) and A_k is the ideal symbol.

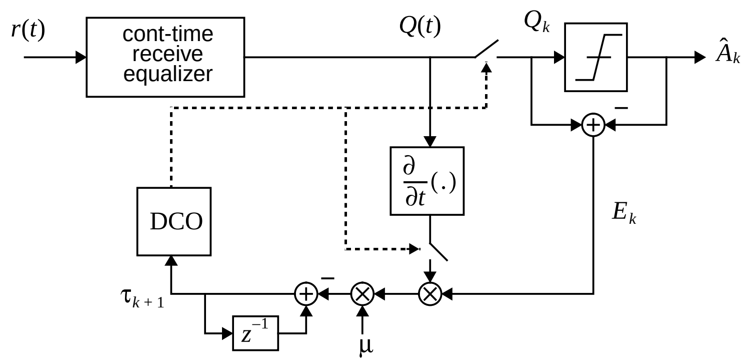
- Stochastic gradient (as in LMS algorithm) leads to

$$\tau_{k+1} = \tau_k - \mu\left(E_k(\tau_k) \times \frac{\partial Q_k(\tau_k)}{\partial \tau_k}\right) \tag{89}$$

Inductive Timing (MMSE)

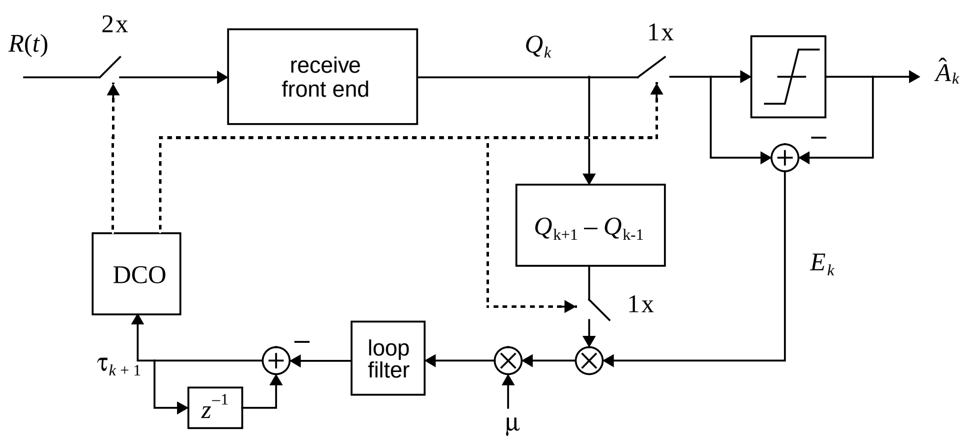
- Can replace derivative wrt τ_k by derivative wrt time since sampled at $t = kT + \tau_k$

$$\frac{\partial Q_k(\tau_k)}{\partial \tau_k} = \left.\frac{\partial Q(t)}{\partial t}\right|_{t = kT + \tau_k} \qquad (90)$$



Inductive Timing (MMSE)

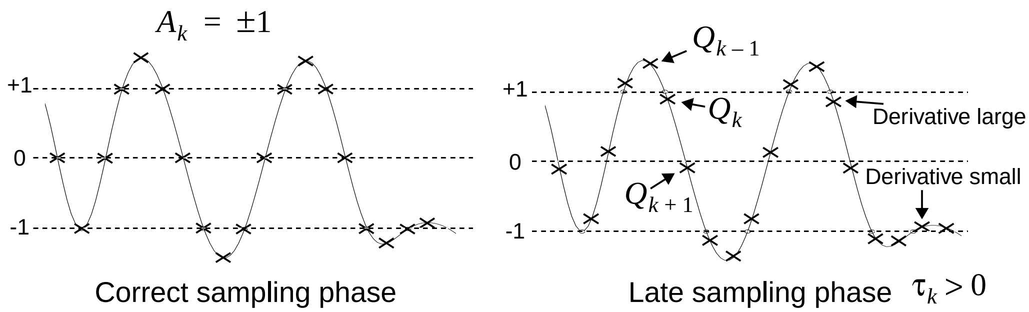
- Can sample at 2X symbol-rate and perform derivative in discrete-time.



2X Timing Example

- Sample at twice symbol-rate

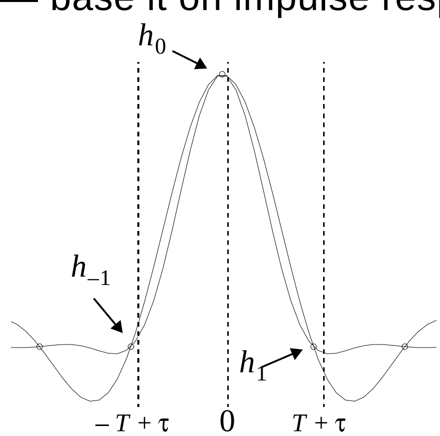
$$\tau_{k+1} = \tau_k - \mu(Q_k - \hat{A}_k) \times (Q_{k+1} - Q_{k-1}) \qquad (91)$$



- At Q_k, slope is neg, E_k is neg, so τ_k is decreased.
- Use absolute values then 50% duty cycle not needed

Inductive Timing — Baud-Rate

- If all sampling done at symbol-rate, MMSE timing can still be used — base it on impulse response.



- Early-late — adjust so $h_1 - h_{-1} = 0$
- Zero-crossing — adjust so $h_1 = 0$

Inductive Timing — Baud-Rate

- To obtain impulse response estimates, cross correlate received signals with received symbols.
- Recall

$$Q(t) = \sum_m A_m h(t - mT) + n(t) \qquad (92)$$

- Sampled at time $kT + \tau$, we have

$$Q_k \equiv Q(kT + \tau)$$

$$= \ldots + A_{k-1} h(kT + \tau - (k-1)T) + A_k h(kT + \tau - kT) + \ldots$$

$$= \ldots + A_{k-1} h_1(\tau) + A_k h_0(\tau) + A_{k+1} h_{-1}(\tau) + \ldots \qquad (93)$$

where $h_k(\tau) \equiv h(kT + \tau)$

Inductive Timing — Baud-Rate

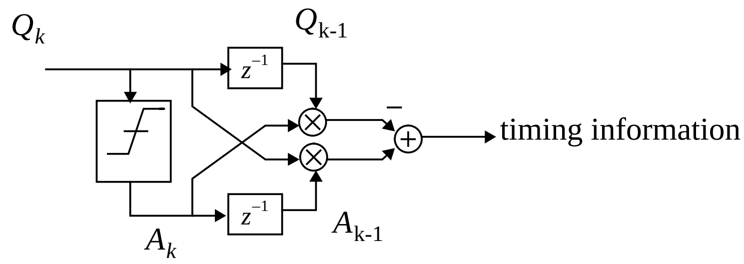
- To estimate $h_1(\tau)$, use $Q_k \times A_{k-1}$
- ***All other terms go to zero since*** A_{k-1} ***is uncorrelated with*** A_j ***when*** $k \neq j$
- To estimate $h_{-1}(\tau)$, we need to use a delayed version of Q_k

$$Q_{k-1} = \ldots + A_{k-1} h_0(\tau) + A_k h_{-1}(\tau) + A_{k+1} h_{-2}(\tau) + \ldots \qquad (94)$$

- To estimate $h_{-1}(\tau)$, use $Q_{k-1} \times A_k$

Inductive Timing — Baud-Rate

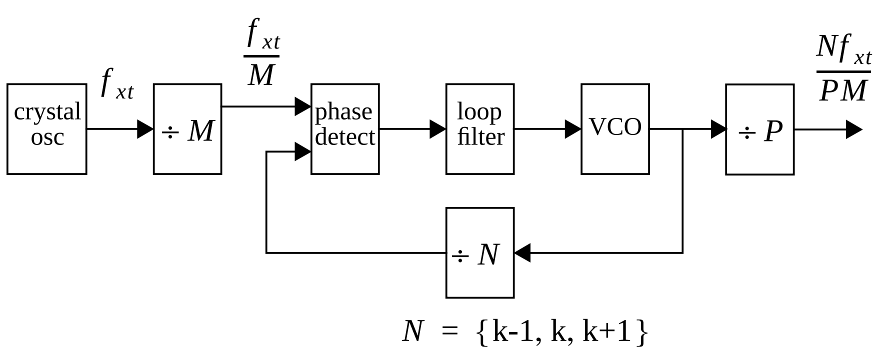
- To build early-late scheme,



- Early-late is insensitive to amplitude distortion.
- Zero-crossing is better where phase distortion dominates
- h_0 factor should be known otherwise adaptation gain will vary (can divide it out in algorithm).

A Fractional-N Frequency Synthesiz er

- Often need a low jitter clock that can have arbitrary frequency.
- A voltage-controlled crystal oscillator is expensive.
- Use oversampling within a PLL



A digital controlled oscillator

Elastic Buffer

- Used to deal with low frequency input clock jitter
- Allows attenuation of clock jitter to next stage

Example

- Input clock rate — 1MHz but varies from 0.9MHz to 1.1MHz in sinusoidal fashion at 1kHz
- Output clock rate — fixed at 1MHz
- Input clock high — 16 extra bits stored in buffer
- Input clock low — 16 bits removed from buffer

- Keep elastic buffer half-full on-average through feedback

Other Modulation Schemes

QAM, CAP and DMT

Complex Signals

- Concept useful for describing a pair of real signals
- Let $j = \sqrt{-1}$

Two Important Properties of Real Signals

- Amplitude is symmetric ($|A(j\omega)| = |A(-j\omega)|$)
- Phase is anti-symmetric ($\angle A(j\omega) = -1 \times \angle A(-j\omega)$)

Two Important Complex Relationships

- Continuous-time

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \tag{95}$$

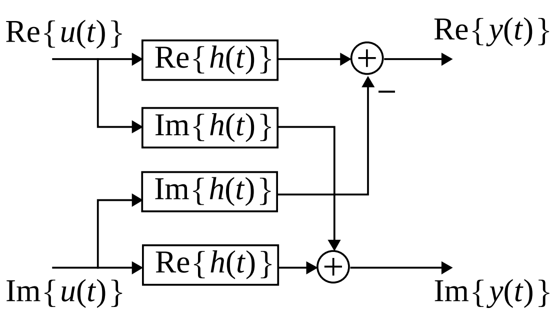
- Discrete-time

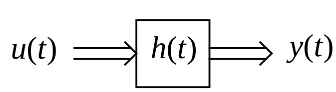
$$e^{j\omega nT} = \cos(\omega nT) + j\sin(\omega nT) \tag{96}$$

Complex Transfer Function

- Let $h(t)$ be a complex impulse response

$$h(t) = \mathrm{Re}\{h(t)\} + j\mathrm{Im}\{h(t)\} \tag{97}$$





- 4 systems needed if both $h(t)$ and $u(t)$ complex
- 1 system needed if both $h(t)$ and $u(t)$ real
- 2 systems needed if one is complex and other real

Hilbert Transform

- Often need a complex signal with all negative frequency components zero — use Hilbert transform
- Hilbert transform is a ***real*** filter with response

$$h_{bt}(t) = \frac{1}{\pi t} \tag{98}$$

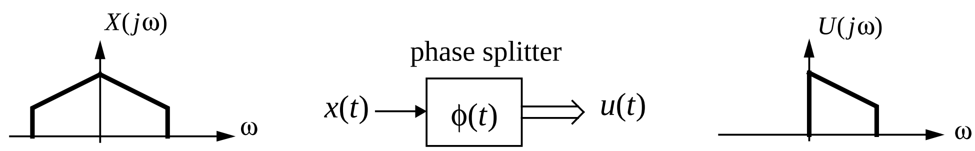
$$H_{bt}(j\omega) = -j\,\mathrm{sgn}(\omega) \tag{99}$$

- The Hilbert transform of a signal $x(t)$ is denoted as $\hat{x}(t)$ and can be found using filter in (99)

$$\hat{X}(j\omega) = -j\,\mathrm{sgn}(\omega)X(j\omega) \tag{100}$$

- Shift phase of signal by -90 degrees at all frequencies — allpass filter with phase shift
- Recall $j = e^{-j(\pi/2)}$

Phase Splitter



- A complex system, $\phi(t)$, that removes negative frequency components referred to as a ***phase splitter***.

$$\Phi(j\omega) = \begin{cases} 1, & \omega \ge 0 \\ 0, & \omega < 0 \end{cases} \tag{101}$$

- A phase splitter is built using a Hilbert transform (hence the name phase splitter)

Phase Splitter

- To form a signal, $u(t)$, having only positive freq components from real signal, $x(t)$

$$u(t) = 0.5(x(t) + j\hat{x}(t)) \tag{102}$$

- $u(t)$ is two real signals where we think of signals as

$$x(t) = \mathrm{Re}\{2u(t)\} \tag{103}$$

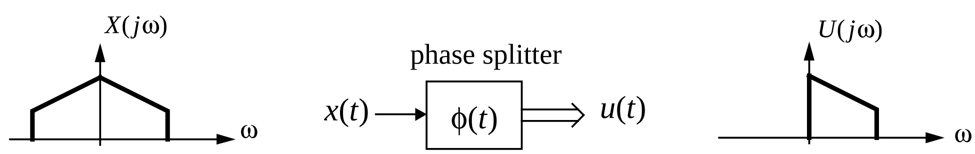
$$\hat{x}(t) = \mathrm{Im}\{2u(t)\} \tag{104}$$

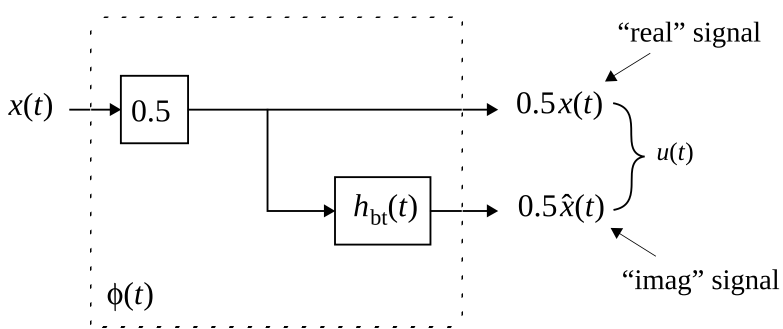
- To see that only positive frequency components remain — use (100) and (102)

$$U(j\omega) = 0.5(X(j\omega) + j \times (-j\,\mathrm{sgn}(\omega) X(j\omega))) \tag{105}$$

$$U(j\omega) = 0.5(X(j\omega) + \mathrm{sgn}(\omega) X(j\omega)) \tag{106}$$

Phase Splitter

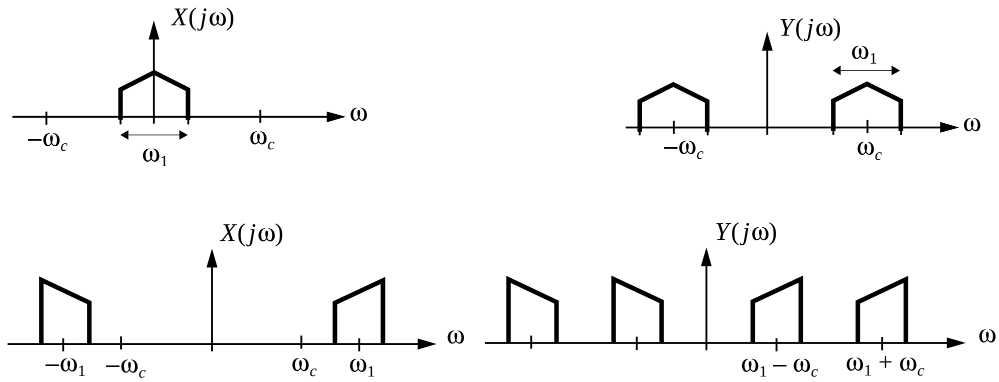




Real-Valued Modulation

$$y(t) = x(t)\cos(\omega_c t) \qquad (107)$$

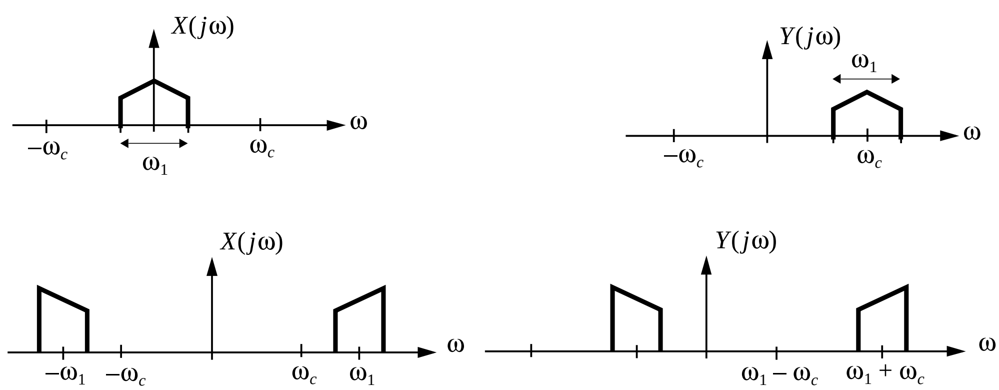
- Multiplication by $\cos(\omega_c t)$ results in convolution of frequency spectrum with two impulses at $+\omega_c$ and $-\omega_c$



Complex Modulation

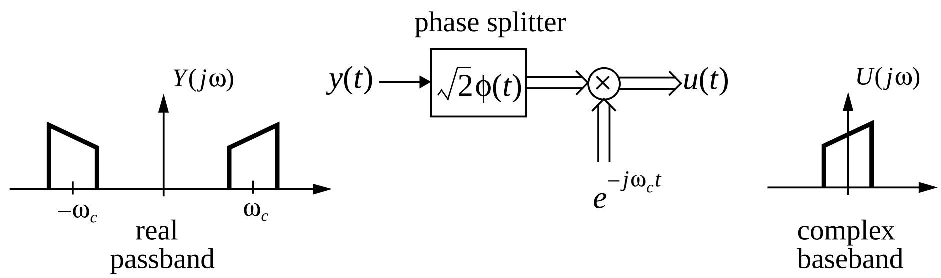
$$y(t) = e^{j\omega_c t} x(t) \qquad (108)$$

- Mult a signal by $e^{j\omega_c t}$ shifts spectrum by $+\omega_c$



Passband and Comple x Baseband Signals

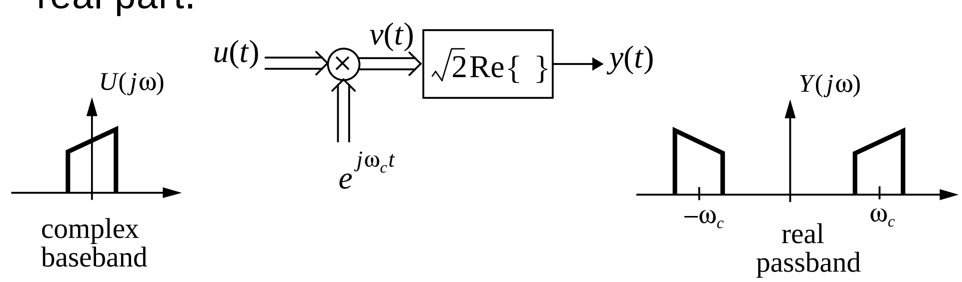
- Can represent a passband signal as a complex baseband signal.
- Need complex because passband signal may not be symmetric around ω_c



- $\sqrt{2}$ factor needed to keep the same signal power.

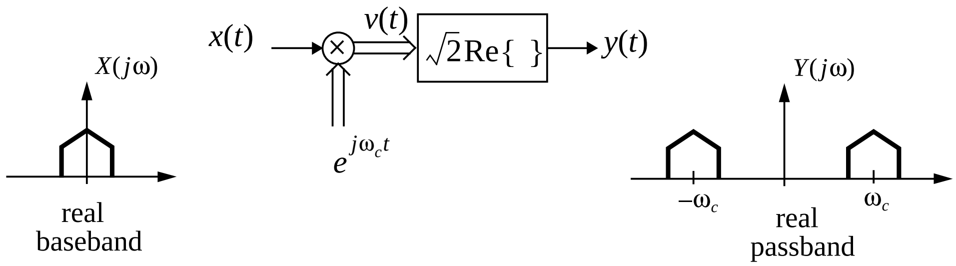
Modulation of Complex Baseband

- It is only possible to send ***real*** signals along channel
- Can obtain passband modulation from a complex baseband signal by complex modulation then taking real part.



- Works because $v(t)$ has only positive freq. therefore its imag part is its Hilbert transform and taking real part restores negative frequencies.

Double Sideband

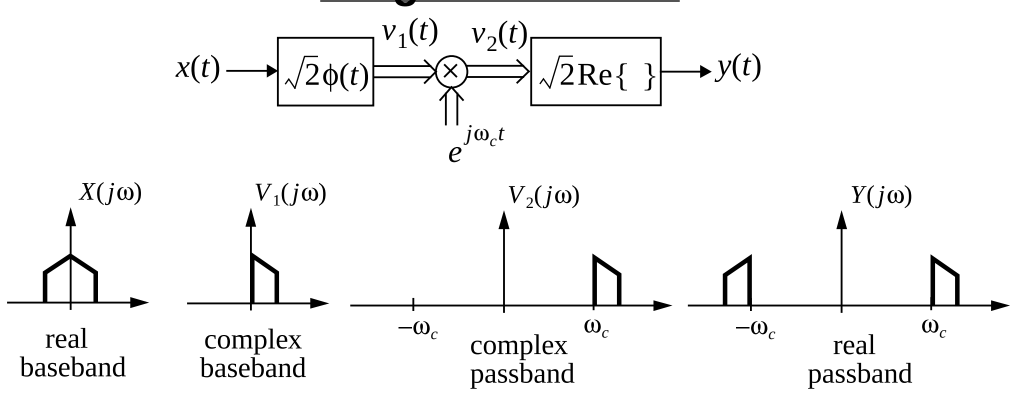


$$v(t) = x(t) \times (\cos(\omega_c t) + j\sin(\omega_c t)) \tag{109}$$

$$y(t) = \sqrt{2}x(t)\cos(\omega_c t) \tag{110}$$

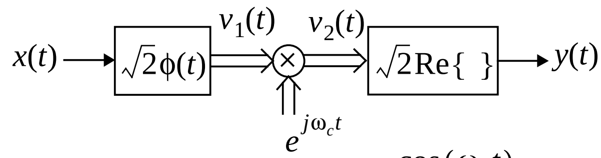
- $x(t)$ is a real signal so positive and negative frequencies symmetric
- Modulated signal, $y(t)$, has symmetry above and below carrier freq, ω_c — using twice minimum bandwidth necessary to send baseband signal.

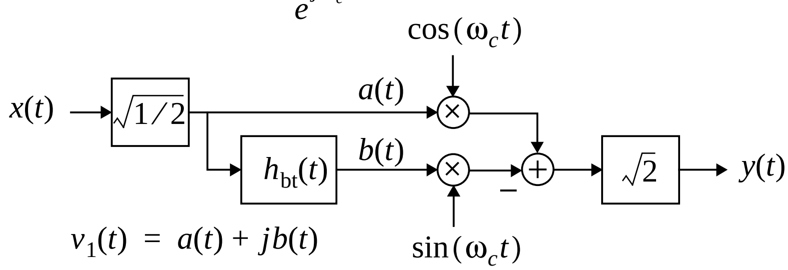
Single Sideband



- Twice as efficient as double sideband
- Disadvantage — requires a phase-splitter good to near dc (difficult since a phase discontinuity at dc)

Single Sideband





- If $v_1(t) = a(t) + jb(t)$, then $y(t) = \mathrm{Re}\{e^{j\omega_c t} v_1(t)\}$ becomes

$$y(t) = \sqrt{2}\mathrm{Re}\{(\cos(\omega_c t) + j\sin(j\omega_c t)) \times (a(t) + jb(t))\} \quad (111)$$

$$y(t) = \sqrt{2}a(t)\cos(\omega_c t) - \sqrt{2}b(t)\sin(\omega_c t) \quad (112)$$

Quadrature Amplitude Modulation (QAM)

- Start with two independent real signals

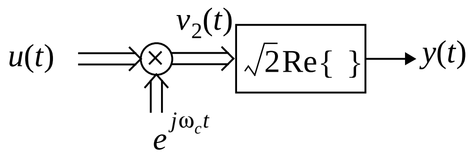
$$u(t) = a(t) + jb(t) \quad (113)$$

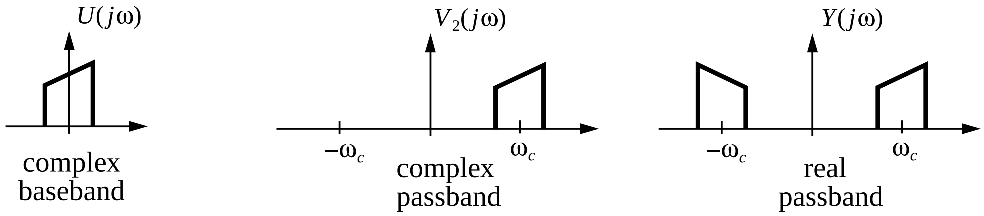
- In general, they will form a complex baseband signal
- Modulate as in single-sideband case

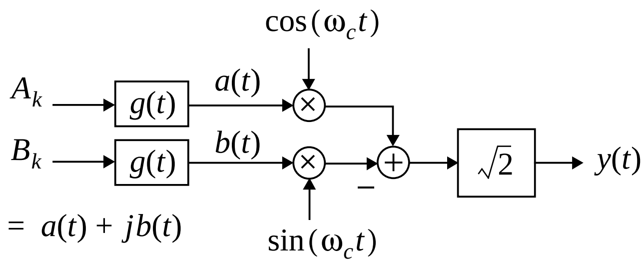
$$y(t) = \sqrt{2}a(t)\cos(\omega_c t) - \sqrt{2}b(t)\sin(\omega_c t) \quad (114)$$

- For data communications let $a(t)$ and $b(t)$ be the output of two pulse shaping filters with multilevel inputs, A_k and B_k
- While QAM and single sideband have same spectrum efficiency, QAM does not need a phase splitter
- Typically, spectrum is symmetrical around carrier but information is twice that of double-side band.

QAM



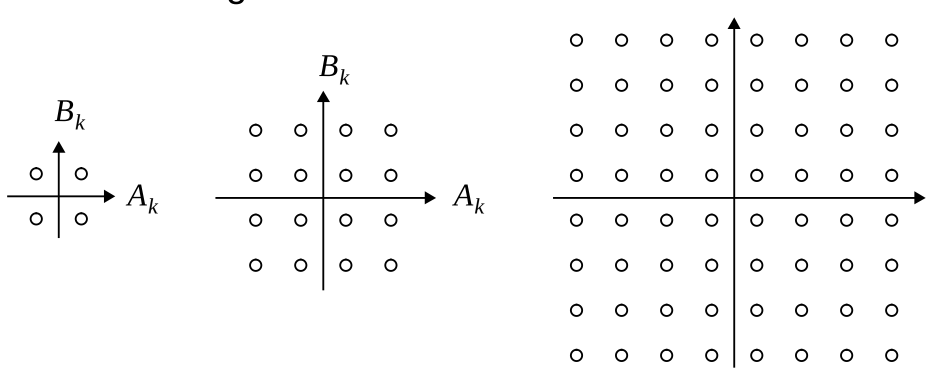




$u(t) = a(t) + jb(t)$

QAM

- Can draw signal constellations

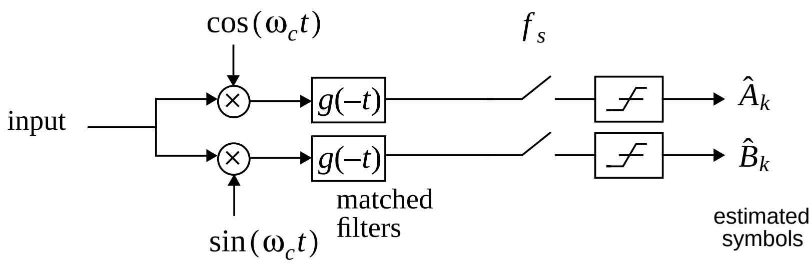


QAM 4 QAM 16 QAM 64

- Can Gray encode so that if closest neighbor to correct symbol chosen, only 1 bit error occurs

QAM

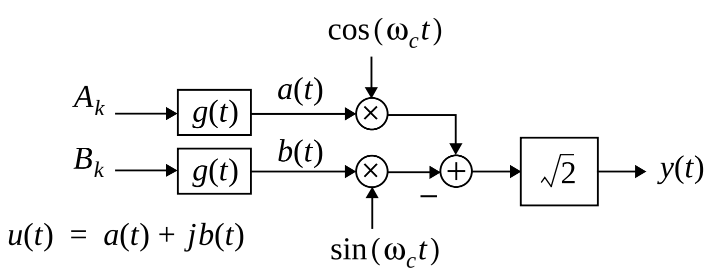
- To receive a QAM signal, use correlation receiver

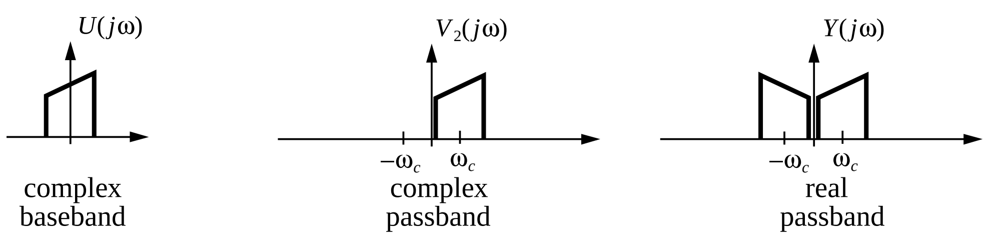


- When transmitting a small bandwidth (say 20kHz) to a large carrier freq (say 100MHz), often little need for adaptive equalization — use fixed equalizer

CAP

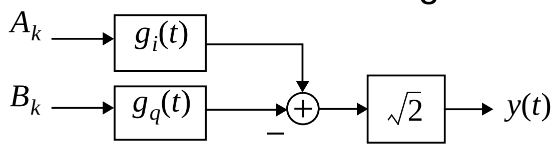
- Carrierless AM-PM modulation
- Essentially QAM modulated to a low carrier, f_c





CAP

- BIG implementation difference — can directly create impulse response of two modulated signals.



where

$$g_i(t) = g(t)\cos(\omega_c t) \quad (115)$$

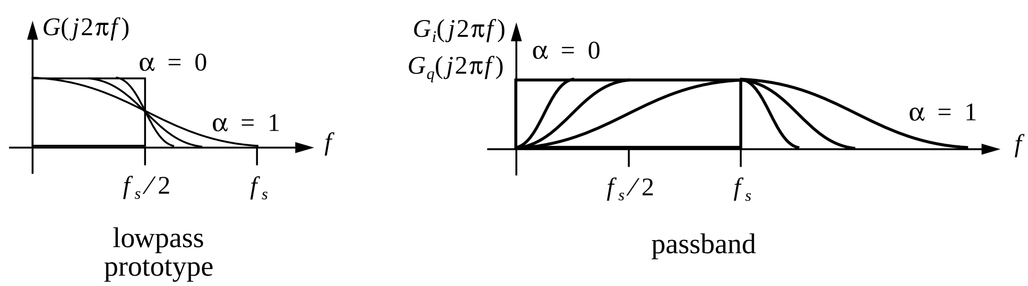
$$g_q(t) = g(t)\sin(\omega_c t) \quad (116)$$

- Not feasible if ω_c is much greater than symbol freq
- Two impulse responses are orthogonal

$$\int_{-\infty}^{\infty} g_i(t) g_q(t) dt = 0 \quad (117)$$

CAP

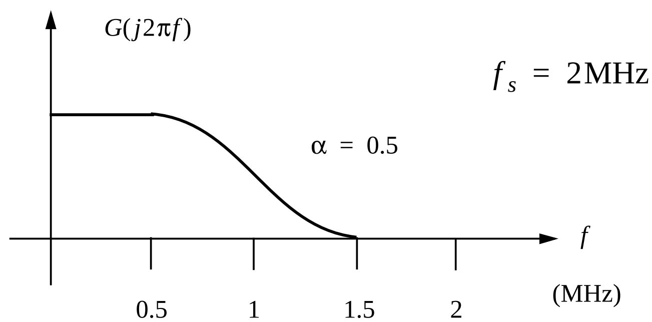
- The choice for ω_c depends on excess bandwidth



- Excess bandwidth naturally gives a notch at dc
- For 100% excess bandwidth $\omega_c = f_s$
- For 0% excess bandwidth $\omega_c = f_s/2$

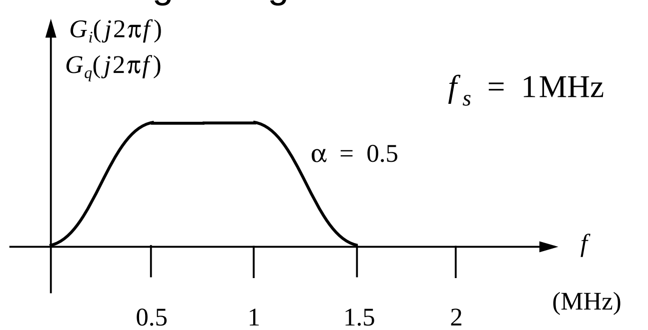
Example — Baseband PAM

- Desired Rate of 4Mb/s — Freq limited to 1.5MHz
- Use 50% excess bandwidth ($\alpha = 0.5$)
- Use 4-level signal (2-bits) and send at 2MS/s



Example — CAP

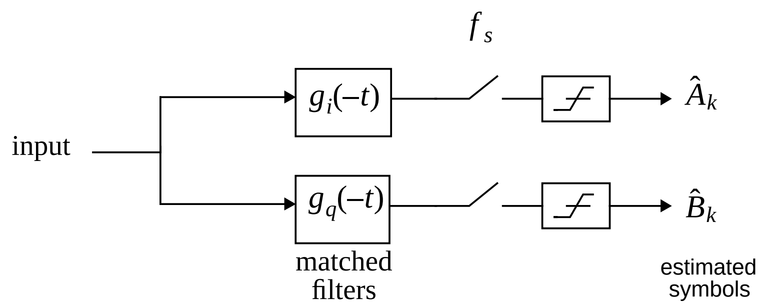
- Desired Rate of 4Mb/s — Freq limited to 1.5MHz
- Use 50% excess bandwidth ($\alpha = 0.5$)
- Use CAP-16 signalling and send at 1MS/s



- Note faster roll-off above 1MHz
- Area under two curves the same

CAP

- Two matched filters used for receiver



- When adaptive, need to adapt each one to separate impulse — should ensure they do not converge to same impulse

CAP vs. PAM

- Both have same spectral efficiency
- Carrier recovery similar? (not sure)
- CAP is a passband scheme and does not rely on signals near dc
- More natural for channels with no dc transmission
- Can always map a PAM scheme into CAP
  — 2-PAM $\leftrightarrow$ 4-CAP   4-PAM $\leftrightarrow$ 16-CAP   8-PAM $\leftrightarrow$ 64-CAP
- Cannot always map a CAP scheme into PAM
  — cannot map 32-CAP into PAM since $\sqrt{32}$ is not an integer number

DMT Modulation

- Discrete-MultiTone (DMT)
- A type of multi-level orthogonal multipulse modulation
- More tolerant to radio-freq interference
- More tolerant to impulse noise
- Can theoretically achieve closer to channel capacity
- Generally more complex demodulation
- Generally more latency

ADSL (Asymmetric DSL)

- 6Mb/s to home, 350kb/s back to central office over existing twisted-pair
- POTS splitter so telephone can coexist

Multipulse Modulation

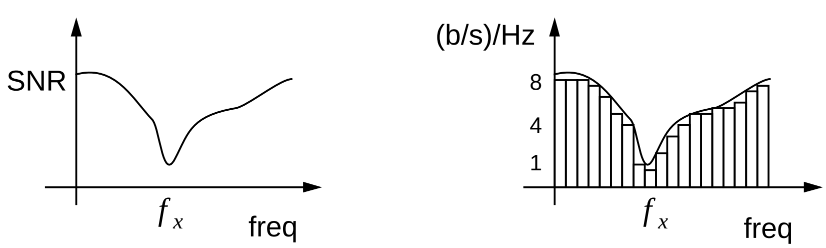
- Consider the two orthogonal signals from CAP — one transmission scheme is to transmit $g_i(t)$ for a binary 1 and $g_q(t)$ for a binary 0.
- Use a correlation receiver to detect which one was sent.
- Spectral efficiency (if $\alpha = 0$) is only 1 (symbols/s)/Hz rather than 2 (symbols/s)/Hz in the case of PAM
- In general, need $N\pi/T$ bandwidth to send N orthogonal pulses
- PAM, $N = 1$, minimum bandwidth: π/T
- QAM and CAP, $N = 2$, minimum bandwidth: $2\pi/T$

Combined PAM and Multipulse

- Changing scheme to sending $\pm g_i(t)$ and $\pm g_q(t)$ becomes a 2-level for each 2 orthogonal multipulses which is same as 4-CAP
- Multitone uses many orthogonal pulses as well as multi-levels on each (each pulse may have different and/or varying number of multi-levels)
- In discrete-form, it makes use of FFT — called Discrete MultiTone (DMT)
- Also called MultiCarrier Modulation (MCM)

Bit Allocation

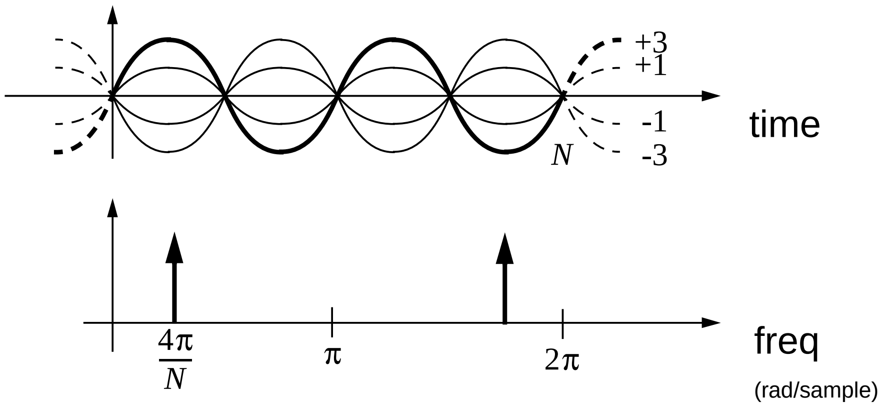
- Allocate more bits where SNR is best



- A radio interferer causes low SNR at f_x
- Perhaps send only 1 b/s/Hz in those bands
- At high SNR send many b/s/Hz

FFT Review

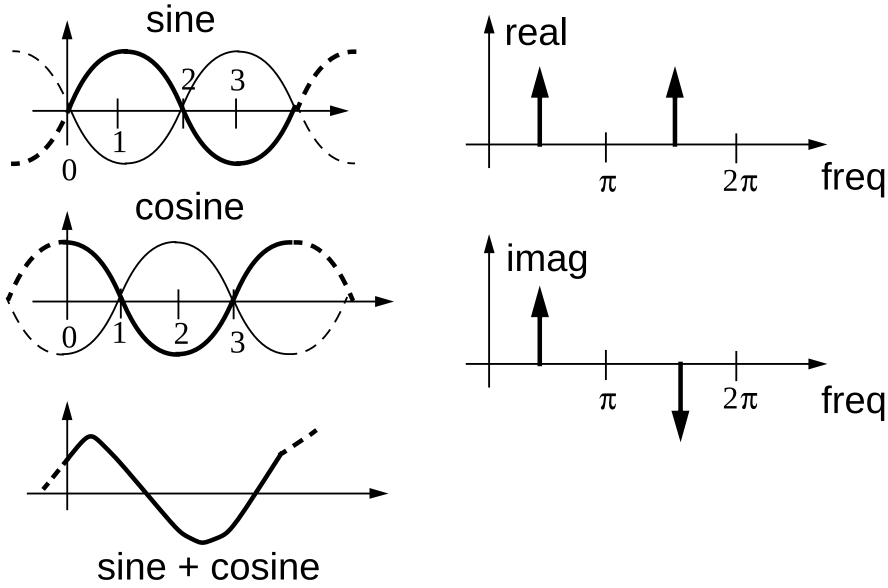
- FFT is an efficient way to build a DFT (Discrete Fourier Transform) when number of samples $N = 2^M$
- If rectangular window used and time-domain signal periodic in N, then FFT has impulses in freq domain



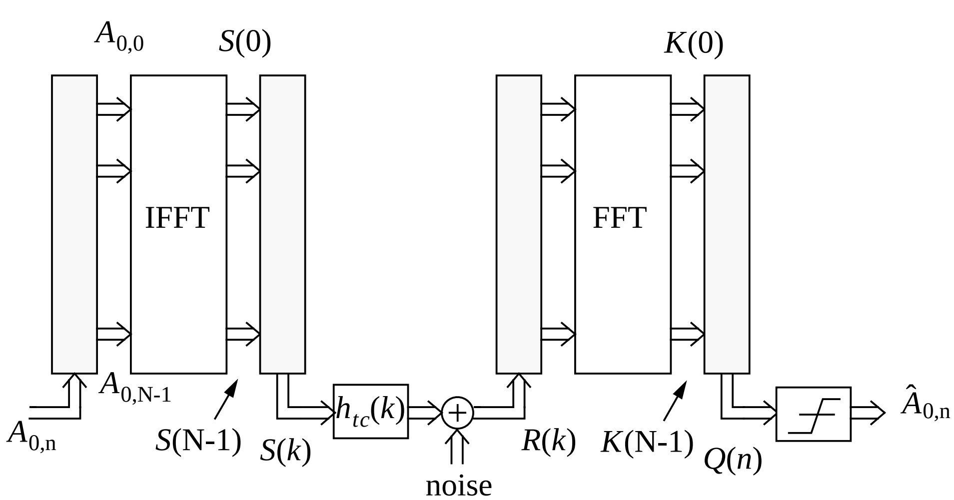
DMT Generation

- Input to IFFT (inverse FFT) is quantized impulses at each freq (real and imag)
- Forced symmetric around π (complex conjugate)
- Output is real and is sum of quantized amplitude sinusoids
- Quantized real - quantized amplitude cosine
- Quantized imag - quantized amplitude sine
- Symbol-rate is much lower than bandwidth used

Example — N=4



DMT Modulation



DMT Modulation

- Symbol Length, T
  — make symbol length as long as tolerable
  — typically need 3 symbol periods to decode
- If max channel bandwidth is $f_{\max}$, sampling rate should be $f_{\text{samp}} > 2f_{\max}$
- Choose $N = 2^M > f_{samp}T$ where M is an integer

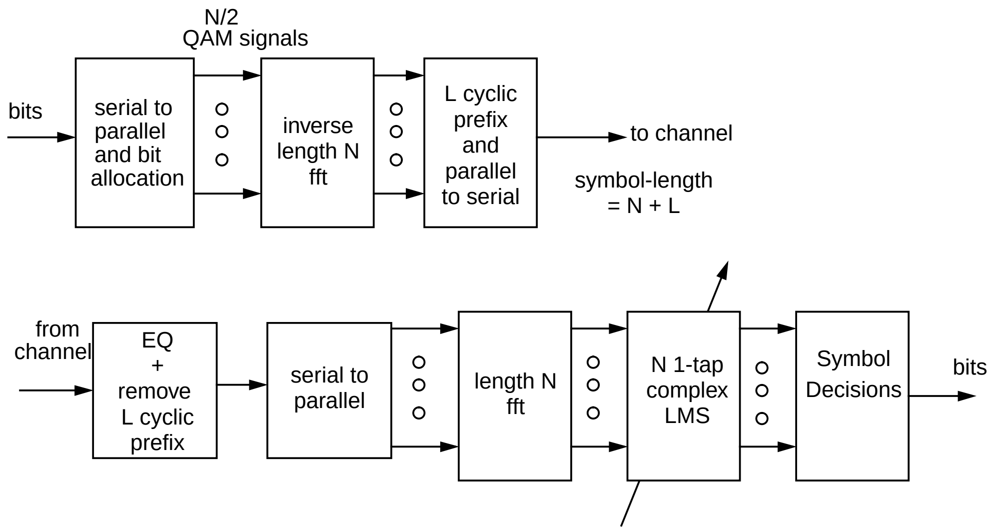
Example

- Max channel bandwidth is 1MHz,
- $f_{samp} = 2\text{MHz}$, $N = 512$ results in $M = 9$, $T = 1/3.9\text{kHz}$
- Channel bandwidths are $\Delta f = f_{\max}/(N/2) = 3.9\text{kHz}$

Cyclic Prefix

- If channel is modelled as having a finite impulse response on length L, send last L samples at beginning to ignore transient portion of channel
- Could send much more but no need
- When receiving, ignore first L samples received (purge out transient part of channel)
- Each FFT bin will undergo phase and magnitude change, equalize out using a complex multiplication
- If channel model too long, pre-equalize to shorten signficant part of channel impulse response

DMT Modulation



DMT Modulation

- Clock sent in one frequency bin
- More tolerant to impulse noise because of long symbol length
  — expect around $10\log(N)$ dB improvement
  $N = 512$ implies $27\ \mathrm{dB}$ improvement
- Longer latency
- Can place more bits in frequency bins where more dynamic range occurs (achieve closer to capacity)
- Transmit signal appears more Gaussian-like
  — a large Crest factor
  — more difficult line driver
  — need channel with less distortion or clipping

Coding

Coding

Scrambling (Spectrum control)

- "Whiten" data statistics
- Better for dc balance and timing recovery

Line Coding (Spectrum control)

- Examples: dc removal or notch

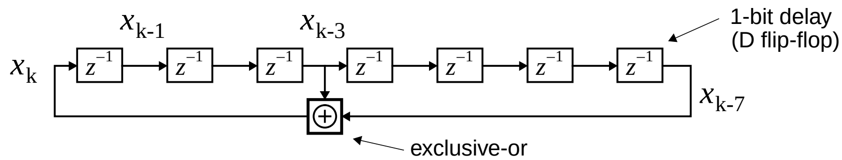
Hard-Decoding (Error Control)

- Error detection or correction — received bits used

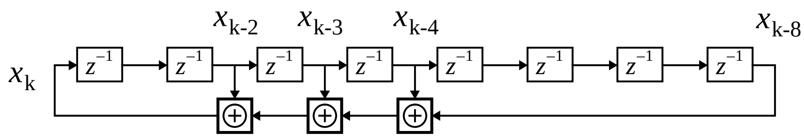
Soft-Decoding (Error Control)

- Error prevention
- Most likely sequence — received samples used

PN Sequence Generators



7-bit PN Sequence (sequence length = 127)



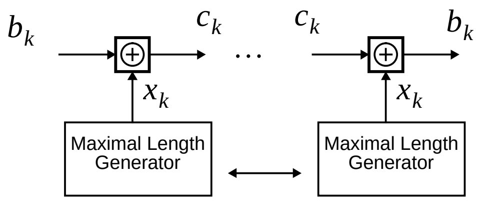
8-bit PN Sequence (sequence length = 255)

- Use n-bit shift register with feedback
- If all-zero state occurs, it remains in that state forever
- Maximal length if period is $2^n - 1$

Maximal-Length PN Sequences

Delay Length	Feedback Taps	Delay Length	Feedback Taps	Delay Length	Feedback Taps
2	1,2	13	1,3,4,13	24	1,2,7,24
3	1,3	14	1,6,10,14	25	3,25
4	1,4	15	1,15	26	1,2,6,26
5	2,5	16	1,3,12,16	27	1,2,5,27
6	1,6	17	3,17	28	3,28
7	3,7	18	8,18	29	2,29
8	2,3,4,8	19	1,2,5,19	30	1,2,23,30
9	4,9	20	3,20	31	3,31
10	3,10	21	2,21	32	1,2,22,32
11	2,11	22	1,22	33	13,33
12	1,4,6,12	23	5,23	34	1,2,27,34

Side-Stream Scrambler



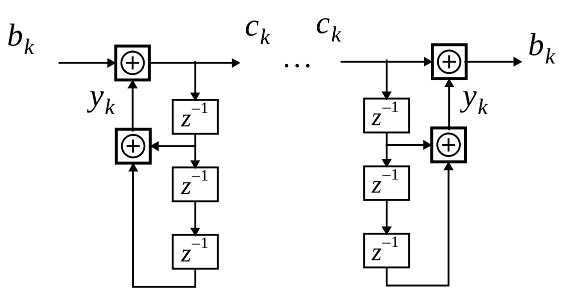
- Also called “frame-synchronized”

$$c_k = b_k \oplus x_k \tag{118}$$

$$c_k \oplus x_k = b_k \oplus x_k \oplus x_k = b_k \oplus 0 = b_k \tag{119}$$

- Advantage: no error propagation
- Disadvantage: need to synchronize scramblers
- Note that c_k would be all zeros if $b_k = x_k$ (unlikely)

Self-Synchronized Scrambler

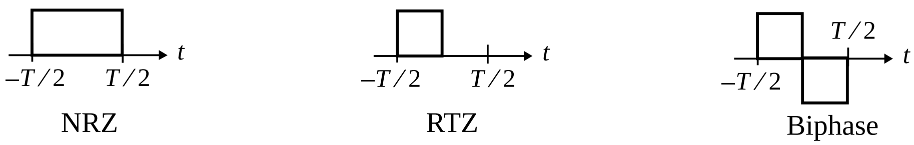


example 3-bit scrambler

- Similar to side-stream, b_k recovered since $y_k \oplus y_k = 0$
- Advantage: no need for alignment of scramblers.
- Disadvantage: one error in received value of c_k results in three errors (one for each XOR summation)
- Can also have more problems with periodic inputs.

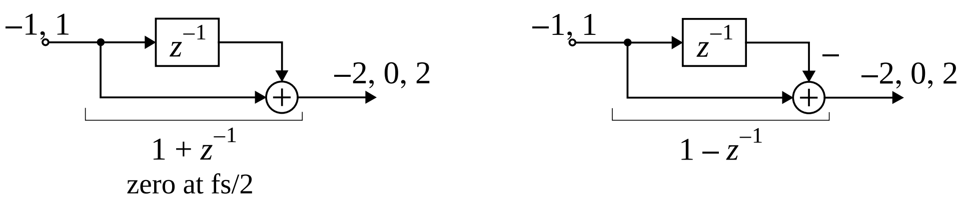
Line Coding

Change pulse shape



- Remains a 2-level signal but more high-freq content

Filter data signal



Line Coding

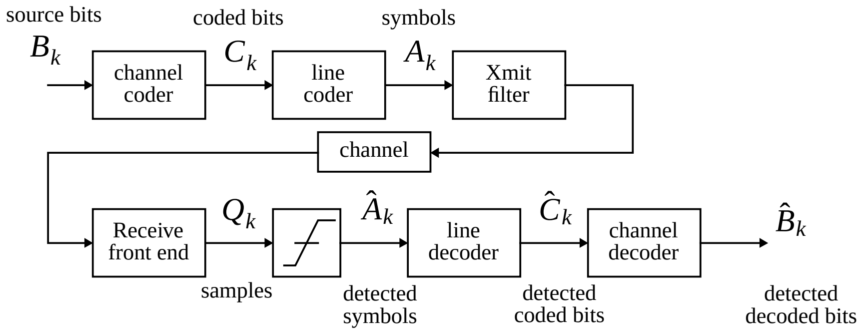
Filter data signal

- Results in more signal levels than needed for bit transmission — “correlated level coding”
- Loose 3dB in performance unless maximal likelihood detector used.

Block Line Codes

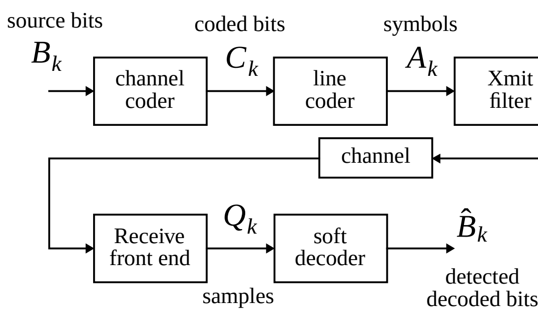
- Map block of k bits into n data symbols drawn from alphabet of size L.
- When $2^k < L^n$, redundancy occurs and can be used to shape spectrum.
- Example: blocks of 3 bits can be mapped to blocks of 2 3-level symbols.

Hard-Decoding



- Redundancy by adding extra bits
- Error detection and/or correction performed by looking ***after*** quantizer
- Examples: parity check, Reed-Solomon

Soft-Decoding



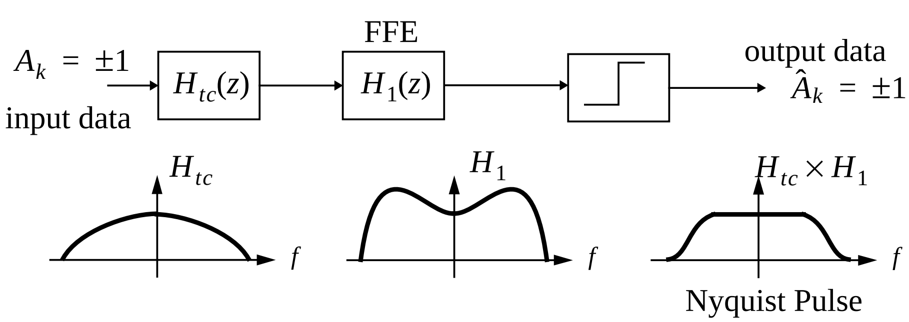
- Makes direct decisions on info bits without making intermediate decisions about transmitted symbols.
- Processes Q_k directly — combines slicing and removal of redundancy
- Can achieve better performance than hard decoding

Partial-Response and Viterbi Detection

Partial-Response Motivation

Disadvantage — Feed-Forward Equalizer

- An FFE boosts the noise in areas where received signal power is low
- Example:



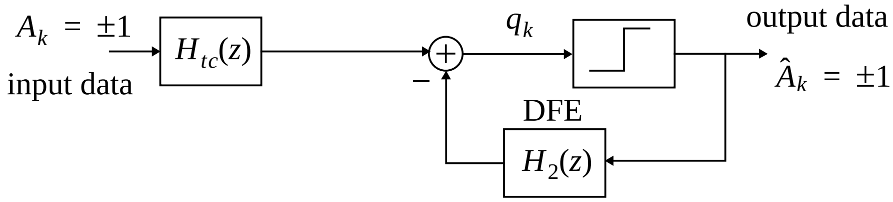
- Noise is boosted at high frequencies.

Partial-Response Motiv ation

Disadvantage — Decision Feedback Equalizer

- A DFE does not make use of all the impulse response.

$$H_{tc}(z) = 1 + 0.9z^{-1} + 0.1z^{-2}$$



$$H_2(z) = 0.9z^{-1} + 0.1z^{-2}$$

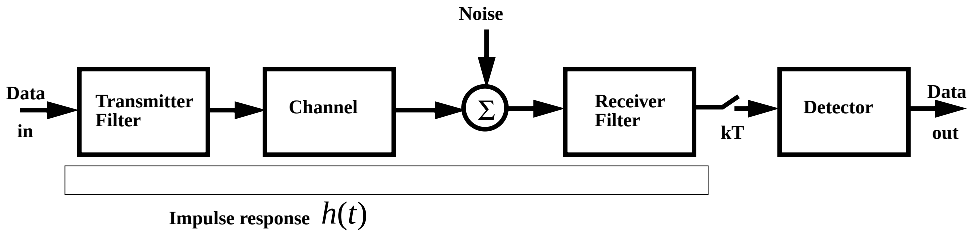
- Since $\hat{A}_k = A_k$, impulse response is $\delta(k)$
- If single input, better to look for $1, 0.9, 0.1, 0, 0$ than $1, 0, 0, 0, 0, 0$
- Postcursor ISI may have significant signal power.

Partial-Response Motiv ation

- Rather than equalizing to a Nyquist pulse, equalize to a partial-response signal
- Equalize to $1 + z^{-1}$ in DFE example
- Less noise boost
- More of the impulse response used to determine transmitted signal
- Need to look at a string of received symbols rather than symbol-by-symbol detection — MLSD

- Disadvantage — extra complexity and may not recover full dynamic range loss

Nyquist Criterion for Zero ISI



- ***Nyquist's First Criterion*** for zero ISI

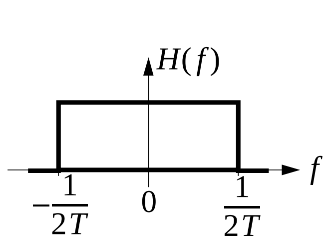
$$h(kT) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

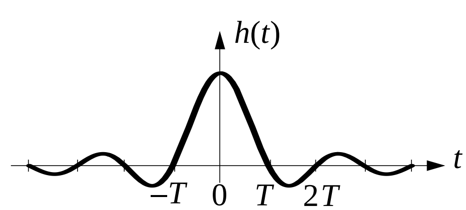
$$\sum_n H\left(\omega - \frac{2\pi n}{T}\right) = T$$

Minimum Bandwidth System with Zero ISI

$$H(\omega) = \begin{cases} T & |\omega| \le \pi / T \\ 0 & \text{elsewhere} \end{cases}$$

- A brickwall low-pass spectrum with a cutoff frequency of $1/(2T)$ ("*sinc*" impulse response)





- However — impulse response decays at a rate of $1/t$ due to the frequency discontinuity in $H(f)$.

- Excessive ISI if ***any*** timing perturbation occurs

Non-Minimum Bandwidth System

- One way to overcome jitter problem is to use more than the minimum bandwidth.
- A popular class of non-minimum bandwidth solutions are — *Cosine Roll-Off Filters*
- Can still transmit and receive only one of two symbols.

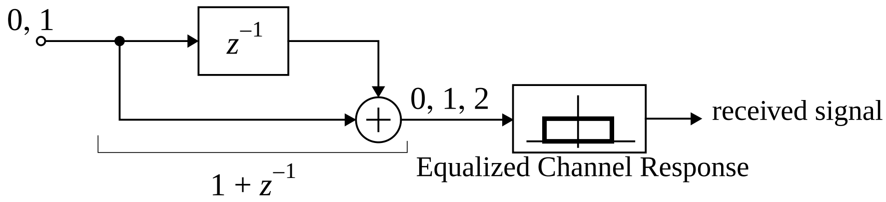
- But are minimum bandwidth systems practical? ***Yes***. — use ***partial-response signaling***.

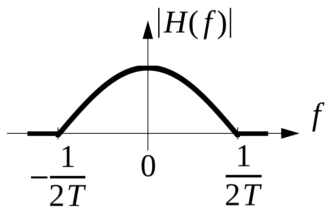
Partial-Response Signaling

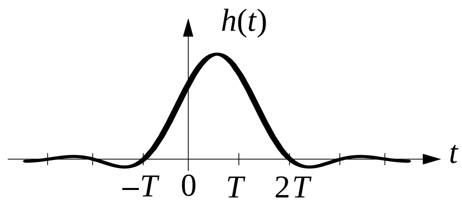
- By relaxing the ***zero-ISI criterion*** of Nyquist, the maximum symbol rate of 2 symbols/hertz can be achieved.
- Allow a ***controlled*** amount of ISI by digitally FIR filtering the data — results in more signal levels.
- Three popular FIR filters:

$1 + z^{-1}$	duobinary - class 1	zero at $f_s/2$
$1 - z^{-1}$	dicode	zero at dc
$1 - z^{-2}$	modified duobinary - class 4 (also called PR4 or PRIV)	zeros at dc, $f_s/2$

Duobinary ($1+z^{-1}$)

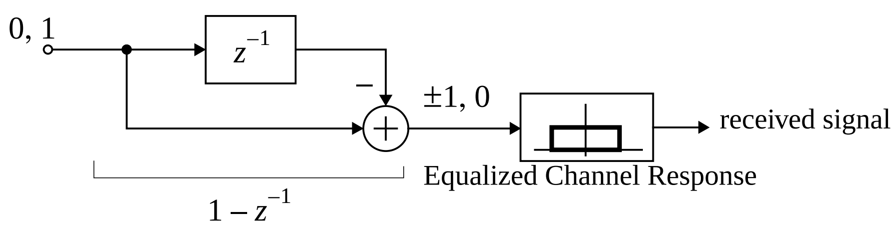


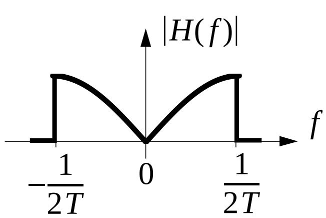


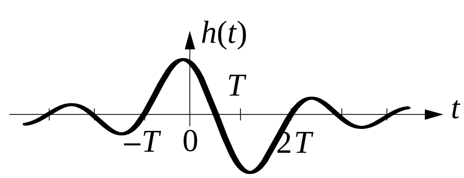


- Impulse response decays at a rate of $1/t^2$ since $H(f)$ is continuous but its first derivative is not.
- However, it transmits signal power at dc.

Dicode ($1-z^{-1}$)

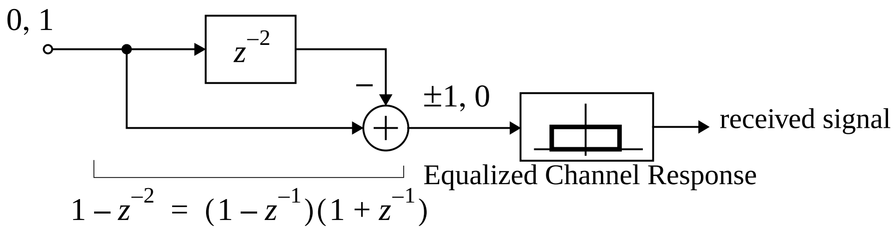


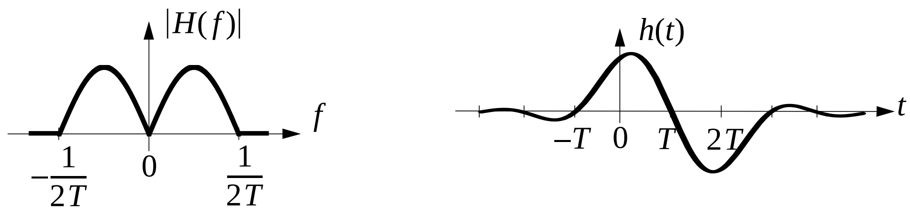




- Impulse response decays at a rate of $1/t$ due to the frequency discontinuity in $H(f)$.
- However, it does not transmit any signal power at dc.

Modified Duobinary ($1-z^{-2}$) — class 4

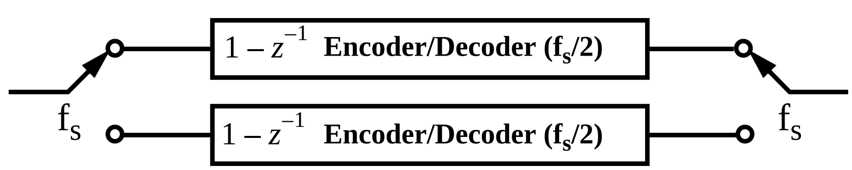




- $h(t)$ decays at a rate of $1/t^2$ since $H(f)$ is continuous.
- It does not transmit signal power at either dc or $f_s/2$.

Class-4 Partial Response Signaling Scheme

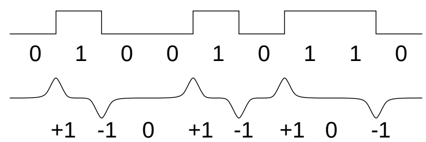
- Spectral nulls at DC and $f_s/2$
- Can be encoded/decoded by two interleaved dicode encoder/decoder each operating at half the rate.



- Thus, ***we need only decode a dicode*** and use two interleaved identical blocks to decode PRIV.
- If binary inputs, 3 level output — BPR4 or BPRIV
- (If 4 level inputs, 9 level output — QPR4 or QPRIV)

Magnetic Recording Similarities

- At low densities, a magnetic read signal is inherently 1-D encoded (i.e. a dicode).

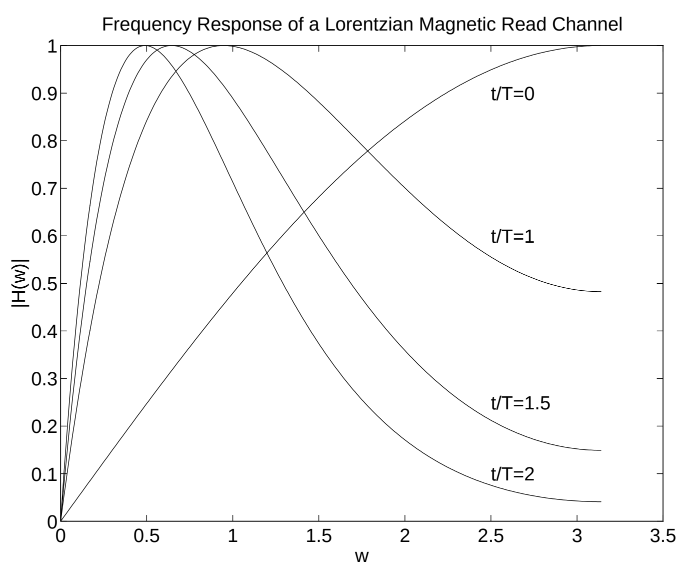


- At higher densities, high-frequency roll-off important (modelled as a Lorentzian pulse).
- If equalized to a 1-D channel, high-frequency noise is amplified.

- ***Find a good approximation to channel so that the boost required by equalizer is kept small.***

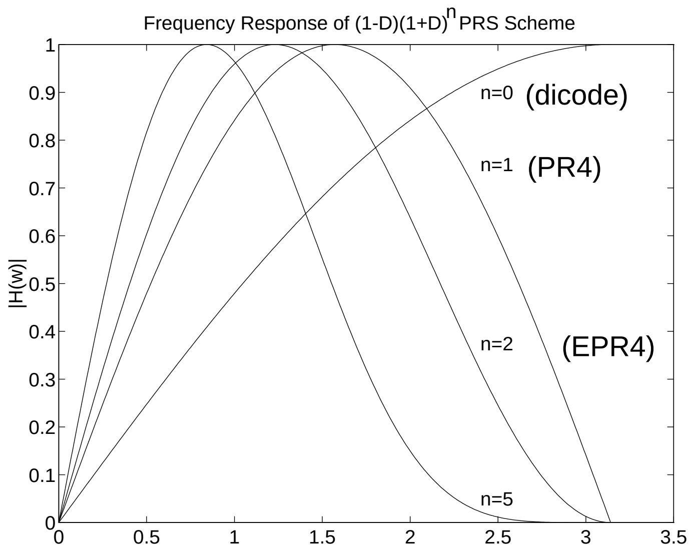
Magnetic Recording Similarities

- Magnetic recording channel often modelled as Lorentzian pulse



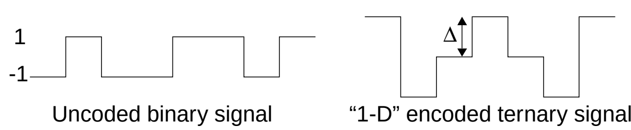
Magnetic Recording Similarities

- Similar to $(1 - z^{-1})(1 + z^{-1})^n$ partial-response channel.



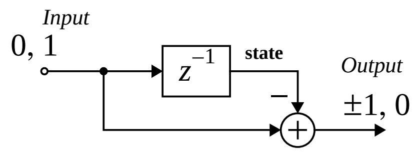
SNR Degradation for Dicode

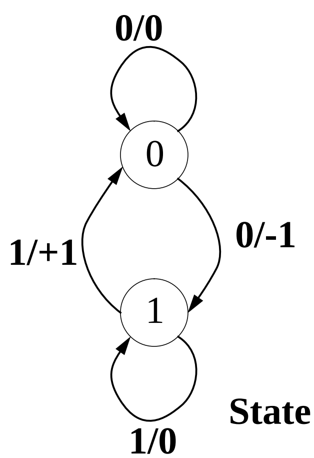
- Now 3 levels being sent rather than just two.



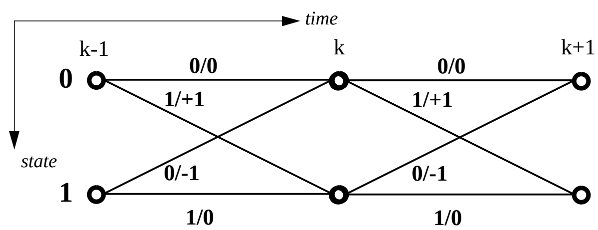
- Thus, a bit-by-bit detection results in SNR performance degradation (about 2-3 dB loss).
- However, the 3 levels have some redundancy included.
- SNR performance can be recovered in detection by employing Maximum-Likelihood Sequence Estimation ***(MLSE) detection*** schemes
- The ***Viterbi Algorithm*** is an efficient way of realizing MLSE detection

Trellis Introduction





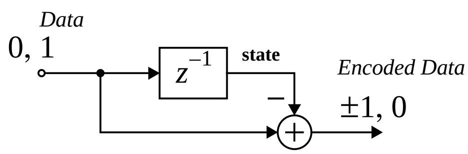
State Diagram

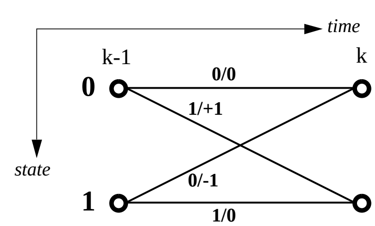


Trellis Diagram

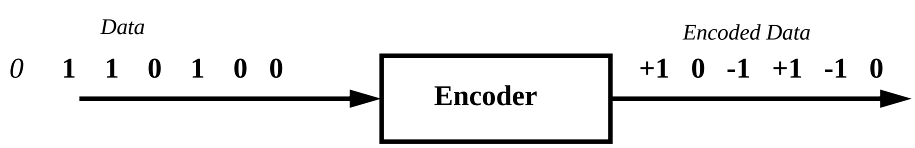
Trellis Representation of Dicode (1-z^{-1})

- A trellis can be used to describe an encoder.
- Example:

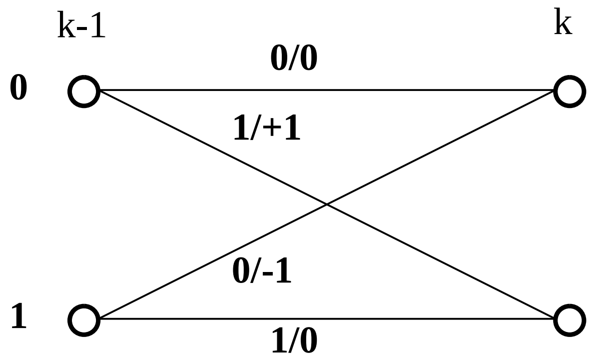




Trellis

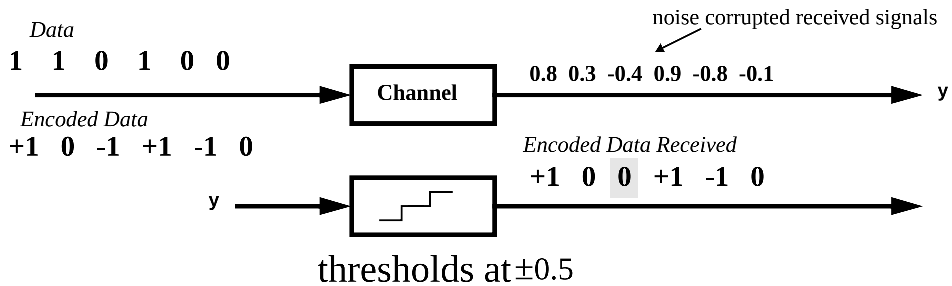


Transmit Trellis for Dicode ($1-z^{-1}$)



- Note that following a ‘+1’ output, there can be an arbitrary number of zeros followed by a ‘-1’.
- In other words, if two ‘+1’ symbols are detected with no ‘-1’ between them, an error occurred in transmission.
- Similar for a ‘-1’ output.

Conventional Bit-by-Bit Detection

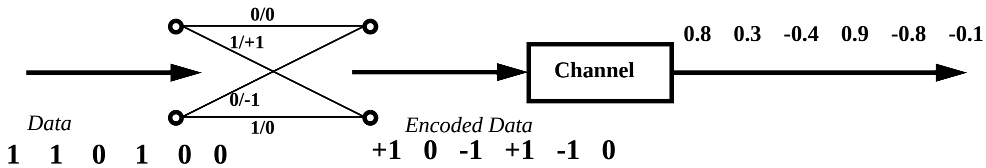


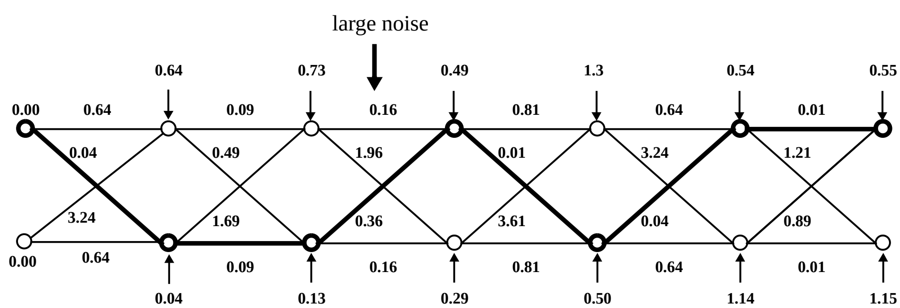
- Note the error in bit 3 received.
- Error can be detected since a “-1” must be next non-zero symbol after a “+1”.
- Did the error most likely occur in symbol 2, 3 or 4?
-

Viterbi Algorithm (VA)

- VA is an iterative method for determining the most likely sequence sent — maximum likelihood detector.
- Accomplished by creating a ***receive trellis*** having ***branch metrics*** proportional to the difference squared between received signal and each ideal symbol value.
- ***The most likely sequence is the shortest path through the receive trellis.***
- ***State metrics*** and ***path memory*** also stored to reduce search time through trellis — they are the length of shortest path and path taken at each node.

Viterbi Algorithm Example



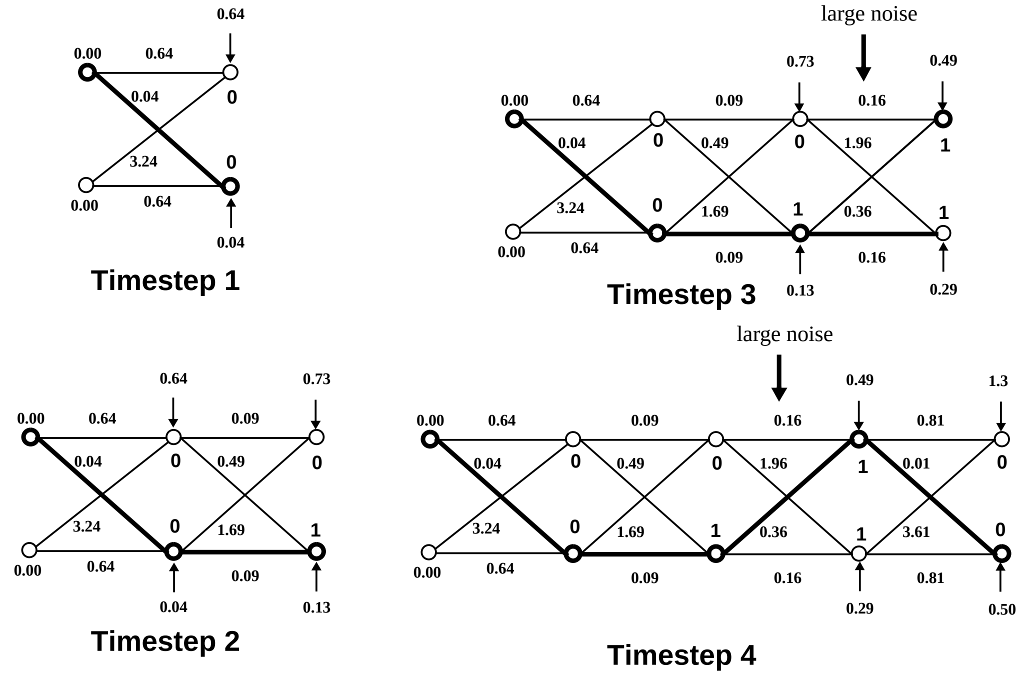


Data Received	**1**	**1**	**0**	**1**	**0**	**0**

Encoded Data Received

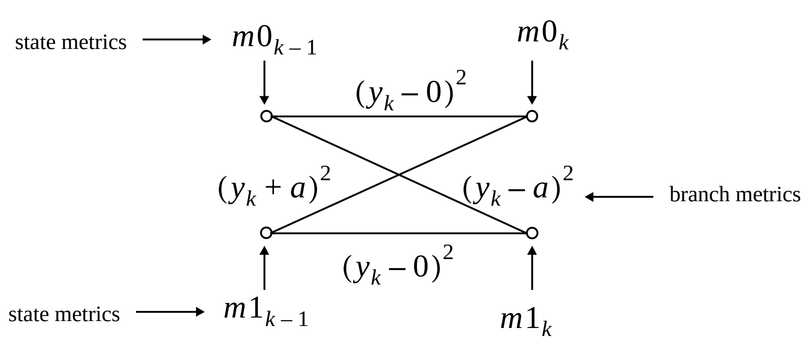
0	**+1**	**0**	**-1**	**+1**	**-1**	**0**

Viterbi Algorithm Example



Detailed Received Trellis Description (BPR4)

- Transmitted signal — one of three values, $\pm a$, 0
- Received signal — y_k.



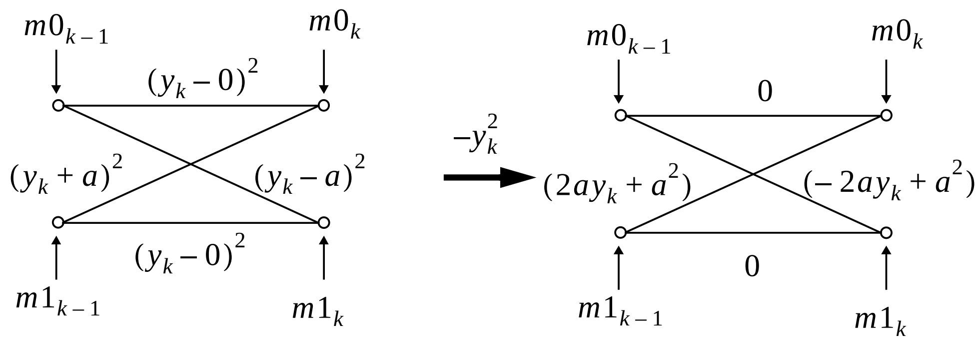
- Equations:

$$m0_k = min\{(m0_{k-1} + y_k^2)\ ,\ (m1_{k-1} + (y_k + a)^2)\}$$

$$m1_k = min\{(m1_{k-1} + y_k^2)\ ,\ (m0_{k-1} + (y_k - a)^2)\}$$

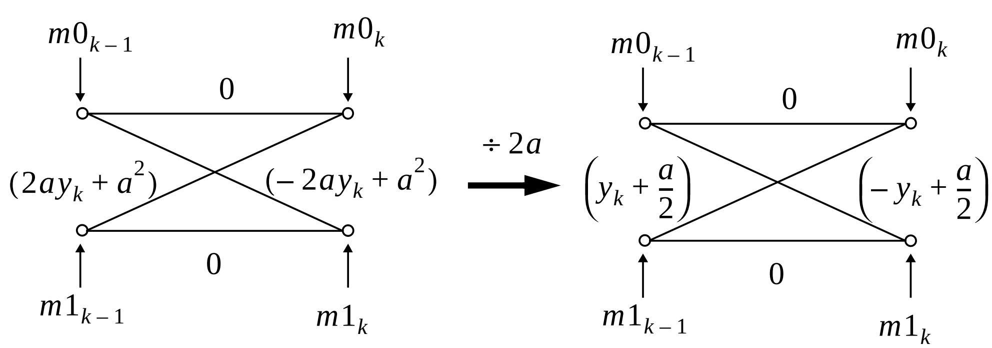
Simplifications to Remove Multiplications

- Remove y_k^2 terms since it occurs in both terms and we are only interested in finding the minimum path (don't need the absolute length of the path).
- State-metrics can now be either positive or negative.



Simplifications to Remove Multiplications

- Divide all branches by $2a$ (assume $a > 0$)
- Simply scales state metrics.



- Equations:

$$m0_k = min\{m0_{k-1}\ ,\ (m1_{k-1} + y_k + a)\}$$
$$m1_k = min\{m1_{k-1}\ ,\ (m0_{k-1} - y_k + a)\}$$

Difference Metric Algorithm

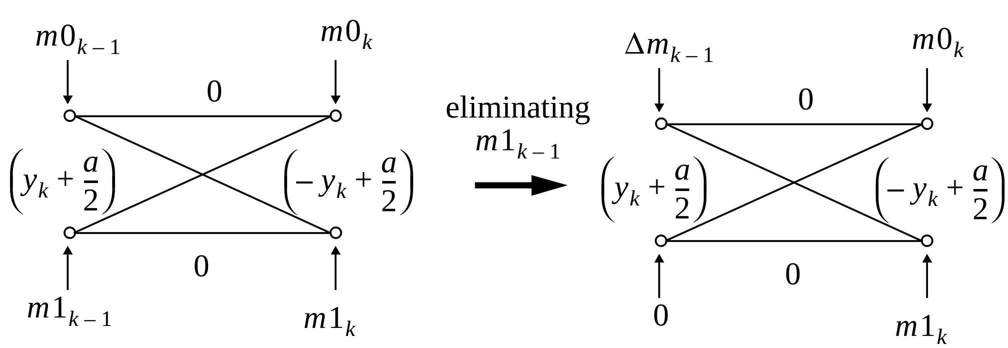
- [Wood and Peterson, Trans. on Comm., May 1986]
- We are not interested in absolute state-metric values — only which state-metric is smaller.
- Store only the difference in the state-metrics, Δm_k

$$\Delta m_k \equiv m0_k - m1_k$$

- We shall see that while absolute state-metric values increase in time, their difference does not.
- This “difference metric algorithm” results in less complex realizations for both digital and analog realizations in cases where there are only two state-metrics.

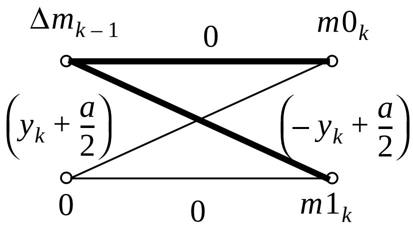
Difference Metric Algorithm

- Subtract off $m1_{k-1}$ from input state-metrics and add it into each branch metric instead.
- Now, $m1_{k-1}$ can be subtracted off branch metrics since it is the same in all branches.



Difference Metric Algorithm

- Different path choices.

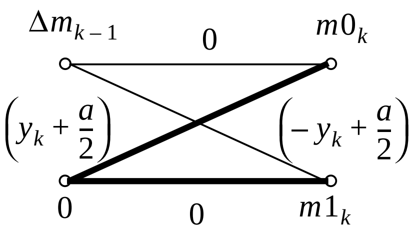


Condition

$$\left(\Delta m_{k-1} < y_k + \frac{a}{2}\right) \text{ and } \left(\Delta m_{k-1} < y_k - \frac{a}{2}\right)$$

Result

$$\Delta m_k = y_k - \frac{a}{2}$$



Condition

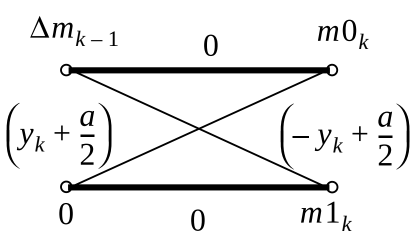
$$\left(\Delta m_{k-1} > y_k + \frac{a}{2}\right) \text{ and } \left(\Delta m_{k-1} > y_k - \frac{a}{2}\right)$$

Result

$$\Delta m_k = y_k + \frac{a}{2}$$

Difference Metric Algorithm

- Different path choices.

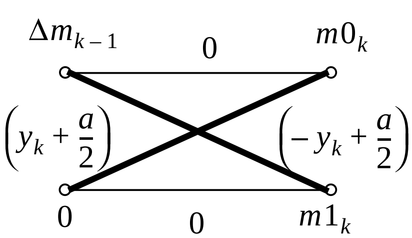


Condition

$$\left(\Delta m_{k-1} < y_k + \frac{a}{2}\right) \text{ and } \left(\Delta m_{k-1} > y_k - \frac{a}{2}\right)$$

Result

$$\Delta m_k = \Delta m_{k-1}$$



Condition

$$\left(\Delta m_{k-1} > y_k + \frac{a}{2}\right) \text{ and } \left(\Delta m_{k-1} < y_k - \frac{a}{2}\right)$$

IMPOSSIBLE

Result

do nothing

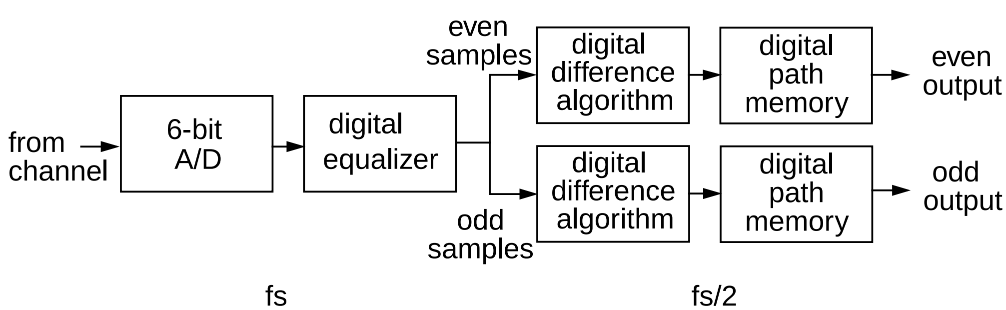
Difference Metric Algorithm

- Equations:

$$\Delta m_k = \begin{cases} y_k + \dfrac{a}{2} & \Delta m_{k-1} > y_k + \dfrac{a}{2} \\ \Delta m_{k-1} & y_k - \dfrac{a}{2} < \Delta m_{k-1} < y_k + \dfrac{a}{2} \\ y_k - \dfrac{a}{2} & \Delta m_{k-1} < y_k - \dfrac{a}{2} \end{cases}$$

- These equations describe an adjustable threshold device.
- Used in digital PR4 implementations.
- They are also simple to implement in ***analog***.

Typical Digital Implementation

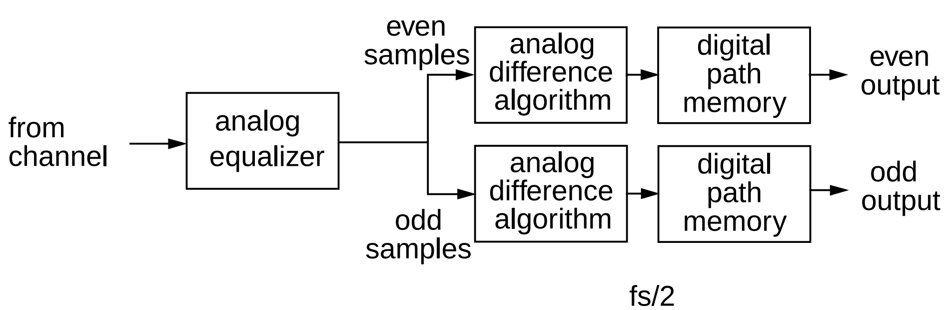


- 6-bit flash A/D requires 63 comparators + decoding logic.
- A/D converter might consume around 300mW (or more)
- FIR equalizer might be 1mW/MHz/tap
  — 8-tap at 100MHz = 800mW

Typical Digital Implementation

- Digital equalizer requires multi-bit multiplies in feed-forward equalizer (power hungry)
- If decision feedback is used, it will need to use early estimates of output (cannot wait for MLSE to finish).
- Digital difference algorithm requires some minor adders and digital comparators.
- Digital path memory logic consists of about 16 serial/ parallel shift registers.

Typical Analog Implementation

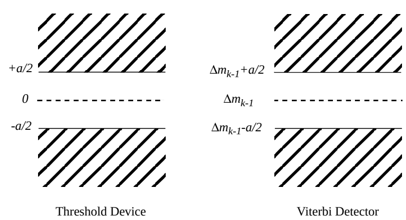


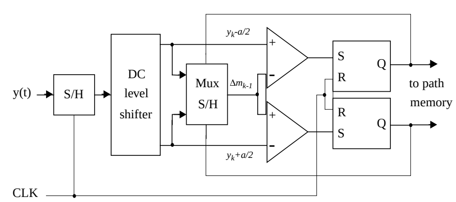
- Analog equalizer needed (less power than digital but more challenging)
- Digital path memory is the same as in fully digital realization.

Typical Analog Implementation

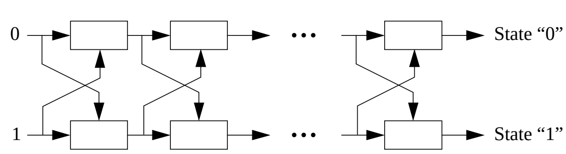
- Analog difference algorithm is very small (about the size of 2 comparators)
- Thus, power is saved and speed can be increased over 6-bit A/D converter.
- Note that dynamic range in analog parts need only be around 6 bits (i.e. 40dB)

Analog Implementation





- Path memory consists of two serial/parallel in/out shift registers.



Some Practical Limitations

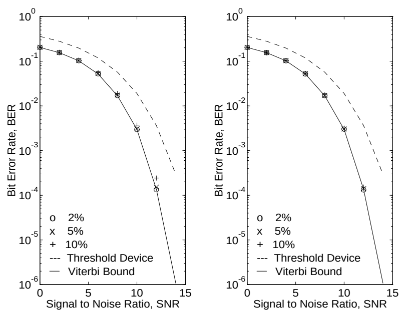
- In a digital implementation, performance is degraded by limiting the number of bits used in A/D conversion
- Typically use about 6-bit A/D converters (easily achievable in an all analog implementation).
- Truncating the trace-back length (path memory) also degrades the performance.
- Typically use length of 16 for little loss in performance.

Why an Analog Implementation?

- Avoids using a pre-stage A/D converter.
- Combines the A/D and VA into one stage with a complexity near to a 2-bit A/D (*Special-purpose A/D* converter).
- Consumes less power.
- Operates faster.
- 6-bit accuracy is enough (Moderate Precision Circuitry).
- Low-dynamic range requirement (Low-Voltage Operation).
- The difference algorithm updates only one sampled data without using previous samples (no *accumulative* analog errors)

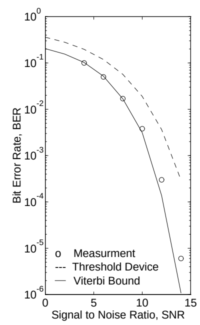
Simulation and Experimental Results

- Simulation and experimental (discrete prototype) results confirm validity of the analog approach and its robustness against imperfections.



DC Offset or Absolute Gain Error — Gain Mismatch

Offsets and mismatches are described in percentage of a



Measured BER Performance of the Detector (Path Memory = 17)

Input-Interleaved Algorithm

- The implementation above updates Δm_k once the comparator outputs are known.
- Thus, ***critical speed path is 2 sample-and-holds***. (Sample input and compare, then, perhaps, update Δm_k with another sample-and-hold).
- The input-interleaved algorithm reduces the critical speed path to a single sample-and-hold (i.e. can operate at twice the speed).
- It uses two sample-and-holds at the input and switches which one the input goes to if Δm_k needs to be updated.

Input Interleaved Algorithm

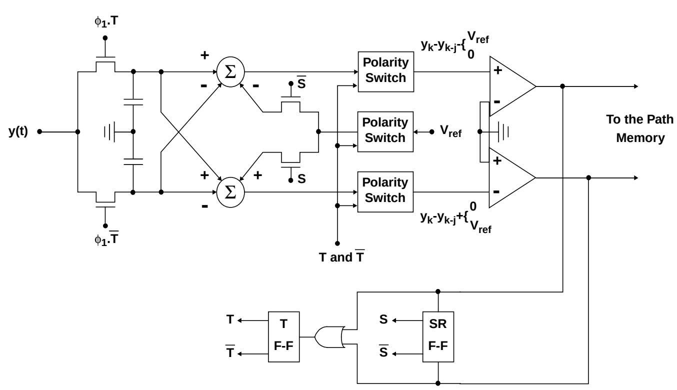
- According to the update mechanism, Δm_{k-1} is a DC-shifted version of a previously-sampled input signal

$$\Delta m_{k-1} = y_{k-j} \pm 0.5$$

- We can use the previously held input signal plus appropriate sign of DC shift for Δm_k.
- When Δm_k needs to be updated, switch input on to other sample-and-hold capacitor and use the just sampled input and a sign-bit for new Δm_k

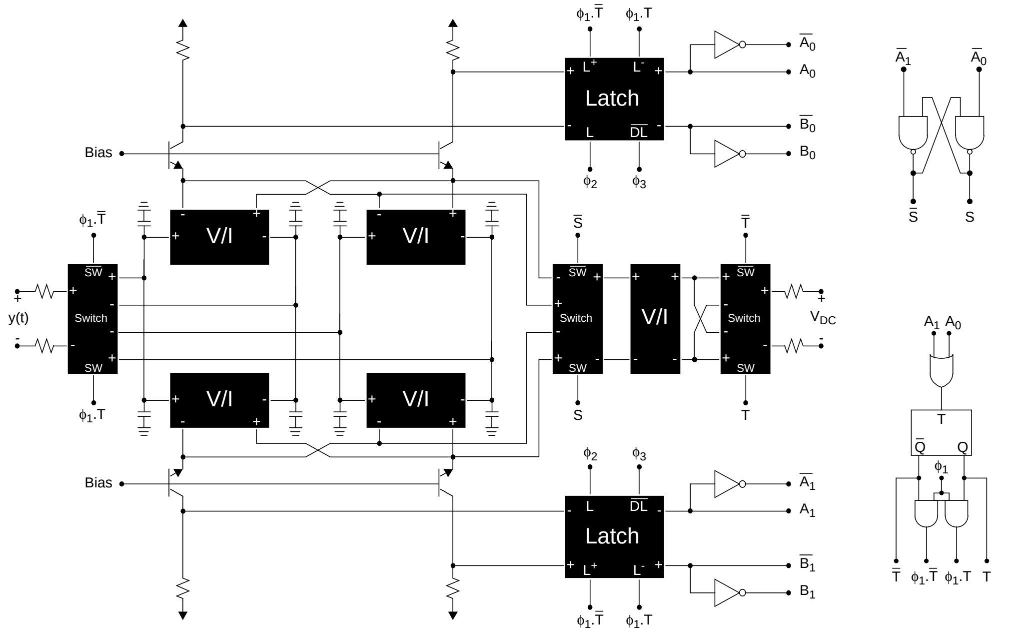
Input-Interleaved Algorithm

- Toggle between two S/Hs which store y_k and y_{k-j}
- Use a flip-flop to properly switch the DC signal

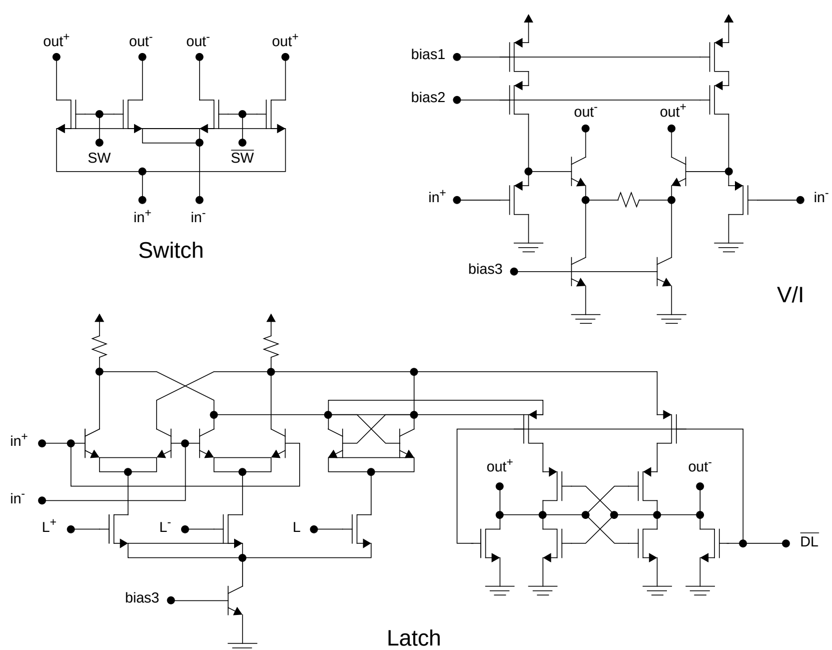


- Speed improvement, as no additional S/H is required (y_{k-j} has already been stored)

BiCMOS Integrated-Circuit Implementation

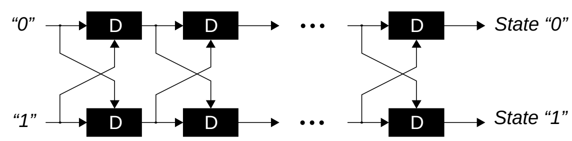


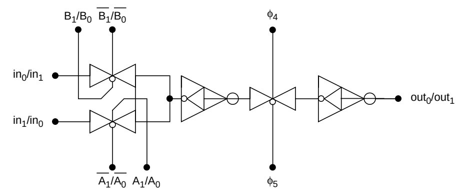
BiCMOS Integrated-Circuit Implementation



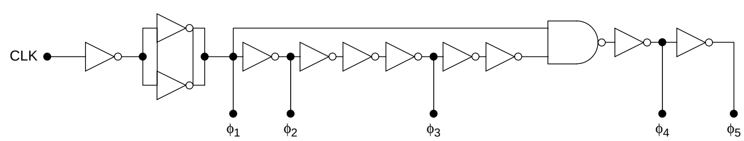
BiCMOS Integrated-Circuit Implementation

- Path memory consists of 2x12 multiplexed-input D flip-flops





- Clock phases



BiCMOS Integrated-Circuit Implementation

- Compared to other analog implementations

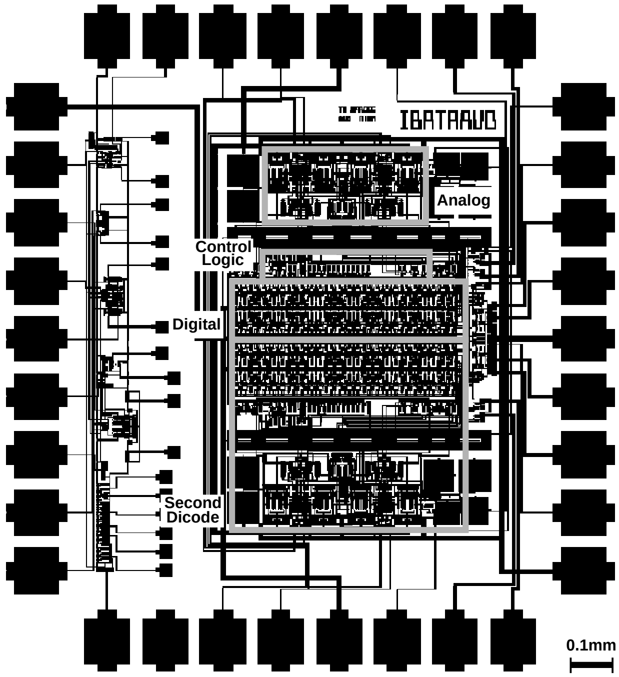
  [Matthews and Spencer, JSSC, Dec. 93]

  - Less complex (Individual state metrics are not calculated)
  - Less prone to imperfections (Feedback signals are only digital)
  - Fully differential
  - Faster (Master-slave S/Hs are not used)

  [Yamasaki, ISSCC, 1994]

  - No details given

Integrated-Circuit Implementation

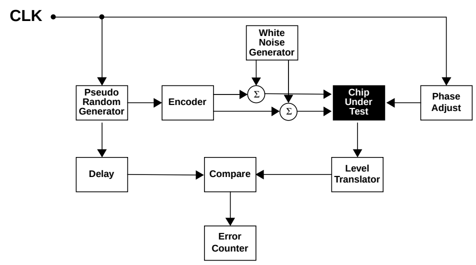


Experimental Results

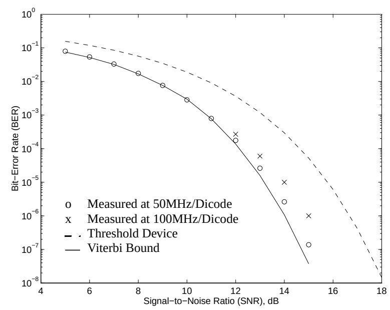
- Process: 0.8 μm BiCMOS
- Area (dicode): ~0.25 mm^2
  - Analog: ~0.06 mm^2
  - Digital: ~0.1 mm^2
  - Bypass capacitors, ...
- Power consumption (dicode):
  - 3.3V power supply
  - ~12mW at 50MHz
  - ~15mW at 100MHz

Experimental Results

- Setup



- Measured Bit-Error-Rate (BER) performance



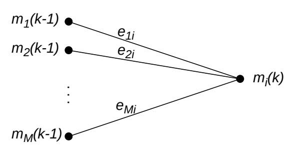
A General Implementation Approach

- Analog implementations are useful if the preceding signal processing is simple
  - Magnetic recording
  - Data transmission over unshielded cables
- Simplifications are only possible in some special cases (i.e. PR4)
- This general approach can be used in
  - More general PRS schemes (i.e. EPR4, EEPR4)
  - Convolutional codes
  - Multi-level digital communication
  - Irregular trellises

A General Implementation Approach

- This approach takes full advantage of the ability of simple analog circuits in realizing the ACS function

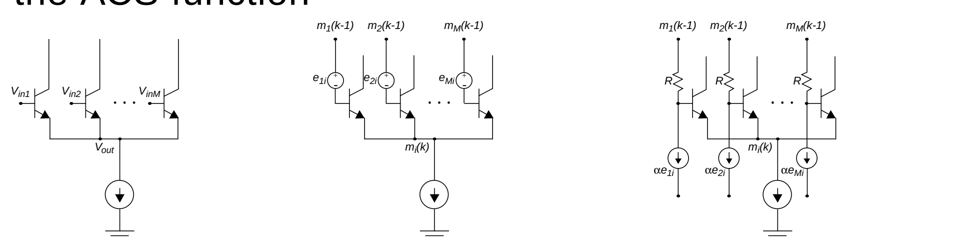
$$m_i(k) = \underset{j}{Max}\ \{m_j(k-1) - e_{ji}(k)\} \qquad \begin{array}{l} i = 1, 2, \ldots, N \\ j = 1, 2, \ldots, M \end{array}$$



- Branch metrics, e_{ij}, are usually expressed in terms of linear combinations of the received samples and DC signals

Circuit Realization

- A generalized differential cell is employed to realize the ACS function

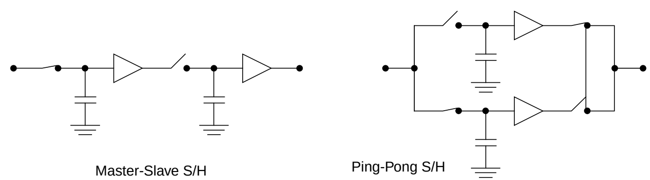


$_{ut} = \underset{j}{Max}\ \{v_{in\ j}\} - v_{BE}$ $\quad m_i(k) = \underset{j}{Max}\ \{m_j(k-1) - e_{ji}(k)\} - v_{BE}$ $\quad m_i(k) = \underset{j}{Max}\ \{m_j(k-1) - \alpha R e_{ji}(k)\} -$

- Using degenerated differential pairs in V/I conversions makes the linear combinations simple to realize
- Optional DC currents added to the error signals reduce the unnecessary DC voltage drops across the resistors

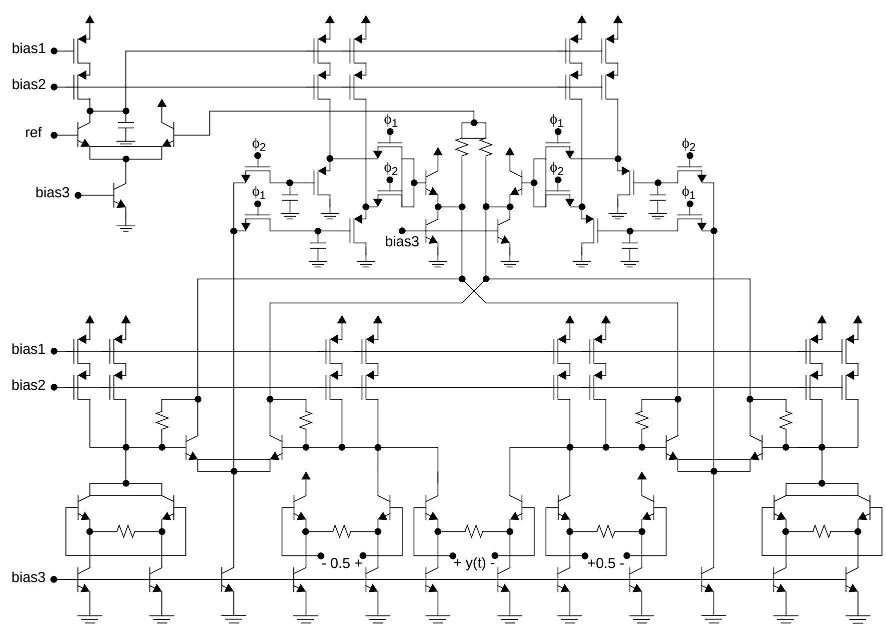
Circuit Realization

- Branch currents in the differential cells are comparison results
- To achieve high speeds, ping-pong S/Hs are preferred to master-slave S/Hs in feeding the state metrics back



- Algorithmic growth of state metrics is overcome by a fast Common-Mode FeedBack (CMFB) circuit
- Fast CMFB minimizes the signal swings of the state metrics — this approach is usually not practical in digital realizations

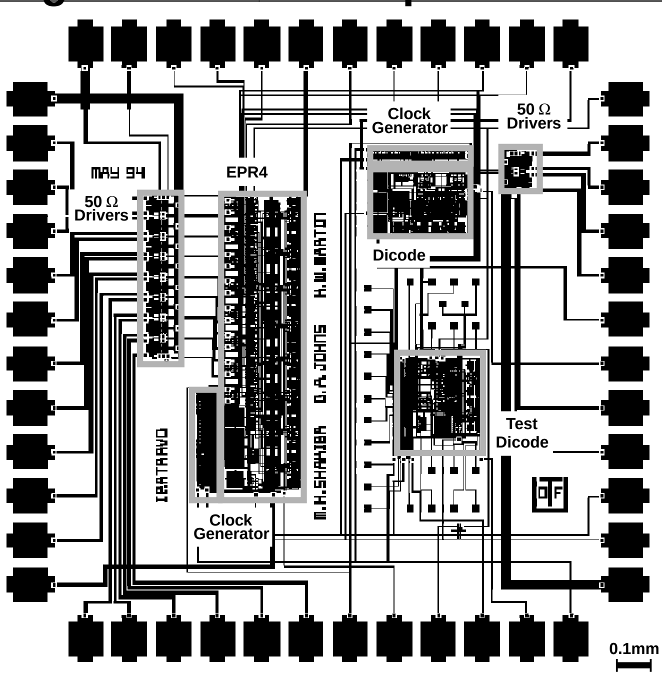
Design Example: *Binary Dicode*



Integrated Circuit Implementation

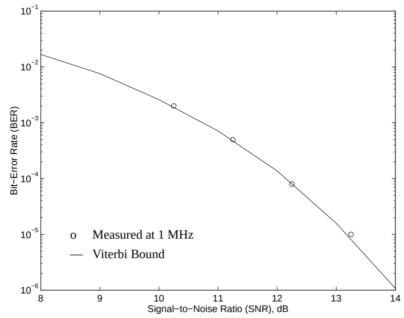
- A chip containing Viterbi decoders for a binary dicode and an Extended PR4 (EPR4, $(1-D)(1+D)^2$) has been fabricated in a 0.8μm BiCMOS process
- Based on simulations, fast speed (>100 MHz) can be achieved with ~15mW/state (Excluding path memory)
- The area is ~$0.03mm^2$/state (Excluding path memory)
- In a CMOS implementation, lower g_m causes some degradation in:
  - Obtaining simple low-impedance nodes
  - ACS performance due to the high dependency of v_{GS} to the drain current

Integrated Circuit Implementation



Preliminary Experimental Results

- Results on dicode



- High-frequency tests have been conducted up to 80 MHz (Off-chip path memory)

Summary

- The use of partial-response signals allows one to send closer to the maximum rate of 2 symbols/hertz.
- Making use of partial-response signalling reduces the need for large equalization boost.
- The difference algorithm is efficient for PR4 signals
- Analog realizations of the difference algorithm save silicon and power over digital realizations (however, an analog equalizer is needed)
- Input-interleaved algorithm increases the speed for an analog implementation
- A general analog Viterbi approach was discussed. (However, it makes use of bipolar transistors).

Infrared Channels

Infrared Channels

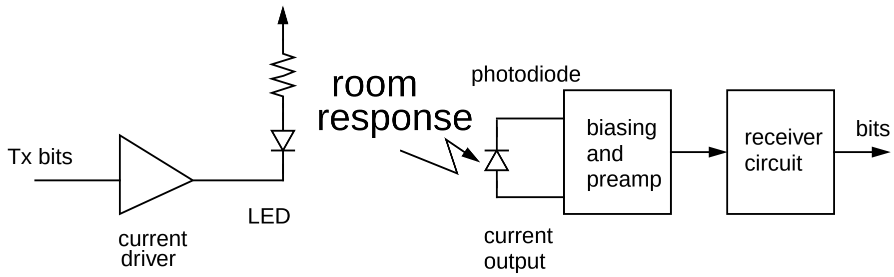
Advantages

- Free from regulation, low cost
- Blocked by walls — reduces eavesdropping and inter-cell interference
- Abundance of bandwidth in directed line-of-sight
- **Disadvantages** — lower range and bandwidth than radio

Types of Links

- Line-of-sight (does not rely on reflections)
  — higher bandwidth, easily blocked
- Diffuse (disperse Tx and wide-angle Rx)
  — lower bandwidth, very tolerant to shadowing

Infrared Channels



Transmit LED

- GaAs LED emission match peak of silicon photodiode sensitivity (850 nm wavelength)
- 10-20% efficient — transmit current up to 0.5A
- Laser diodes also used — 30-70% efficient
- Laser diodes must be rendered eye-safe — diffuser

Receive Photodiode

- Reverse-biased — light creates electron-hole pairs in depletion region
- R is responsivity of diode — 0.6 implies 60% of photons collected result in current flow
- Optical filters and concentrators can be used (reduces required size of photodiode)
- Usually off-chip photodiode used since received power is proportional to photodiode area
- Large photodiode results in large diode capacitance
- Large input dynamic range (perhaps 100 dB) required.

Infrared Channels

Eye safety

- Need to limit optical power for eye safety
- Optical power proportional to Tx signal current
- Receive signal current proportional to optical power
- Optical power proportional to ***square*** of receive signal power
- Makes design different that conventional channels

Example

- 10 dB loss in optical power ($100\mu W \rightarrow 10\mu W$)
- 20 dB loss in Rx current ($100nA \rightarrow 10nA$)
- If noise remains unchanged, twice dB loss
- Reason for low range operation

Noise

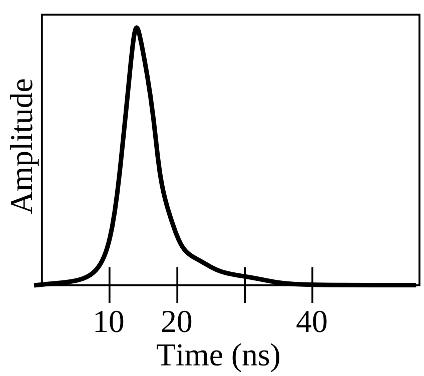
- Ambient light typically much larger than infrared light resulting in a large dc bias Rx current
- Main source of noise — shot noise from ambient light on photodiode
- Modelled as white but can be shaped by preamp (increases at higher freq)
- Independent of Rx signal
- Also noise from fluorescent lamp ballasts — 20kHz and harmonics
- Fluorescent lamp noise likely to become worse as ballast frequencies increase

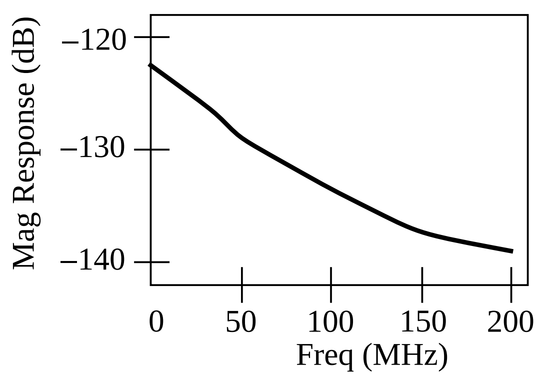
Diffuse Channel

- Infrared behaves same as visible light
- No worry about multipath fading since wavelength is so small.
- Multpath dispersion does exist and limits channels to 10-50MHz
- Most reflective walls are modelled as Lambertain reflectors — incident light re-radiated in all directions
- Results in multipath that is hard to shadow
- Results in time-dispersion (i.e. lowpass filtering)
- Use an LED with spatial dispersion
- Use photodiode with optical filter + concentrator

Diffuse Channel

- Typical room responsel

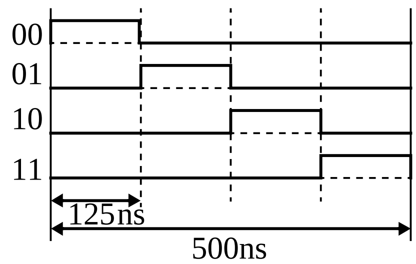




- Diffuse system with no LOS
- Rolls off steadily at high frequencies

Pulse-Position Modulation

- In 1993, IrDA (Infrared Data Association) formed
- 4.0 Mb/s standard uses 4-PPM



- Type of orthogonal modulation
- In general, L-PPM has each symbol having L time slots
- Power transmitted in one time slot and zero otherwise

Pulse-Position Modulation

Advantage

- Average power requirement decreases with increasing L

Disadvantages

- Higher bandwidth required
- Increased peak-power requirement
- Requires both time-slot and symbol-level synchronization

Soft-Decoding

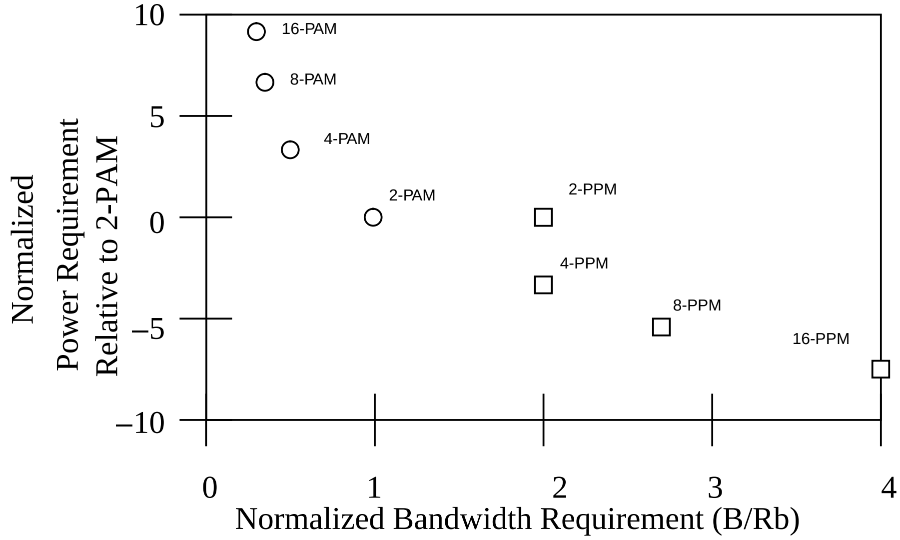
- Choose largest of L samples

Hard-Decoding

- Each sample quantized to 0 or 1 (1.5 dB penalty)

Different Modulation Sc hemes

- For optical channel (intensity modulation)



IR Systems

Present Systems

- 4Mb/s 4-PPM over LOS systems
- dc rejection to combat fluorescent lighting noise
- Little equalization
- Silicon PIN diode roughly 1 cm^2

Future Trends

- Lower cost laser diodes with diffusers
- PAM modulation for higher data-rates
- 50Mb/s diffuse system demonstrated but not integrated (2-PAM)
- DFE and Max Likelihood Sequence Detectors (MLSD) used to combat intersymbol interference

Analog Equalization

Analog Filters

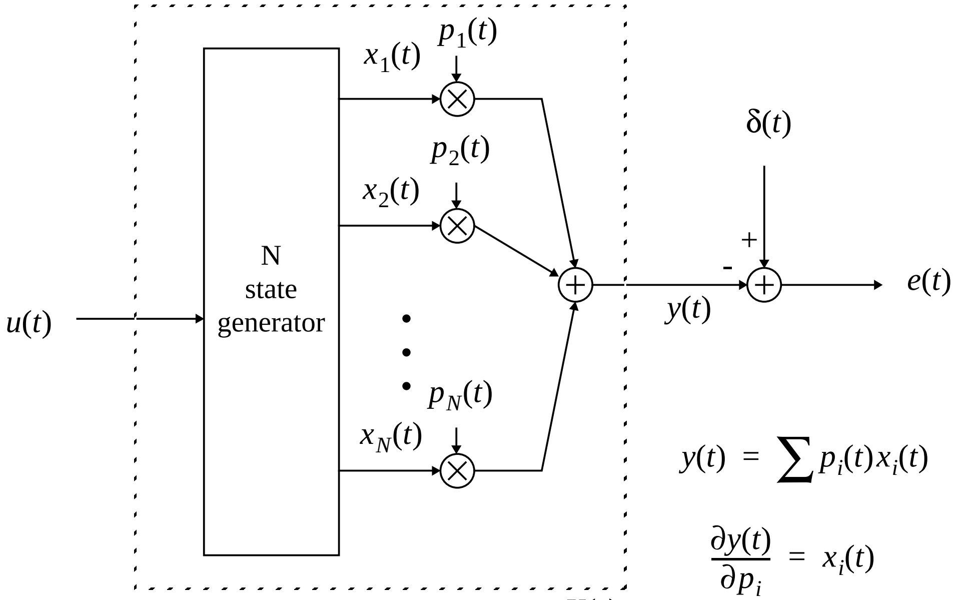
Switched-capacitor filters

+ Accurate transfer-functions
+ High linearity, good noise performance
- Limited in speed
- Requires anti-aliasing filters

Continuous-time filters

- Moderate transfer-function accuracy (requires tuning circuitry)
- Moderate linearity
+ High-speed
+ Good noise performance

Adaptive Linear Combiner



$$y(t) = \sum p_i(t)x_i(t)$$

$$\frac{\partial y(t)}{\partial p_i} = x_i(t)$$

$$H(s) = \frac{Y(s)}{U(s)}$$

Adaptive Linear Combiner

- The gradient signals are simply the state signals
- If coeff are updated in discrete-time

$$p_i(n+1) = p_i(n) + 2\mu e(n)x_i(n) \tag{120}$$

- If coeff are updated in cont-time

$$p_i(t) = \int_0^{\infty} 2\mu e(t)x_i(t)dt \tag{121}$$

- Only the zeros of the filter are being adjusted.
- There is no need to check that for filter stability (though the adaptive algorithm could go unstable if μ is too large).

Adaptive Linear Combiner

- The ***performance surface is guaranteed unimodal*** (i.e. there is only one minimum so no need to worry about being stuck in a local minimum).
- The performance surface becomes ill-conditioned as the state-signals become correlated (or have large power variations).

Analog Adaptive Linear Combiner

- Better to use input summing rather than output summing to maintain high speed operation
- Requires extra gradient filter to obtain gradients

Analog Adaptive Filters

Analog Equalization Advantages

- Can eliminate A/D converter
- Reduce A/D specs if partial equalization done first
- If continuous-time, no anti-aliasing filter needed
- Typically consumes less power and silicon for high-frequency low-resolution applications.

Disadvantages

- Long design time (difficult to “shrink” to new process)
- More difficult testing
- DC offsets can result in large MSE (discussed later).

Analog Adaptive Filter Structures

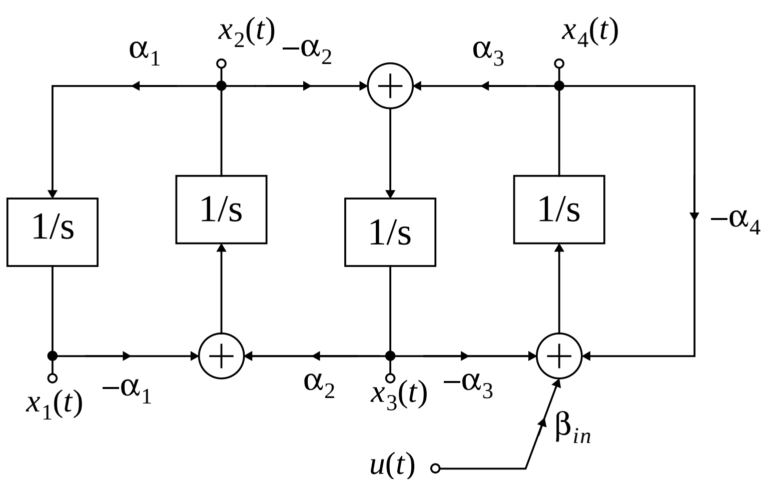
- Tapped delay lines are difficult to implement in analog.

To obtain uncorrelated states:

- Can use Laguerre structure — cascade of allpass first-order filters — poles all fixed at one location on real axis

- For arbitrary pole locations, can use orthonormal filter structure to obtain uncorrelated filter states [Johns, CAS, 1989].

Orthonormal Lad der Structure



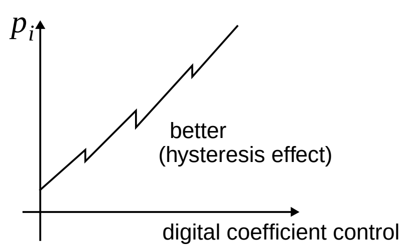
- For white noise input, all states are uncorrelated and have equal power.

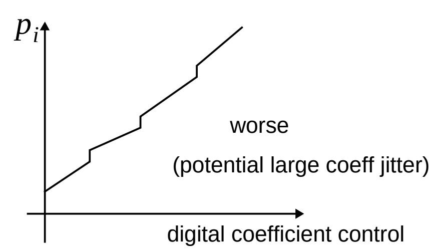
Analog's Big Advantage

- In digital filters, programmable filter has about same complexity as a fixed filter (if not power of 2 coeff).
- In analog, arbitrary fixed coeff come for free (use element sizing) but programming adds complexity.
- In continuous-time filters, frequency adjustment is required to account for process variations — relatively simple to implement.
- ***If channel has only frequency variation — use arbitrary fixed coefficient analog filter and adjust a single control line for frequency adjustment.***
- Also possible with switched-C filter by adjusting clock frequency.

Analog Adaptive Filters

- Usually digital control desired — can switch in caps and/or transconductance values
- Overlap of digital control is better than missed values

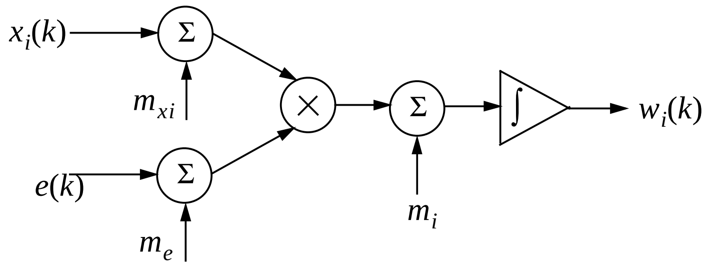




- In switched-C filters, some type of multiplying DAC needed.
- Best fully-programmable filter approach is not clear

Analog Adaptive Filters — DC Offsets

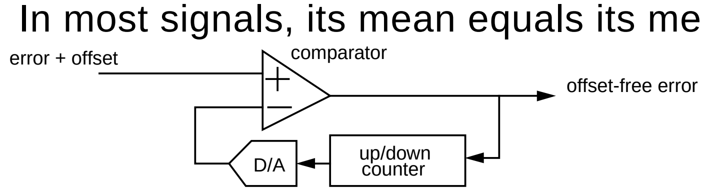
- DC offsets result in partial correlation of data and error signals (opposite to opposite DC offset)



- At high-speeds, offsets might even be larger than signals (say, 100 mV signals and 200mV offsets)
- DC offset effects worse for ill-conditioned performance surfaces

Analog Adaptive Filters — DC Offsets

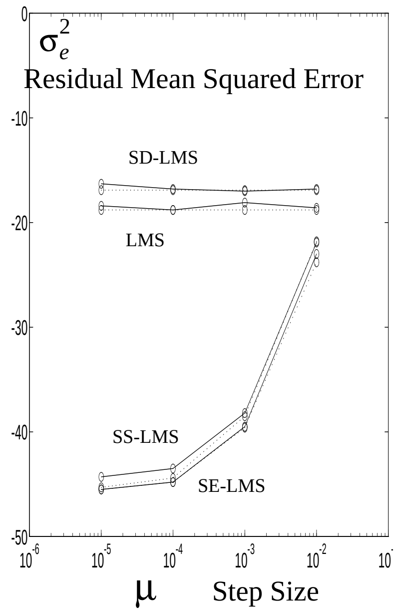
- Sufficient to zero offsets in either error or state-signals (easier with error since only one error signal)
- For integrator offset, need a high-gain on error signal
- Use ***median-offset cancellation*** — slice error signal and set the median of output to zero
- In most signals, its mean equals its median



- Experimentally verified (low-frequency) analog adaptive with DC offsets more than twice the size of the signal.

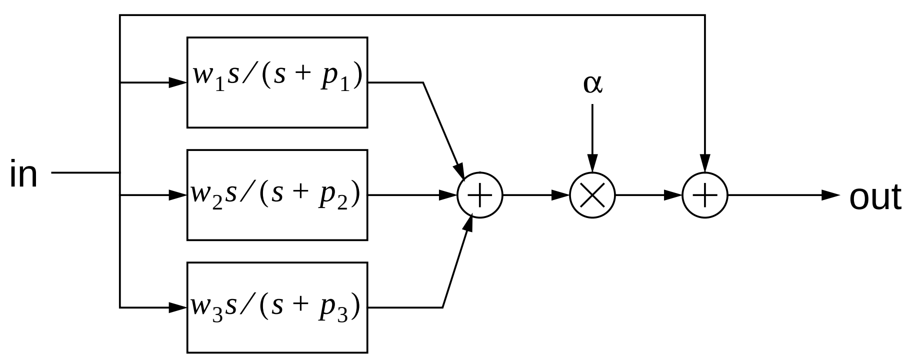
DC Offset Effects for LMS Variants

Test Case	LMS	SD-LMS	SE-LMS	SS-LMS
input power	$\sigma_e^2 \propto 1/\sigma_x^2$	no effect	$\sigma_e^2 \propto 1/ln[\sigma_x^2]$	no effect
no offsets	$\sigma_e^2 \to 0$ for $\mu \to 0$	$\sigma_e^2 \to 0$ for $\mu \to 0$	$\sigma_e^2 \propto \mu^2 \sigma_x^4$	$\sigma_e^2 \propto \mu^2 \sigma_x^2$
	σ_e^2 weakly depends on μ		σ_e^2 strongly depends on μ	
algorithm circuit complexity	1 multiplier/tap 1 integrator/tap	1 slicer/tap 1 trivial multiplier/tap 1 integrator/tap	1 trivial multiplier/tap 1 integrator/tap 1 slicer/filter	1 slicer/tap 1 XOR gate/tap 1 counter/tap 1 DAC/tap 1 slicer/filter
convergence	no gradient misalignment	gradients misaligned	no gradient misalignment	gradients misaligned



Coax Cable Equalizer

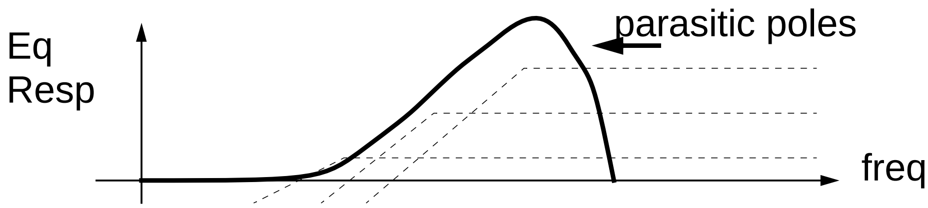
- Analog adaptive filter used to equalize up to 300m
- Cascade of two 3'rd order filters with a single tuning control



highpass filters

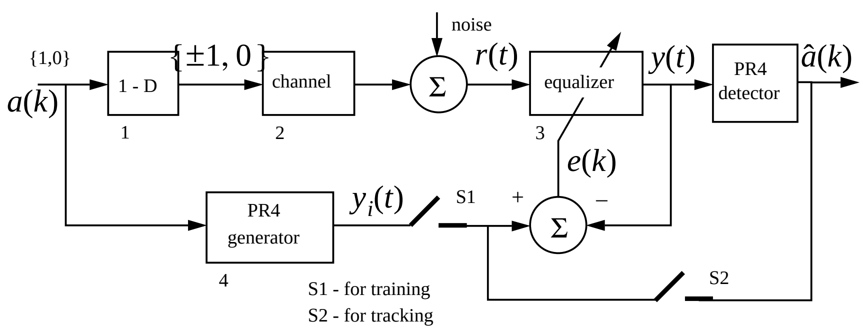
- Variable α is tuned to account for cable length

Coax Cable Equaliz er



- Equalizer optimized for 300m
- Works well with shorter lengths by tuning α
- Tuning control found by looking at slope of equalized waveform
- Max boost was 40 dB
- System included dc recovery circuitry
- Bipolar circuit used — operated up to 300Mb/s

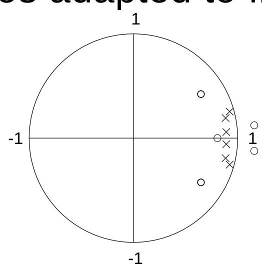
Analog Adaptive Equalization Simulation

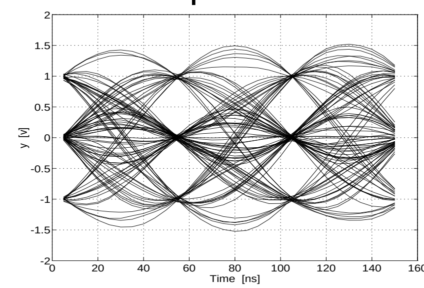


- Channel modelled by a 6’th-order Bessel filter with 3 different responses — 3MHz, 3.5MHz and 7MHz
- 20Mb/s data
- PR4 generator — 200 tap FIR filter used to find set of fixed poles of equalizer
- Equalizer — 6’th-order filter with fixed poles and 5 zeros adjusted (one left at infinity for high-freq roll-off)

Analog Adaptive Equalization Simulation

- Analog blocks simulated with a 200MHz clock and bilinear transform.
- Switch S1 closed (S2 open) and all poles and 5 zeros adapted to find a good set of fixed poles.





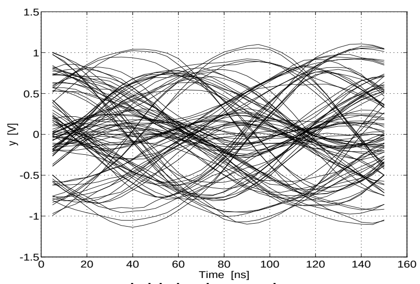
- Poles and zeros depicted in digital domain for equalizer filter.
- Residual MSE was -31dB

Equalizer Simulation — Decision Directed

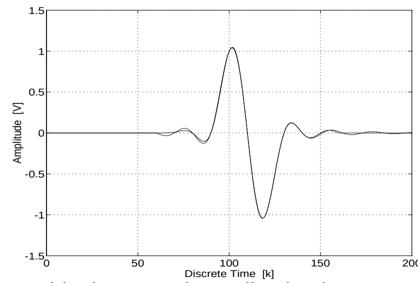
- Switch S2 closed (S1 open), all poles fixed and 5 zeros adapted using
  - $e(k) = 1 - y(t) \quad \text{if} \quad (y(t) > 0.5)$
  - $e(k) = 0 - y(t) \quad \text{if} \quad (-0.5 \le y(t) \le 0.5)$
  - $e(k) = -1 - y(t) \quad \text{if} \quad (y(t) < -0.5)$
- all sampled at the decision time — assumes clock recovery perfect
- Potential problem — AGC failure might cause $y(t)$ to always remain below ± 0.5 and then adaptation will force all coefficients to zero (i.e. $y(t) = 0$).
- Zeros initially mistuned to significant eye closure

Equalizer Simulation — Decision Directed

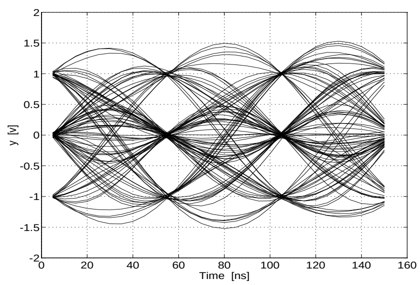
- 3.5MHz Bessel



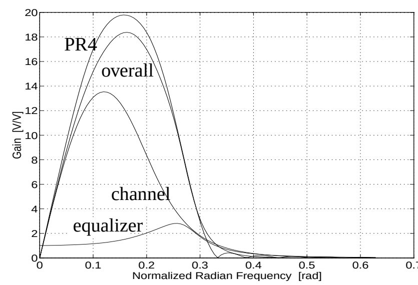
initial mistuned



ideal PR4 and equalized pulse outputs



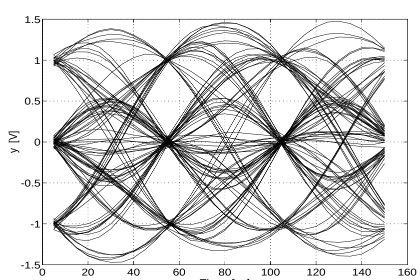
after adaptation (2e6 iterations)



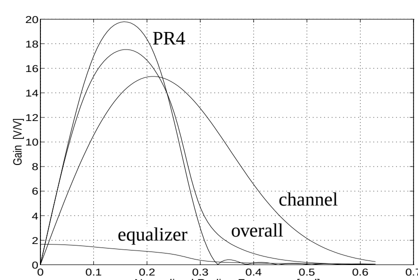
after adaptation (2e6 iterations)

Equalizer Simulation — Decision Directed

- Channel changed to 7MHz Bessel
- Keep same fixed poles (i.e. non-optimum pole placement) and adapt 5 zeros.



after adaptation (2e6 iterations)

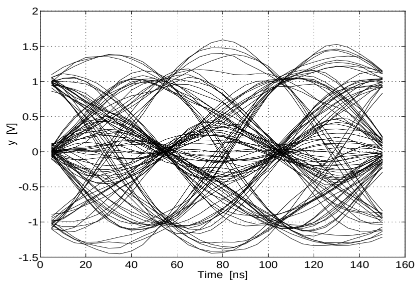


after adaptation (2e6 iterations)

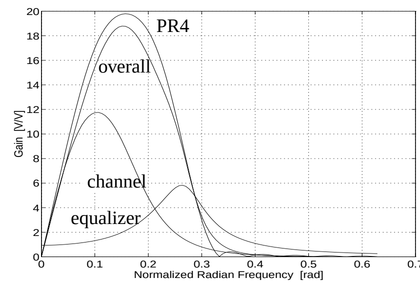
- Residual MSE = -29dB
- Note that no equalizer boost needed at high-freq.

Equalizer Simulation — Decision Directed

- Channel changed to 3MHz Bessel
- Keep same fixed poles and adapt 5 zeros.



after adaptation (3e6 iterations)



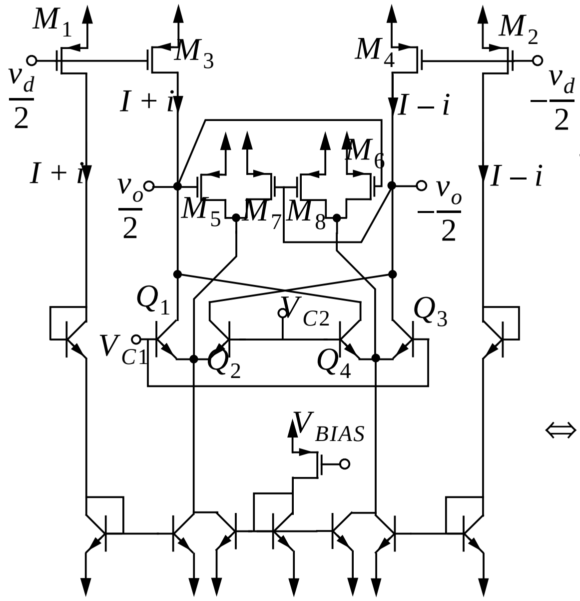
after adaptation (3e6 iterations)

- Residual MSE = -25dB
- Note that large equalizer boost needed at high-freq.
- Probably needs better equalization here (perhaps move all poles together and let zeros adapt)

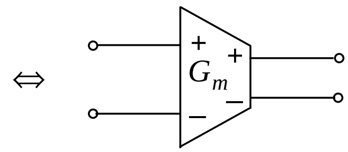
BiCMOS Analog Adaptive Filter Example

- Demonstrates a method for tuning the pole-frequency and Q-factor of a 100MHz filter — adaptive analog
- Application is a pulse-shaping filter for data transmission.
- One of the fastest reported integrated adaptive filters — it is a Gm-C filter in 0.8um BiCMOS process
- Makes use of MOS input stage and translinear-multiplier for tuning
- Large tuning range (approx. 10:1)
- All analog components integrated (digital left off)

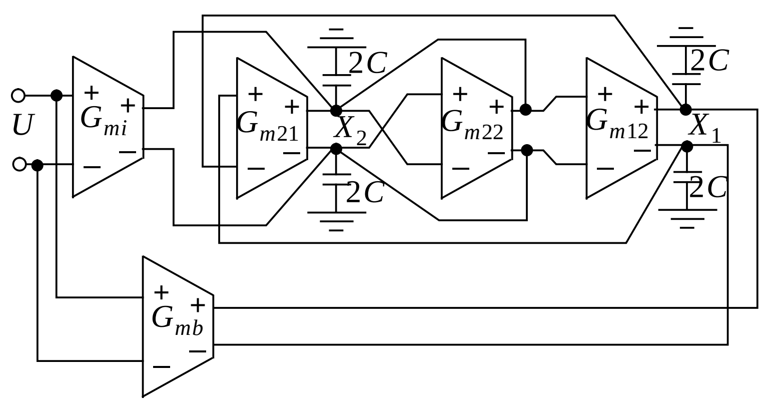
BiCMOS Transconductor



Two styles implemented:
"2-quadrant" tuning (F-Cell),
"4-quadrant" tuning by cross-coupling top input stage (*Q*-Cell)



Biquad Filter

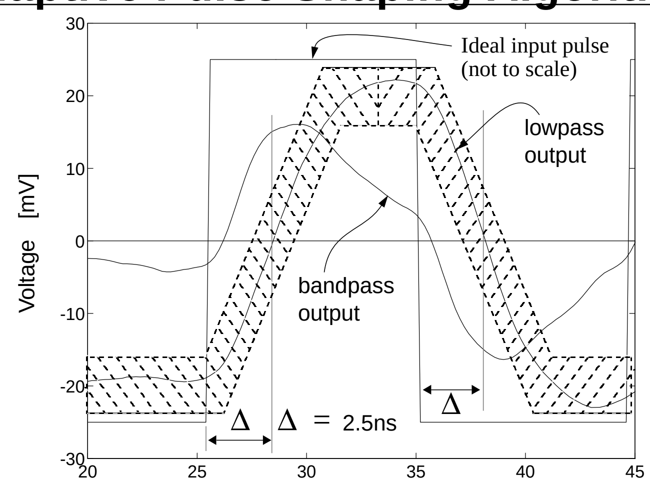


- fo and Q not independent due to finite output conductance
- Only use 4 quadrant transconductor where needed

Experimental Results Summary

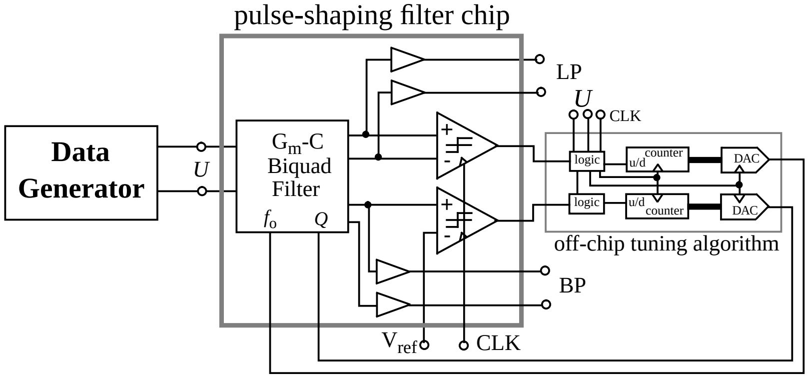
Transconductor (T.) size	0.14mm x 0.05mm	
T. power dissipation	10mW @ 5V	
Biquad size	0.36mm x 0.164mm	
Biquad worst case CMRR	20dB	
Biquad f_c tuning range	10MHz-230MHz @ 5V, 9MHz-135MHz @ 3V	
Biquad Q tuning range	1-Infinity	
Bq. inpt. ref. noise dens.	$0.21\mu V_{rms}/\sqrt{Hz}$	
Biquad PSRR+	28dB	
Biquad PSRR-	21dB	
Filter Setting	**Output 3rd Order Intercept Point**	**SFDR**
100MHz, $Q = 2$, Gain = 10.6dB	23dBm	35dB
20MHz, $Q = 2$, Gain = 30dB	20dBm	26dB
100MHz, $Q = 15$, Gain = 29.3dB	18dBm	26dB
227MHz, $Q = 35$, Gain = 31.7dB	10dBm	20dB

Adaptive Pulse Shaping Algorithm



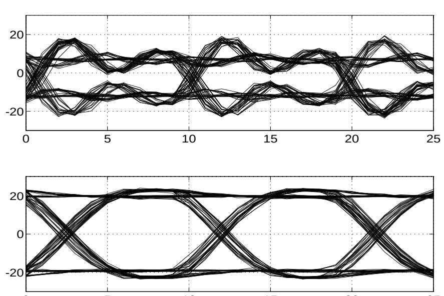
- Fo control: sample output pulse shape at nominal zero-crossing and decide if early or late (cutoff frequency too fast or too slow respectively)
- Q control: sample bandpass output at lowpass nominal zero-crossing and decide if peak is too high or too small (Q too large or too small)

Experimental Setup

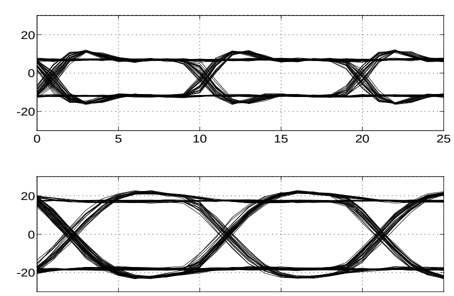


- Off-chip used an external 12 bit DAC.
- Input was 100Mb/s NRZI data 2Vpp differential.
- Comparator clock was data clock (100MHz) time delayed by 2.5ns

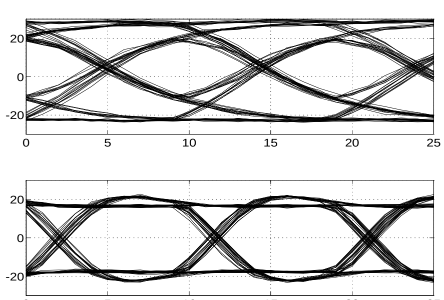
Pulse Shaper Responses



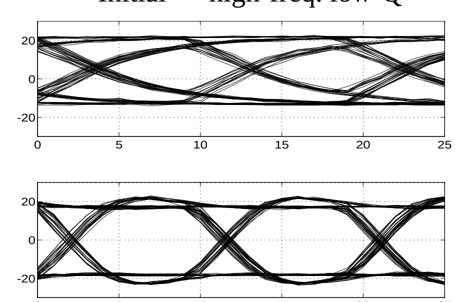
Initial — high-freq. high-Q



Initial — high-freq. low-Q



Initial — low-freq. high-Q



Initial — low-freq. low-Q

Summary

- Adaptive filters are relatively common
- LMS is the most widely used algorithm
- Adaptive linear combiners are almost always used.
- Use combiners that do not have poor performance surfaces.
- Most common digital combiner is tapped FIR

Digital Adaptive:

- more robust and well suited for programmable filtering

Analog Adaptive:

- best suited for high-speed, low dynamic range.
- less power
- very good at realizing arbitrary coeff with frequency only change.
- Be aware of DC offset effects

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