Electromigration Median Time-to-Failure based on a Stochastic Current Waveform*

by

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Abstract

This paper deals with the estimation of the median time-to-failure (MTF) due to electromigration in the power and ground busses of VLSI circuits. In our previous work [4, 5], we had presented a novel technique for MTF estimation based on a *stochastic current waveform* model. We had derived the *mean (*or *expected) waveform* (not a time average) of such a current model and conjectured that it is the appropriate current waveform to be used for MTF estimation. This paper proves that conjecture, and presents new theoretical results which show the exact relationship between the MTF and the statistics of the stochastic current. This leads to a more accurate technique for deriving the MTF which requires the *variance waveform* of the current, in addition to its mean waveform. We then show how the variances of the bus branch currents can be derived from those of the gate currents, and describe several simplifying approximations that can be used to maintain efficiency and, therefore, make possible the analysis of VLSI circuits.

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1 Introduction

Reliability is becoming a major concern in integrated circuit design. As higher levels of integration are used, metal line width and line separation will decrease, thereby increasing a chip's susceptibility to failures resulting from line shorts or opens. This indicates that the importance of reliability can only increase in the future.

This paper addresses *electromigration* [1, 2] (EM), which is a major reliability problem caused by the transport of atoms in a metal line due to the electron flow. Under persistent current stress, EM can cause deformations of the metal lines which may result in shorts or open circuits. The failure rate due to EM depends on the current density in the metal lines and is usually expressed as a median time-to-failure (MTF). There is a need for CAD tools that can predict the susceptibility of a given design to EM failures.

We focus on the power and ground busses. To estimate the bus MTF, an estimate of the current waveform in each branch of the bus is required. In general, the MTF is dependent on the *shape* of the current waveform [3], and not simply on its time-average. However, a very large number of such waveform shapes are possible, depending on what inputs are applied to the circuit. This is especially true for CMOS circuits, which draw current only during switching. It is not clear, therefore, which current waveform(s) should be used to estimate the MTF.

The argument presented in [4, 5] is that the correct current waveform to be used for MTF estimation is one that combines (in some sense) the effects of all possible logic input waveforms. By considering the set of logic waveforms allowed at the circuit inputs as a *probability space* [6], the current in any branch of the bus becomes a *stochastic process* [6]. CREST derives the *mean waveform* (not a time-average) of this process, which we call an *expected current waveform*. This is a waveform whose value at a given time is the weighted average of all possible current values at that time, as shown in Fig. 1. CREST uses *statistical* information about the inputs to directly derive the expected current waveform. The resulting methodology is what we call a *probabilistic simulation* of the circuit.

The feasibility of deriving the expected current waveform was established in [4, 5], where we also conjectured that such a waveform is the appropriate current waveform to be used for MTF estimation. In section 2 of this paper, we prove this conjecture and derive the exact relationship between the MTF and the statistics of the stochastic current. This leads to an efficient and more accurate technique for deriving the MTF, which requires the *variance waveform* of the stochastic current as well.

The derivation of the variance waveform is discussed in section 3. We present a methodology by which the variance waveforms of the bus branch currents can be obtained from those of the gate currents. We also present several approximations that can be used when handling large chips to simplify the variance computations, and thus make it possible to handle VLSI circuits. Section 4 draws some conclusions.

2 Stochastic Current Waveforms and the MTF

Consider a metal line of uniform width and thickness carrying a *constant* current. The relationship between the MTF, t_{50}, due to electromigration in the metal line and the current density, j, has been extensively studied, and shown to be a complex nonlinear function [7], as shown in Fig. 2. We will consider the MTF to be $t_{50} \propto 1/f(j)$ where j is in A/cm^2, and f is a dimensionless nonlinear function, whose plot is shown in Fig. 3, which was derived from Fig. 2.

If a metal line carries a *varying* current, of density $j(t)$, then the MTF is $t_{50} \propto 1/J_{\text{eff}}$, where J_{eff} depends both on f and on the waveform shape of $j(t)$. It has been suggested [3] that, if the waveform is periodic with period T and consists of a train of pulses $k = 1, \ldots, m$ of heights j_k and duration t_k, then :

$$J_{\text{eff}} = \sum_{k=1}^{m} \frac{t_k}{T} f(j_k). \tag{1}$$

Since this strategy is conservative, as explained in [3], we will adopt it. For a general periodic waveform, we take the summation to the limit and write :

$$J_{\text{eff}} = \frac{1}{T} \int_0^T f(j)dt. \tag{2}$$

If the current waveform is not periodic, then better estimates of J_{eff} are obtained by using larger values of T so that more features of the waveform are included. Therefore one can write :

$$J_{\text{eff}} = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(j)dt. \tag{3}$$

Now suppose that the current waveform is *stochastic*, i.e., it is a stochastic process $\mathbf{j}(t)$, that represents a family of deterministic (real) current waveforms $j_k(t)$, with associated probabilities P_k, $k = 1, \ldots, N$, over the (finite) interval $[0, t_0]$. Based on this information, we can build a (non-stochastic) current waveform $j(t)$, over $[0, T]$ as $T \to \infty$, that is indicative of the current during typical operation as follows. Consider a random sequence of the waveforms $j_k(t)$, each being shifted in time, spanning an interval of length t_0, and occurring with its assigned probability P_k, as shown in Fig. 4. Let $n_k(T)$ be the (integer) number of occurrences of the waveform $j_k(t)$ in $[0, T]$, and let $n_T = \lfloor T/t_0 \rfloor$. If J_k, $k = 1, \ldots, N$ are defined as follows :

$$J_k \triangleq \frac{1}{t_0} \int_0^{t_0} f(j_k)dt, \tag{4}$$

then :

$$J_{\text{eff}} = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(j)dt = \lim_{n_T\to\infty} \sum_{k=1}^{N} J_k \frac{n_k(T)}{n_T} = \sum_{k=1}^{N} J_k \lim_{n_T\to\infty} \left[\frac{n_k(T)}{n_T}\right]. \tag{5}$$

By the law of large numbers [6], $\lim_{n_T\to\infty}[n_k(T)/n_T] = P_k$, which leads to :

$$J_{\text{eff}} = \sum_{k=1}^{N} \left[\frac{1}{t_0} \int_0^{t_0} f(j_k)dt\right] P_k = \frac{1}{t_0} \int_0^{t_0} \left[\sum_{k=1}^{N} f(j_k)P_k\right] dt \tag{6}$$

and, finally :

$$J_{\text{eff}} = \frac{1}{t_0} \int_0^{t_0} E\left[f(\mathbf{j})\right] dt \tag{7}$$

where $E[\ ]$ denotes the expected value operator [6]. This is an important result; it says that *the MTF due to a stochastic current depends only on the expected waveform of a nonlinear function of the current.*

Since f is nonlinear, $E\left[f(\mathbf{j})\right]$ is not easy to evaluate. At low current values, where f is linear (Fig. 3), $E\left[f(\mathbf{j})\right] = f(E[\mathbf{j}])$. If this is substituted in (7) and compared with (2) it shows that, when f is linear, the expected current waveform $E[\mathbf{j}]$ derived in [4, 5] may itself be used as the current waveform $j(t)$ in (2) for MTF estimation. This establishes the importance of the expected current waveform for electromigration failure analysis. In general, f is nonlinear, and a generalized approach will be developed below.

At any time t, the process $\mathbf{j}(t)$ can be thought of as a random variable $\mathbf{j}$ with mean $\eta_j \triangleq E[\mathbf{j}]$, and variance $\sigma_j^2 \triangleq E\left[(\mathbf{j}-\eta_j)^2\right]$. In general, the p^{th} moment of $\mathbf{j}$ is $\mu_{j,p} \triangleq E\left[(\mathbf{j}-\eta_j)^p\right]$. To estimate the mean of $f(\mathbf{j})$, $E\left[f(\mathbf{j})\right]$, we resort to a Taylor series expansion of f, which leads to equation (5-54) in [6], reproduced here for convenience :

$$E\left[f(\mathbf{j})\right] \approx f(\eta_j) + f''(\eta_j)\frac{\sigma_j^2}{2} + \cdots + f^{(p)}(\eta_j)\frac{\mu_{j,p}}{p!}. \tag{8}$$

It is evident that, when f is linear, (8) reduces to :

$$E\left[f(\mathbf{j})\right] = f(E[\mathbf{j}]), \tag{9}$$

as observed above. Hence using the expected current waveform as an actual current waveform for MTF estimation based on (2) amounts to making a first-order approximation in (8). Naturally, higher order approximations would lead to better results. In particular, if f is approximated by a quadratic in the neighborhood of η_j, then :

$$E\left[f(\mathbf{j})\right] \approx f(\eta_j) + f''(\eta_j)\frac{\sigma_j^2}{2}. \tag{10}$$

This second-order approximation becomes exact if $f(j)$ is represented by the straight lines corresponding to j^1 and j^2 in Fig. 3. It is more accurate than (9) since it covers a wider range of currents. As a result, *equations (10) and (7) offer a new, more accurate technique for computing the MTF*. In order to make use of this technique, we need to derive the variance of the current waveform in addition to its expected value. As pointed out in the introduction, the estimation of the expected current waveform has already been described in our previous work [4, 5]; the next section will discuss the derivation of the variance.

3 Estimating the Variance

Since the current density $\mathbf{j}(t)$ in any branch of the power or ground bus is directly proportional to the current $\mathbf{i}(t)$ in that branch, then, to simplify the presentation, we will discuss the derivation of σ_i^2 rather than σ_j^2. The variance of the current waveform is a time function $\sigma_i^2(t)$ which we will refer to as the *variance waveform*. Furthermore, we will discuss the power bus only since the ground bus analysis is similar.

The current in a branch of the bus, $\mathbf{i}(t)$, is a function of the currents being drawn off the bus contacts, $\mathbf{i_j}(t), j = 1, \ldots, n$. Each of these is, in-turn, simply the sum of the individual gate currents tied to each contact :

$$\mathbf{i_j}(t) = \mathbf{i_{j1}}(t) + \cdots + \mathbf{i_{jk}}(t). \tag{11}$$

Thus, in the framework of our probabilistic simulation technique, the process of deriving the variance waveforms consists of three steps :

-1- Using the statistics of the signals at the inputs to each logic gate, derive the variance waveform for its current.

-2- Combine these at each contact point to derive the variances of the contact currents.

-3- Using the bus topology, and the variances of the contact currents, derive the variances of the bus branch currents.

Due to lack of space, the details of step 1, along with its implementation in the probabilistic simulator CREST, will be presented in a forth-coming paper [8] (as well as in [9]). The other two steps will be described below.

The critical issue is the correlation between the different current waveforms. Since such correlation is too expensive to derive for VLSI circuits, we will occasionally be making conservative approximations to simplify the problem. Our experience with the probabilistic simulation approach [4, 5, 9] suggests that neglecting the correlation between different current waveforms gives good results in most cases.

Based on this, we assume that the gate currents tied to the same contact are *uncorrelated.* This immediately provides a simple solution for step 2, using (11), as follows :

$$\sigma^2_{i_j}(t) = \sigma^2_{i_{j1}}(t) + \cdots + \sigma^2_{i_{jk}}(t). \tag{12}$$

The remainder of this section will be devoted to the more difficult task of solving step 3, i.e., deriving the bus current variance waveforms from those of the contact currents.

The metal bus can be modeled as a multi-input multi-output, causal, *linear*, time-invariant, (LTI) system with causal inputs $\mathbf{x_j}$ and outputs $\mathbf{y_i}$. The inputs $\mathbf{x_j}(t), j = 1, \ldots, n$ represent the contact currents, and carry the stochastic processes $\mathbf{i_j}(t)$ of known variance waveforms $\sigma^2_{i_j}(t)$. The outputs $\mathbf{y_i}(t)$, $i = 1, \ldots, m$ represent the bus branch currents at which the variance waveforms, $\sigma^2_{y_i}(t)$, are required. Let $h_{ij}(t)$ be the impulse response function relating $\mathbf{y_i}(t)$ to $\mathbf{x_j}(t)$:

$$\mathbf{y_i}(t) = \sum_{j=1}^{n} h_{ij}(t) * \mathbf{x_j}(t), \qquad i = 1, \ldots, m \tag{13}$$

where "$*$" denotes the *convolution* operation.

It is well known (see [6], page 209) that the variances of the system inputs are not enough to derive the variances of its outputs. The *auto-correlation* of each input, $R_{x_j x_j}(t_1, t_2) \triangleq E\left[\mathbf{x_j}(t_1)\mathbf{x_j}(t_2)\right]$, is also required. Since the input processes are not *wide-sense stationary* [6], an exact analytical solution can be quite complex, even if the auto-correlation were known. Therefore, as is often necessary, we will make certain simplifying assumptions about the structure of $R_{x_j x_j}$.

We will assume that the correlation between $\mathbf{x_j}(t)$ and $\mathbf{x_j}(t+\tau)$ goes to zero as $\tau \to \infty$. In terms of the *auto-covariance*, $C_{x_j x_j}(t_1, t_2) \triangleq R_{x_j x_j}(t_1, t_2) - \eta_{x_j}(t_1)\eta_{x_j}(t_2)$, this will be formulated as :

$$C_{x_j x_j}(t_1, t_1) = \sigma^2_{x_j}(t_1), \text{ and } C_{x_j x_j}(t_1, t_2) = 0 \text{ for } |t_1 - t_2| \geq T, \tag{14}$$

where T is a (typically small) time interval.

Consider the *discrete time* system obtained by sampling, with period T, the continuous time system defined by (13). If $\mathbf{x_j}[k] \triangleq \mathbf{x_j}(kT)$ are the discrete processes at the inputs, and $\mathbf{y_i}[k] \triangleq \mathbf{y_i}(kT)$ are the discrete output processes, then :

$$\mathbf{y_i}[k] = \sum_{j=1}^{n} h^{(d)}_{ij}[k] * \mathbf{x_j}[k], \qquad i = 1, \ldots, m \tag{15}$$

where $h^{(d)}_{ij}[k]$ is the discrete impulse response function relating $\mathbf{y_i}[k]$ to $\mathbf{x_j}[k]$. As shown below, the discretized output variance waveforms can be derived irrespective of the shape of $C_{x_j x_j}(t_1, t_2)$ for $|t_1 - t_2| < T$. The continuous variance waveforms can then be obtained by interpolation. Strictly speaking, therefore, the sampling period T should be small : $1/T$ should be larger than the largest frequency component of the inputs. However, since fine waveform details are not of paramount importance in this work, we need only restrict T to be small enough so that waveform features in so small an interval are inconsequential.

To simplify the notation, define $\mathbf{y_{ij}}[k] \triangleq h_{ij}^{(d)}[k] * \mathbf{x_j}[k]$. Furthermore, as pointed out above, we will neglect the correlation between the contact currents. Hence the $\mathbf{x_j}$ inputs are uncorrelated, and :

$$\sigma_{y_i}^2[k] = \sum_{j=1}^{n} \sigma_{y_{ij}}^2[k], \qquad i = 1, \ldots, m. \tag{16}$$

We have thus reduced the problem to analyzing a single-input single-output discrete LTI system :

$$\mathbf{y_{ij}}[k] = h_{ij}^{(d)}[k] * \mathbf{x_j}[k] = \sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa]\mathbf{x_j}[k-\kappa]. \tag{17}$$

Let $\tilde{\mathbf{x}}_{\mathbf{j}}[k] \triangleq \mathbf{x_j}[k] - \eta_{x_j}[k]$ and $\tilde{\mathbf{y}}_{\mathbf{ij}}[k] \triangleq \mathbf{y_{ij}}[k] - \eta_{y_{ij}}[k]$. Then $\sigma_{y_{ij}}^2[k] = E[\tilde{\mathbf{y}}_{\mathbf{ij}}[k]^2]$ and $\tilde{\mathbf{y}}_{\mathbf{ij}}[k] = h_{ij}^{(d)}[k] * \tilde{\mathbf{x}}_{\mathbf{j}}[k]$, hence :

$$\sigma_{y_{ij}}^2[k] = E\left[\left(\sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa]\tilde{\mathbf{x}}_{\mathbf{j}}[k-\kappa]\right)^2\right] = \sum_{\kappa_1=0}^{\infty} h_{ij}^{(d)}[\kappa_1] \sum_{\kappa_2=0}^{\infty} h_{ij}^{(d)}[\kappa_2] E\Big[\tilde{\mathbf{x}}_{\mathbf{j}}[k-\kappa_1]\tilde{\mathbf{x}}_{\mathbf{j}}[k-\kappa_2]\Big]. \tag{18}$$

Furthermore, it is easy to see that $E\big[\tilde{\mathbf{x}}_{\mathbf{j}}[k_1]\tilde{\mathbf{x}}_{\mathbf{j}}[k_2]\big] = C_{x_j x_j}(k_1, k_2)$, which, using (14), gives :

$$\sigma_{y_{ij}}^2[k] = \sum_{\kappa=0}^{\infty} \left|h_{ij}^{(d)}[\kappa]\right|^2 \sigma_{x_j}^2[k-\kappa] = \left|h_{ij}^{(d)}[k]\right|^2 * \sigma_{x_j}^2[k]. \tag{19}$$

And, finally, the variance waveforms for the system outputs are, using (16) :

$$\sigma_{y_i}^2[k] = \sum_{j=1}^{n} \left|h_{ij}^{(d)}[k]\right|^2 * \sigma_{x_j}^2[k], \qquad i = 1, \ldots, m. \tag{20}$$

In other words, *the variances of the system outputs (bus branch currents) can be obtained from the convolution of the variances of its inputs (contact currents) with the squares of its discrete impulse response functions.* This discrete convolution can be easily performed once the discrete impulse response functions are found. Of course the summation need not be taken to infinity, and may be conveniently truncated after $\left|h_{ij}^{(d)}[\kappa]\right|$ is less than some small value. To obtain the discrete impulse response functions, note that if a unit-step input current is applied at contact j, with all other contact currents held at zero, and if the resulting outputs $y_i(t)$ are monitored, then :

$$h_{ij}^{(d)}[k] = y_i(kT) - y_i\big((k-1)T\big) = \int_{(k-1)T}^{kT} h_{ij}(\tau)d\tau, \qquad i = 1, \ldots, m. \tag{21}$$

This suggests two methods for deriving $h_{ij}^{(d)}[k]$. The first uses a simulation program such as SPICE to simulate the bus with unit-step input currents applied at each contact (one at a time), while monitoring the bus branch currents. This gives the mn functions $h_{ij}^{(d)}[k]$ using (21). Another (approximate) method would be to make use of the second equality in (21) : if the continuous impulse response functions are approximated using some RC time-constant analysis of the bus, then the discrete impulse response functions can be obtained from them.

For very large chips, it may be prohibitively expensive to perform the required convolutions. One can simplify the calculations by making an additional assumption as follows. If the bus is known to be "fast", i.e., if $h_{ij}^{(d)}[k]$ dies down faster than changes in $\sigma_{x_j}^2[k]$, then (19) reduces to :

$$\sigma_{y_{ij}}^2[k] \approx \sigma_{x_j}^2[k] \sum_{\kappa=0}^{\infty} \left| h_{ij}^{(d)}[\kappa] \right|^2. \tag{22}$$

So the convolutions in (20) can be replaced by simple multiplications, and the constants $\sum_{\kappa=0}^{\infty} \left| h_{ij}^{(d)}[\kappa] \right|^2$ can be derived in a pre-processing step from the impulse response functions and stored in a single $m \times n$ constant matrix.

If the chip is too big to even derive $h_{ij}^{(d)}[k]$, then one further simplification can be made as follows. If $h_{ij}^{(d)}[k]$ dies down faster than changes in $\mathbf{x_j}[k]$ then (17) reduces to $\mathbf{y_{ij}}[k] = \mathbf{x_j}[k] \sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa]$, and so :

$$\sigma_{y_{ij}}^2[k] \approx \sigma_{x_j}^2[k] \Big(\sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa] \Big)^2. \tag{23}$$

The constants $\left(\sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa] \right)^2$ can be very easily obtained as follows. Note that $\sum_{\kappa=0}^{\infty} h_{ij}^{(d)}[\kappa]$ is the *steady state* current in branch i in response to a unit-step input current at contact j, with all other contact currents held at zero. If the bus is modeled as a resistive network, then the steady state node voltages in response to such inputs are the entries of the driving point impedance matrix. So if the node-admittance matrix is built by simple inspection of the bus and then inverted to produce the driving point impedance matrix, the steady state currents are immediately available.

4 Conclusions

In conclusion, we have looked at the problem of estimating the MTF due to electromigration in the power and ground busses of VLSI circuits. In our previous work [4, 5], we had presented a novel technique for MTF estimation based on a stochastic current waveform model. We had derived the mean (or expected) waveform (not a time average) of such a current model and conjectured that it is the appropriate current waveform to be used for MTF estimation. This paper proves that conjecture and presents new theoretical results which show the exact relationship between the MTF and the statistics of the stochastic current. We prove the following two main results. The first, equation (7), relates the J_{eff}, required for MTF estimation, to the mean waveform of a nonlinear function of the stochastic current. Coupled with equation (10), it provides an efficient and more accurate technique for computing the median time-to-failure. The second result, equation (20), provides a technique by which the variances of bus branch currents can be derived from those of the contact currents. Such variances are required for MTF estimation in (10). Several simplifying approximations are presented that make it possible to handle VLSI circuits.

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Figure Captions :

Figure 1 : Actual current waveforms (dashed), and the corresponding expected current waveform (solid).
Figure 2 : The dependence of MTF on current density, reproduced for convenience from [7]. The dashed lines show the results of the approximation $t_{50} \propto j^{-n}$ for $n = 1$, $3/2$, and 2.
Figure 3 : A plot of $f(j)$, obtained from Fig. 2 by inverting and appropriately scaling the ordinate axis.
Figure 4 : A (non-stochastic) current waveform, $j(t)$, built as a sequence of the waveforms $j_k(t)$, each occurring with its assigned probability P_k.