Stabilization of Unstable and Unintuitive Plants by Fuzzy Control

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***Abstract*—A heuristically derived stabilization strategy for an unstable and unintuitive plant by fuzzy control is described. It is shown that the often-used classical fuzzy controller, which is both static and time-invariant, is incapable of stabilizing such types of plants. However, a simple modification to the classical fuzzy-controller architecture that separates the measurement and control phases, together with a hierarchical control strategy, enable the unstable and unintuitive plant to be stabilized. The fuzzy-control strategy, as well as the new fuzzy controller architecture, are based on the consideration of "what a human subject would do when dealing with a physical plant which is both unstable and unintuitive." The stabilization strategy is then generalized to other mathematically-similar systems. While the rules for the stabilization of the plant are heuristically defined, the membership functions associated with the rules are tuned by a simulated-annealing procedure.**

I. INTRODUCTION

MANY typical applications of fuzzy control are based on an intuitive control paradigm which is obviously good only for those plants which behave intuitively. By intuitive control, we mean that the control effort is applied in such a way as to achieve an apparent goal directly. For example, if we want to move an object lying on the floor to a point on the right-hand-side of the object, we push the object to the right. Equivalently, if we want the car in which we are sitting to move, we need to step on the gas pedal (assuming that the car is already in a condition to move).

Furthermore, the intuitive control actions should be monotonic: thus, if we want the object on the floor to move faster, we push harder; and if we want the car to move faster, we step harder on the gas pedal.

All such control actions can be described by a set of intuitive *IF-THEN* rules. This set of intuitive rules manifests itself as a monotonic control surface, when these rules, together with the associated fuzzy attributes, are implemented on a fuzzy controller. Thus, it is sometimes easy to tell, at least for the case of one or two inputs, whether inappropriate rules have been incorporated into a rulebase by inspecting the control surface. Nonmonotonicity of the control surface indicates unintuitive control action.

But, there are plants that do not behave intuitively. An example of an open-loop-stable second-order single-input-single-output linear time-invariant (LTI) plant which exhibits unintuitive behavior is

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = u - \frac{du}{dt} \tag{1.1}$$

the transfer function of which is

$$\frac{Y(s)}{U(s)} = \frac{1-s}{(s+2)(s+3)} \tag{1.2}$$

where $y(t)$ is the output of the plant, and $u(t)$ is the input to the plant, with $Y(s)$ and $U(s)$ the corresponding Laplace-transformed variables. As can be seen from the step response of the plant shown in Fig. 1, the output first undershoots (moves in the "wrong" direction) before settling to the final position. Clearly, this type of response can confuse a simple controller. In linear-control theory, such a plant is said to be stable and of nonminimum phase; it is typically difficult to achieve good feedback-control performance for such plants [1]. There are also other nonminimum-phase plants that are unstable, a well-known example is the cart–pole system which will be discussed in later sections. The unintuitive behavior of an LTI plant is often associated with the nonminimum phase property of the plant; however, not all nonminimum phase LTI plants[1] exhibit an apparent unintuitive behavior such as that shown in Fig. 1.

This paper concerns the general issue of the stabilization of unstable and unintuitive plants[2] with fuzzy control. The counterpart of such plants in linear-control theory are called open-loop unstable nonminimum-phase plants, and various stabilization techniques [2]–[4] exist for them. This paper proposes a new technique for fuzzy control which is possibly somewhat easier to understand and implement than those employed in usual linear-control approaches. The basic assumption made in the technique is that all intuitive goals are, in principle, achievable by employing a set of intuitive rules, and the idea underlying the technique originates from consideration of what a human operator would do when confronted with such

Manuscript received March 18, 1995; revised November 1, 1995.

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Publisher Item Identifier S 1083-4419(97)00018-6.

[1] For instance, the step response of a 3rd-order plant with poles at $-1,\ -2,\ -3$, and zeros at $1+j7,\ 1-j7$ does not show any apparent unintuitiveness.

[2] In this work, an unintuitive plant refers to either a plant whose response to an input signal is unintuitive (such as that shown in Fig. 1), or a plant that requires unintuitive control actions to achieve a given goal.

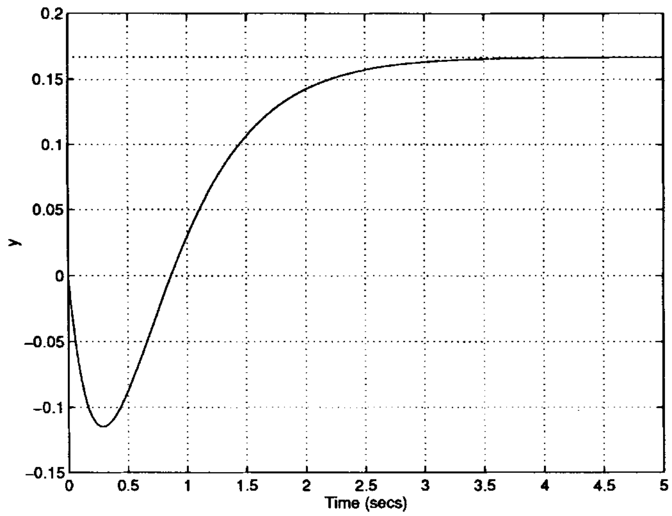

Fig. 1. Step response of (1.1).

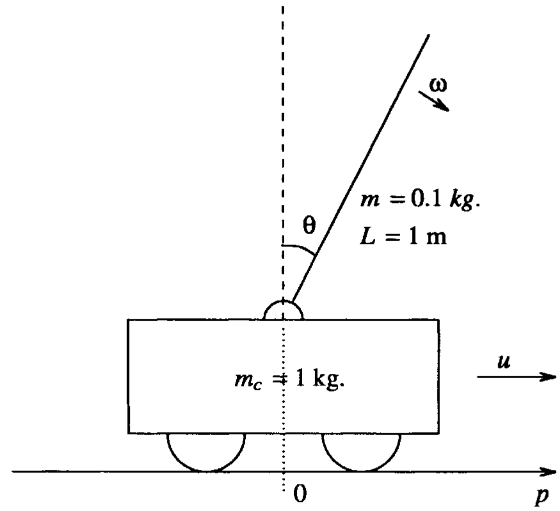

Fig. 2. A cart–pole system.

a plant. This approach, in a way, is very much in accordance with the original spirit of fuzzy control which was premised on a human operator's physical insight and intuition of how a plant should behave. Later in this paper, this fuzzy approach is examined and shown to be reasonable also from an analytical perspective.

In fuzzy control, having a set of linguistic rules to control a plant is not sufficient to implement the fuzzy controller. The fuzzy attributes of the fuzzy rules require quantification and tuning, and one of the techniques most often used for such purposes is trial-and-error. There are, however, other techniques including various adaptive schemes [5], genetic algorithms [6], and neural approaches [7], that can aid in the adjustment process of a fuzzy controller. In this work, the simulated-annealing approach, which is an heuristic optimization scheme, is used to determine the appropriate parameters of a fuzzy controller.

In this paper, an unstable plant which contains both intuitive and unintuitive goals is first described in Section II. In Section III, an unstable plant with only an unintuitive goal is discussed, and it is shown that typical fuzzy controllers are unable to stabilize such a plant directly. A new heuristically-derived fuzzy-control approach that is able to deal with the unstable and unintuitive plant is then described in Section III. An analytical examination of the heuristic approach is also given there; and furthermore, the new fuzzy control approach is generalized for the stabilization of other unintuitive plants. The simulated-annealing algorithm for determining the parameters of the fuzzy controller is described in Section IV. Simulation results are given in Section V. Conclusions are presented in Section VI.

II. An Unstable Plant with Both Intuitive and Unintuitive Goals

If the plant under control has unintuitive characteristics, unintuitive rules will be needed. The unintuitive rules themselves are not necessarily difficult to form once the nature of the "unintuitiveness" of the plant is understood. However, a rulebase which contains purely unintuitive rules is incapable of achieving the control objective. This is because typical unintuitive rules will attempt to direct the plant away from the actual long-term goal. (This is why the rules are called unintuitive!)

Consider the well-known cart–pole system shown in Fig. 2, of which the control objectives are to keep the pole upright and to move the cart to a specified location by applying an appropriate horizontal force. This control problem has a single control input, u, and two outputs, one corresponding to the pole angle, θ, and the other corresponding to the position, p, of the cart. There are two control goals, the first one is to keep $\theta = 0$, while the other is to move the cart to $p = 0$ (or any other location) from any given initial condition.

If the two control objectives for the cart–pole system are examined separately, it is not difficult to see that the goal of balancing the pole requires an intuitive control action; that is, this system embodies an intuitive goal. However in contrast, the goal of moving the cart requires an unintuitive control action. If the pole alone is considered, and if the pole is falling to the right, the control force should be applied in the right direction, and vice versa when the pole is falling to the left. Furthermore, the control action is monotonic—the more the pole has fallen, the larger the control force should be. However, for the second objective (that of moving the cart), to move the cart–pole system to the right, an initial control force acting in the left direction is needed; and vice versa if the need is to move the cart–pole system to the left. The reason for the unintuitiveness in this second case is because the action of the control force not only affects the cart, it also affects other dynamics of the plant, that is, the motion of the pole. Taken together, an unintuitive control force is needed to move the cart to a desired location.

The cart–pole problem is an interesting problem, as it contains goals of both an intuitive and an unintuitive kind. Moreover, it is the unintuitive part of the problem that often makes a control problem difficult to deal with. Nevertheless, the cart–pole problem has been successfully tackled by a fuzzy-control methodology [8] using a prioritized divide-and-conquer approach: in this situation, the typical strategy is to try to balance the pole first, and then, when the pole is almost balanced, the control force is applied in a direction opposite to where the cart is supposed to go. In other words, *the strategy is to consider the intuitive goal as the primary one, and to make the unintuitive goal a sub-goal*. From a position established by the intuitive goal, the unintuitive one can be achieved. This principle of "ordered control" is a crucial control strategy in dealing with unintuitive plants, and it will be referred to again in later sections.

Therefore, it would seem that provided that there are other intuitive goals associated with a control problem, unintuitive control goals are not necessarily difficult to accomplish. But now, if the control problem is such that the unstable plant has only unintuitive goals (and none which are intuitive), how can an unintuitive goal be achieved? This problem is addressed in the following section.

III. An Unstable Plant With an Unintuitive Goal or "Can a Blind Person Balance a Pole?"

As is explained in the previous section, once the nature of the "unintuitiveness" of a plant is understood, it is probably not difficult to form a set of unintuitive rules to deal with it. However, as has been pointed out, a set of purely unintuitive rules cannot achieve a control goal. Therefore, a new way must be found to stabilize an unstable plant having an unintuitive goal. In order for fuzzy control to be successful in this case, deeper knowledge of the behavior of the plant is needed, so that somewhere along the line, some intermediate but intuitive goals can be identified. The intermediate goals can then be used in the same manner described in Section II to help toward achieving the unintuitive goal.

A. Modified Cart–Pole Problem

Consider the cart–pole problem, under the condition that the control problem is modified such that the cart is to move to a specific location, but that the pole-angle information is not available. The new problem, called the *modified cart–pole problem*, becomes that of an unstable plant with an unintuitive goal. Equivalently, this new problem can be visualized as that in Fig. 2 but with the pole portion of the cart–pole system covered up so that it is out of sight, with no pole-angle information available, where it is required to move the cart to a specific location while implicitly keeping the unseen pole upright. An even more graphic description of the problem can be formulated, namely, "Can a blind person balance a pole (on his/her finger tip) while moving around?"

Now, if the modified cart–pole system discussed above is examined analytically, or more specifically, if the linearized model of the system is analyzed, the plant can be seen to belong to a class of systems called open-loop unstable nonminimum-phase systems which is often not easy to stabilize [3]. Specifically, the analytical model of the cart–pole system is [9]

$$m_c\ddot{p} + m\ddot{p} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u \qquad (3.1a)$$

$$J\ddot{\theta} + ml(\ddot{p}\cos\theta + l\ddot{\theta}) - mgl\sin\theta = 0 \qquad (3.1b)$$

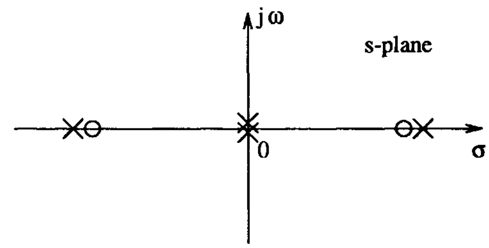

Fig. 3. The locations of the poles and zeros of the linearized cart–pole system.

where m_c is the mass of the cart, p is the horizontal displacement of the cart, m is the mass of the pole, l is the half-length of the pole, u is the control force exerted on the cart, J is the moment of inertia of the pole, and g is the acceleration due to gravity. For small θ and small $\dot{\theta}$, the above equations can be linearized to

$$(m_c + m)\ddot{p} + ml\ddot{\theta} = u \qquad (3.2a)$$

$$ml\ddot{p} + (J + ml^2)\ddot{\theta} - mgl\theta = 0. \qquad (3.2b)$$

For illustrative purposes, let $m_c = 1$ kg, $m = 0.1$ kg, $l = 0.5$ m, $J = 0.0083$ kg/m^2, and $g = 9.81$ m/s^2, in which case, the transfer function from the control input to the cart position can be written as

$$\frac{P(s)}{U(s)} = \frac{0.98s^2 - 14.36}{s^2(s^2 - 15.79)} \qquad (3.3)$$

where $P(s)$ and $U(s)$ are the Laplace-transformed variables of $p(t)$ and $u(t)$, respectively. The linearized cart–pole system has four poles and two zeros as depicted in Fig. 3, where the relative positions of the poles and zeros are independent of the parameter values. The combined location of the poles and zeros is characteristic of an open-loop-unstable nonminimum-phase system.

Before going further in finding a way to stabilize the modified cart–pole problem described by (3.3), we first establish, via proof by contradiction, that the classical fuzzy controller is incapable of achieving this goal.

B. The Modified Cart–Pole Problem Cannot be Stabilized by a Classical Fuzzy Controller

In this section, it is shown that the modified cart–pole problem cannot be stabilized by using a classical fuzzy controller. This is illustrated in two different ways, one based on a descriptive argument, and the other based on an analytical approach. Before discussing the main issues associated with the classical fuzzy controller, some definitions are first given:

Definition 1: A *classical fuzzy controller* is a controller which is static and time-invariant, with further properties of zero-input-zero-output and continuity as described below.

Consider a multi-input-single-output classical fuzzy controller as shown in Fig. 4, and assume that the plant under control admits a unique stationary solution with the input to the plant, the output of the plant, and the higher-order derivatives of the output of the plant all equal to 0. The output of the fuzzy controller is, in general, a nonlinear algebraic function

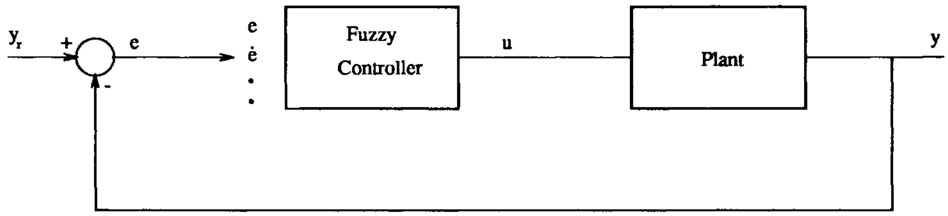

Fig. 4. A fuzzy control system in an output-feedback configuration.

of the inputs to the controller, that is,

$$u = f(z_1, z_2, \ldots, z_n) \tag{3.4}$$

where u is the output from the controller, f is a nonlinear algebraic relationship, and z_1 to z_n, the inputs to the controller, normally represent the error, and the derivatives of the error of the plant.

Property 1: The controller output, u, is zero when all the controller inputs are zero, namely, in (3.4)

$$f(0, 0, \ldots, 0) = 0. \tag{3.5}$$

Property 2: The output, u, is a continuous function of the inputs, i.e., if $\delta z_j, j = 1, 2, \ldots, n$ denote perturbations in the inputs, and δu denotes the corresponding change of the output, then

$$u + \delta u = f(z_1 + \delta z_1, z_2 + \delta z_2, \ldots, z_n + \delta z_n) \tag{3.6a}$$

has the property that

$$\delta u \to 0 \quad \text{as} \quad \|\delta z_1, \ldots, \delta z_n\| \to 0. \tag{3.6b}$$

It is to be noted that the output of the controller about the origin (where $u = 0, z_j = 0, j = 1, 2, \ldots, n$) of the control surface may or may not be a smooth function[3] of the inputs, so that the first derivative of the output with respect to the inputs may or may not be continuous about the origin. In the former case, a linearized expression about the origin of the control surface can be obtained. In the latter case, the region around the origin can be divided into smaller and separate smooth regions, so that regionally-linearized expressions can be obtained about the origin. Some examples of fuzzy control surfaces and perturbed expressions can be found in [10].

Definition 2: A closed-loop system is said to be in a *stabilized condition*, about a given equilibrium point, if it is asymptotically stable, in the sense of Lyapunov, about this equilibrium point. Without loss of generality, for all systems discussed in this work, the stabilized condition that is of interest is assumed to occur at the origin of the state space.

Definition 3: A *not-strongly-stabilizable linear time-invariant (LTI) plant* [11] is a LTI plant with the property that any stabilizing LTI controller for the plant must itself be unstable. (See Appendix A for further details.)

For the stabilization of the modified cart–pole problem, assume that the desired final position of the cart is 0 with respect to some reference point. The external reference input (i.e., desired output), y_r, shown in Fig. 4 can be regarded as zero while the position of the cart corresponds to the output, y, of the plant. The objective of the fuzzy control problem is to establish a set of linguistic rules that can drive the output of the plant and its derivatives to 0 from an initial nonzero condition. Inputs (premises) to the fuzzy controller include the values of the error and its derivatives.

[3] Here, a smooth function refers to a function which has at least a continuous first derivative with respect to the input variables.

To show that the classical fuzzy controller is unable to stabilize the modified cart–pole problem, a descriptive argument is first presented. This is then followed by an analytical argument which is applicable to a more restrictive class of fuzzy controllers. The descriptive argument is based on both a qualitative and analytical knowledge of the plant, whereas the analytical approach is based only on analytical knowledge.

1) A Descriptive Argument: It is known from experience that moving the cart to a desired location requires unintuitive control actions; furthermore, as has been outlined in the previous section, an unintuitive goal cannot be accomplished unless there is an associated intuitive goal which can be used for guidance.

The unintuitiveness associated with the control of the cart movement is due to the interaction of various dynamics in the cart–pole system, and this "unintuitiveness characteristic," which can be attributed to the particular form of the input-output transfer function of (3.3), cannot be removed by differentiation. Therefore, it is not possible to obtain an intuitive goal from the derivatives of the error if the error is itself unintuitive. *Since neither the error nor any of the derivatives of the modified cart–pole system can be stabilized by using an intuitive control action, it is not possible to come up with a set of linguistic rules for the classical fuzzy controller that can stabilize the modified cart–pole system.*

2) An Analytical Approach: The qualitative argument given earlier applies to all classical fuzzy controllers. For cases where the behavior of the fuzzy controller about the origin of the control surface is linearizable, an analytical approach can also be used. The following proposition and proof show that because of the static nature of the classical fuzzy controller, it cannot, at least for the cases where its behavior is linearizable about the origin of the control surface, stabilize the class of so-called "not-strongly-stabilizable plants," such as that described by (3.3).

Proposition 1: The class of classical fuzzy controllers which is linearizable about the origin of the control surface is not capable of stabilizing any not-strongly-stabilizable plant such as that shown in (3.3).

Proof: To show that the above-mentioned fuzzy controller is incapable of stabilizing a plant such as is described by (3.3), it is necessary only to show that the fuzzy-control closed-loop system cannot achieve local asymptotic stability. For local stability analysis of a fuzzy-control closed-loop system, we can first carry out a perturbation analysis of the classical fuzzy controller about the equilibrium point, and linear or regionally-linear expressions describing the behavior of the fuzzy controller can then be obtained [10], [12]. In the former case, the perturbed controller can be viewed as a linear time-invariant (LTI) controller about the equilibrium point. Furthermore, since the controller is static, the linearized fuzzy controller behaves as a stable LTI controller.

According to the results in [11] (see also Appendix A), any LTI controller which is capable of stabilizing not-strongly-stabilizable plants has to be unstable. Since the class of classical fuzzy controllers which is linearizable around the equilibrium point can only behave as a stable LTI controller around the equilibrium point, it is not capable of stabilizing any not-strongly-stabilizable plant such as that described by (3.3). □

Although the proof of Proposition 1 suggests that it is possible to introduce unstable dynamics into the fuzzy controller (e.g., by feeding back previous controller output values), to stabilize the not-strongly-stabilizable plants, there appears to be no published heuristic strategy for achieving such a task. And, even if unstable dynamics can be successfully introduced into a fuzzy controller to stabilize not-strongly-stabilizable plants, the approach is still not desirable, since possible instability in the controller is something to avoid if at all possible.

In the next section, a heuristically-based approach, using a time-varying fuzzy controller, which can stabilize the modified cart–pole problem is discussed. The approach is relatively easy to understand, and easy to implement without the need for an unstable controller. An analysis of the approach is given in a later section.

C. The "Blind-Person" Strategy

Section III-B has shown that the often-used fuzzy controller configuration shown in Fig. 4 cannot directly stabilize the modified cart–pole problem. Noting that fuzzy control has its original roots in human intuition and heuristics, it would seem reasonable to examine how a human subject would deal with the problem. As described earlier, the devised problem can be stated as "Can a blind person balance a pole on his/her finger tip while moving around?"

Unable to see, it would seem that the only way that a blind person can detect where the pole is falling is by sensing the sideward force exerted on his/her finger by the falling pole. If the pole is falling to the left, there is a force pushing his/her finger to the right, and vice versa when the pole is falling to the right. Furthermore, the more the pole has fallen, the larger is the sideward force at the balancing finger (this does not hold if the pole angle is exceedingly large). However, *in order to sense the force of the falling pole, the balancer should not apply any force during the sensing period, otherwise, he/she would have difficulty in obtaining the correct information*. Subsequently, with the knowledge of the force from the falling pole, he/she can then exert a counteracting force to prevent the pole from falling. This sequence of sensing and then controlling can be repeated until the pole is balanced. Notice that the control action whose goal is to minimize the sideward force on the finger is intuitive in the sense described earlier. Being able to stabilize the pole, the blind person can then carry on to move the position of the pole around with appropriate unintuitive control action while at the same time keeping the pole balanced. We believe that if the dynamics of the pole are slow enough for the blind person to react, he/she should be able to balance the pole and also move the pole around. The corresponding fuzzy control rules based on the control strategy described in this section are given in Appendix B.

There are two important ingredients in this "blind-person" strategy. The first is to separate the *measurement (sensing) phase*, during which the control is turned off (i.e., maintained at zero), and the *control phase* in which the control action is formulated from the previous measurement value. The second is to identify a supplementary intuitive goal (here, to minimize the sideward force on the finger). Although in the often-used fuzzy-control configuration shown in Fig. 4, measurements are typically carried out while the plant is being controlled, the configuration can be easily converted to implement the first part of the strategy described above. This is done directly by modulating the output of the original fuzzy controller in Fig. 4 by a pulse signal as shown in Fig. 5, so that the input control signal to the plant becomes a series of pulses. This pulse-control approach renders the new fuzzy controller time-varying; that is, the input-output behavior of the new fuzzy controller now becomes a function of time. The pulse-modulation scheme is a form of pulse-control technique [13].

There are at least two types of pulse control which can be used for constructing the proposed fuzzy controller; one type, which is shown in Fig. 5, has a fixed pulse-width, with the amplitude of the input pulse to the plant determined by the inference process in the usual manner. The other, which is not illustrated, uses a pulse of fixed amplitude whose duration is determined by the inference process. The first approach is usually referred to as a pulse-amplitude-modulation (PAM) scheme, while the second approach is a type of pulse-width modulation (PWM). Provided that the inference period (or sampling period) is sufficiently small, both schemes can produce equivalent results if the plant is linear. Note that pulse-modulation schemes are already used in many control applications, but typically viewed in a much different context than that described here. Simulation results for the modified cart–pole problem based on the new fuzzy controller architecture are shown in Section V.

The heuristic approach described in this section offers an alternative way of solving open-loop-unstable and nonminimum phase problems discussed in [3], [4] which are based on conventional control theory. Compared to conventional approaches, the heuristic approach has the advantage that it is relatively easy to understand (as it is based on physical insight of the plant) and possibly simpler to implement. Hopefully, the proposed approach can extend the capability of a linguistic fuzzy controller while preserving the intuitiveness of the control action. The next section examines the "blind-person" strategy from an analytical perspective.

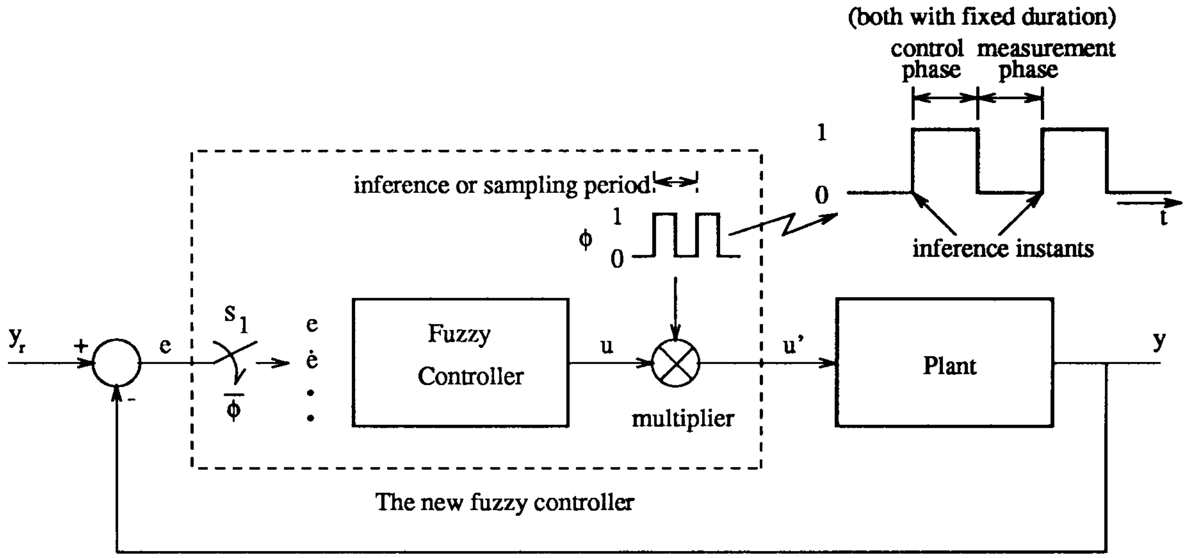

Fig. 5. A modified fuzzy-control system using the pulse-amplitude-modulation scheme.

D. Analysis of the "Blind-Person" Strategy

Consider the state-space representation of a linear time-invariant (LTI) system

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{3.7}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector, $y \in R^r$ is the output vector, A is the $n \times n$ system matrix, B is the $n \times m$ input matrix, and C is the $r \times n$ output matrix.

According to linear control theory, if the system in (3.7) is observable, then the internal state x can be reconstructed [14] from the input u, and the output y, and their derivatives[4]. If u is zero, as in the "blind-person" strategy, then x can be reconstructed by considering only y and its higher-order derivatives. Therefore, *in principle, this separation of measurement and control phases can allow the fuzzy control strategy to reconstruct any state variable of the plant from the output of the plant and its derivatives*. However, there is no guarantee that the necessary heuristics exist for such purposes.

The cart–pole problem is a 4th-order system with four states: $\theta,\ \dot{\theta},\ p$ and $\dot{p}$, where $\dot{p}$ represents the horizontal velocity, and the force which the blind person can sense is proportional to $\ddot{p}$. It can be shown, in the modified cart–pole problem, that the "blind-person" strategy has actually reconstructed the angle information for the pole. Therefore, the strategy makes sense analytically. The next section shows how this control strategy can be generalized to other similar systems.

E. Generalization of the "Blind-Person" Strategy

Given a physical plant which has an unintuitive goal, it may be possible to come up with some particular strategy for achieving this goal if it is possible for an operator to interact with the system and thereby gain experience. If on the other hand, only the mathematical model of the system is given, and there is no operator knowledge, then there may not be much physical insight which can be relied on to draw up a control strategy.

However, the experience that has been gained from dealing with the modified cart–pole problem, in which we have a knowledge of both the physical plant as well as the corresponding mathematical model, may allow us to tackle a similar class of mathematical systems for which the actual physical plant may not be known. Three examples are given below, all of which are 2nd-order, single-input-single-output, unstable and nonminimum-phase systems; the systems are expressed in a canonical state-space realization.

Before discussing the examples, it must be stressed that the plants to be used are well-defined and are LTI, and to emphasize that there exist conventional control techniques [3]–[4] which can stabilize them. The purpose of this paper is *not* to compare the advantages and disadvantages of fuzzy versus conventional techniques, but rather to show, through some examples, how a heuristically based pulse-control fuzzy controller can stabilize such plants. (It is to be emphasized that a classical fuzzy controller cannot accomplish this for Examples 1 and 3, as both of them are not-strongly-stabilizable.)

The technique to be used in stabilizing such plants is based on the principle of separation of the measurement and control phases, which is derived from the "blind-person" strategy. Essentially, the idea is to manipulate the output of the plant and the input to the plant (e.g., by differentiation) so that individual state variables or the sum of the state variables show up as output derivatives, with the property that the plant with these new outputs is now intuitive to control.

Example 1:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.8a}$$

$$y = [-1 \quad 1]x \tag{3.8b}$$

where $x = [x_1 \ \ x_2]'$. The output transfer function is nonmin-

[4] The internal state of a plant can also be reconstructed using a state observer which does not require the use of the derivatives of u and y.

imum phase. The 1st derivative of y is

$$\dot{y} = x_2 + u \tag{3.8c}$$

and the corresponding transfer function is also nonminimum phase. Therefore, the system is not easy to control. However, by letting $u = 0$ (in the measurement phase), $\dot{y}$ is equal to the state variable, x_2, and in this case in the following control phase, the system (with input u, output x_2) is "intuitive," and hence a conventional fuzzy controller can be applied to stabilize the system. Thus, the strategy when using the fuzzy control configuration given in Fig. 5 is first to utilize $\dot{y}$, and then to utilize y in order to stabilize the system. Corresponding fuzzy-control rules are given in Appendix C. Simulation results are shown in Section V.

Example 2:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.9a}$$

$$y = [5 \quad -1]x. \tag{3.9b}$$

Again in this example, the output transfer function is nonminimum phase, and the 1st derivative of y is

$$\dot{y} = 3x_1 + x_2 - u \tag{3.9c}$$

the transfer function of which is also nonminimum phase. However, by setting u to 0 (in the measurement phase), $\dot{y}$ becomes a weighted sum of the two state variables $(3x_1 + x_2)$ that can be utilized to stabilize the system in the same way as described in Example 1. In this example, however, to stabilize the system, it is only necessary to utilize $\dot{y}$ in the controller[5]. The corresponding fuzzy control rules are given in Appendix D. Simulation results are shown in Section V.

Example 3:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.10a}$$

$$y = [2 \quad -1]x. \tag{3.10b}$$

The output transfer function is nonminimum phase, and the 1st derivative of y is

$$\dot{y} = 3x_1 - 2x_2 - u \tag{3.10c}$$

the transfer function of which is also nonminimum phase. If u is set to zero, $\dot{y}$ becomes a weighted difference of the state variables that cannot be stabilized in the manner discussed previously. However, it is possible to turn the problem to one similar to that in Examples 1 or 2 by combining y and $\dot{y}$ linearly to form a new quantity, for instance,

$$2y - \dot{y} = x_1 + u$$

which can then be dealt with in the same manner as before.

[5]This is so because the system consisting of (3.9a) and (3.9c) with $u = 0$ is observable.

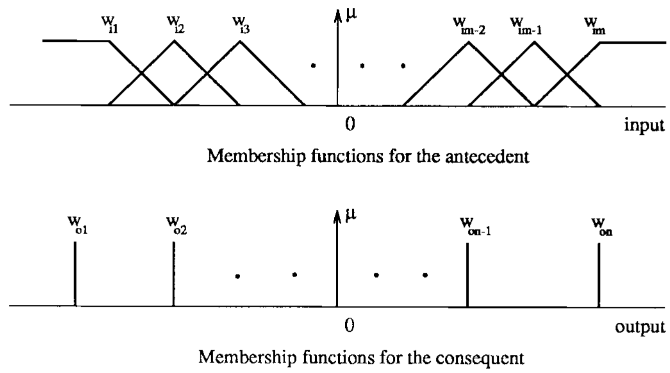

Fig. 6. Membership functions.

IV. Simulated Annealing (SA) for Tuning of Membership Functions

In the modified cart–pole problem discussed in Section III-A, as well as in Examples 1 and 2 in Section III-E, it is possible (based on the previous discussion) to come up with a set of heuristic rules (see Appendixes B, C, and D) to stabilize the plants. However, the fuzzy attributes of the rules still need to be determined before the rules can be implemented. An often-used approach is to roughly estimate the ranges of the input and output variables, and then, on the basis of that information, characterize the membership functions used in the rules. Finally, the membership functions are individually tuned to optimize the results as much as possible. This is largely a trial-and-error procedure.

However, there are other approaches that can aid in the tuning of membership functions: one such approach makes use of the so-called genetic algorithm [6], and another is based on simulated annealing (SA) [15]. Both approaches are probabilistic search techniques which aim at driving down a cost function, and both of them are applicable to a wide range of problems in various disciplines. In the work reported here, the tuning of the membership functions of the fuzzy rules is formulated as a combinatorial optimization problem, and modified SA for continuous parameters is employed to determine the appropriate membership-function combination. To simplify the optimization procedure, only triangular membership functions are used for the antecedent, and singletons for the consequent.

For the combinatorial optimization problem of tuning of the membership functions depicted in Fig. 6, the solution space can be defined as

$$\mathbf{S} = \{w \mid w = (w_{i1}, w_{i2}, \ldots, w_{im}, w_{o1}, w_{o2}, \ldots, w_{on})\} \tag{4.1}$$

where w_{ix} is the location of the apex point of a triangular membership function used in the antecedent, the width of the triangle is determined by adjacent apex points, w_{ox} is the location of a consequent singleton, m is the total number of antecedent parameters, and n is the total number of consequent parameters. Therefore, there are total of $m + n$ degrees of freedom in the problem. The cost function C of the optimization problem maps each solution into a real number $C\colon \mathbf{S} \rightarrow \mathbf{R}$. The optimization problem can be stated as follows:

TABLE I
SA-OPTIMIZED LOCATIONS OF THE APEX POINTS OF THE MEMBERSHIP FUNCTIONS FOR THE MODIFIED CART–POLE PROBLEM. (THE NUMBERS IN BRACKETS ARE INITIAL VALUES)

	PL = -NL	PS = -NS	PVS = -NVS	POS = -NEG	ZE
$\frac{d^3p}{dt^3}$	3.025 (2.0)	1.64 (1.0)	–	–	0
$\frac{d^2p}{dt^2}$	1.177 (1.0)	0.315 (0.5)	–	–	0
$\frac{dp}{dt}$	–	–	–	1.484 (1.0)	0
p	2.417 (2.0)	1.675 (1.0)	–	–	0
u	43.28 (3.0)	32.86 (2.0)	19.66 (1.0)	–	0

Minimize

$$C(w)$$

Subject to the constraints:

1) $w_{i1} < w_{i2} < \cdots < w_{im}$.
2) $w_{o1} < w_{o2} < \cdots < w_{on}$.
3) The rules and the inference method proposed are used.
4) The dynamics of the plant are satisfied.

The SA procedure, which draws on the analogy between a physical-annealing process and a combinatorial optimization problem, applies the Metropolis algorithm [16] to generate a sequence of solutions. Briefly stated, a parameter of $w \in S$, that is, a value of w_{ix} or w_{ox}, is chosen randomly and perturbed, and the corresponding cost is then evaluated. The new solution is accepted if it leads to a reduction in cost, and it is accepted probabilistically if it leads to an increase in cost. This procedure implies that the SA approach can avoid being trapped in local minima. The SA procedure also contains a control schedule, so that as the optimization process progresses, it becomes less and less likely that deteriorations in cost will be accepted. Finally, a near-optimal solution will result. Some detailed examples of the general SA procedures can be found in [17].

In the next section, SA is used to optimize the membership functions associated with the modified cart–pole problem introduced in Section III-A, and in Examples 1 and 2 in Section III-E.

V. SIMULATION RESULTS

Simulation results for the cases previously discussed are given in this section. There are three sets of results; the first set corresponds to the modified cart–pole problem; the second set corresponds to Example 1 in Section III-E; and the third set of results corresponds to Example 2 in Section III-E. The procedures used for obtaining these three sets of results are the same, and all the simulations are based on the fuzzy-control configuration shown in Fig. 5 using the product-product-sum inference method [10]. Some practical considerations of this new fuzzy control scheme can be found in [18].

An inference (or sampling) period of 10 ms, which is much smaller than any of the system dynamics, is used in all three cases. The duty cycle of the switching pulse shown in Fig. 5 affects the gain of the control loop; the greater the pulse, the higher is the effective gain, and vice versa for a smaller pulse width. To produce the same control effect, a smaller pulse width requires a larger control signal amplitude, and vice versa for a wider pulse width. In the simulations, the duty cycle of the pulse signal is set to 50%.

The rulebases for the three cases are given in Appendixes B, C, and D. The initial values of the membership functions associated with the rules are set according to the estimated ranges of the variables, and are then tuned by an SA algorithm. The cost function, C, for all three cases, is defined as

$$C = \int_0^T t|e(t)|dt \tag{4.2}$$

where T is the length of the time interval for evaluating the cost function, t is the time, and $|e(t)|$ is the absolute error between the reference input and the output. This corresponds to the classical performance index of ITAE (integral of time and absolute error). Since the reference input can be regarded as zero in a stabilization problem, $e(t)$ is equal to the output of the plant.

A. The Modified Cart–Pole Problem

As has been discussed in Section III-C, information on the horizontal force due to the pole, as well as on the rate of change of the force, are needed to balance the pole. In the simulations, the horizontal force is represented by the 2nd-order derivative of the horizontal displacement, and the rate of change of force is represented by the 3rd-order derivative of the displacement. It must be pointed out that such a scheme for obtaining the information of the force and the rate of change of force would probably not work in practice because the differentiation process "amplifies" noise, and higher-order derivatives will be easily "drowned" out by noise. Other techniques for obtaining the horizontal force information must be used in practice[6].

[6] A possible candidate may be the accelerometer from Analog Devices [19].

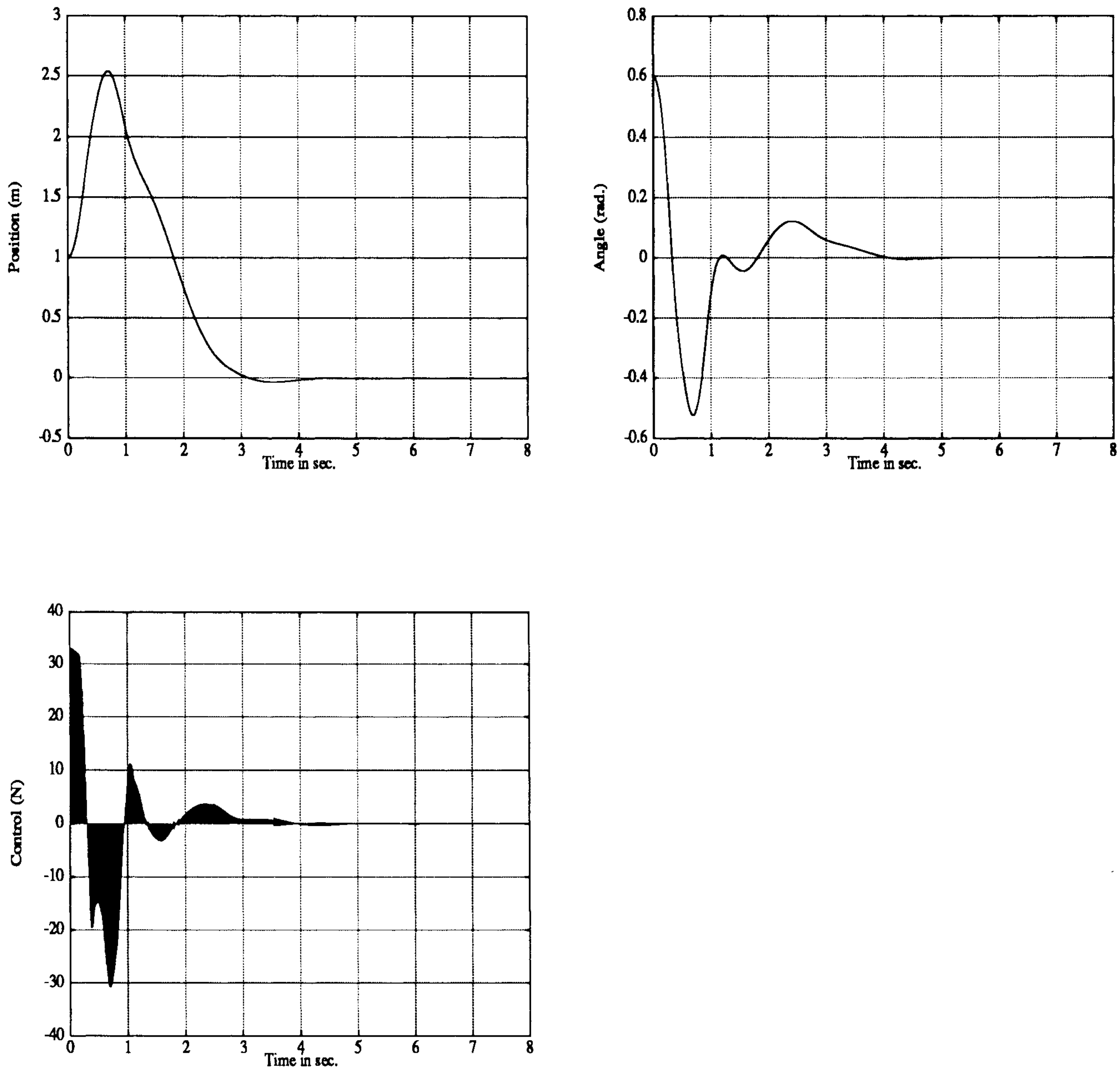

Fig. 7. Simulation results for the modified cart–pole problem. The apparently dark areas in the control waveform are due to fast switching.

In any case, the control strategy is to utilize the higher-order derivative terms first, in attempting to stabilize the system.

Initial conditions of cart displacement of 1 m and a pole angle of 0.6 rad were used in the simulation. The initial and SA-optimized locations of the apex points of the membership functions are shown in Table I. Simulation results based on the optimized membership functions are shown in Fig. 7. The control input to the plant is a switching waveform which alternates every 5 ms (control or measurement interval) between 0 and a particular control value obtained from the fuzzy-inference process. As can be seen in Fig. 7, the fuzzy controller can successfully move the cart to the origin while also keeping the pole upright. The apparently dark areas in the control waveform in Fig. 7 are due to the sampling process.

B. Example 1 in Section III-E

In this case, the control strategy is to utilize the $\dot{y}$ term first, and then utilize the output y in order to stabilize the system. Initial conditions of the plant used in the simulation were $y(0) = -3,\ \dot{y}(0) = -2$. SA-optimized parameters are given in Table II. As can be seen from the simulation results shown in Fig. 8, which are based on the optimized parameters, both $\dot{y}$ and y are successfully regulated to zero.

C. Example 2 in Section III-E

This is the simplest case examined in which it is only necessary to utilize $\dot{y}$ in the resulting controller. Initial plant conditions for the simulation were $y(0) = 7$ and $\dot{y}(0) = 1$.

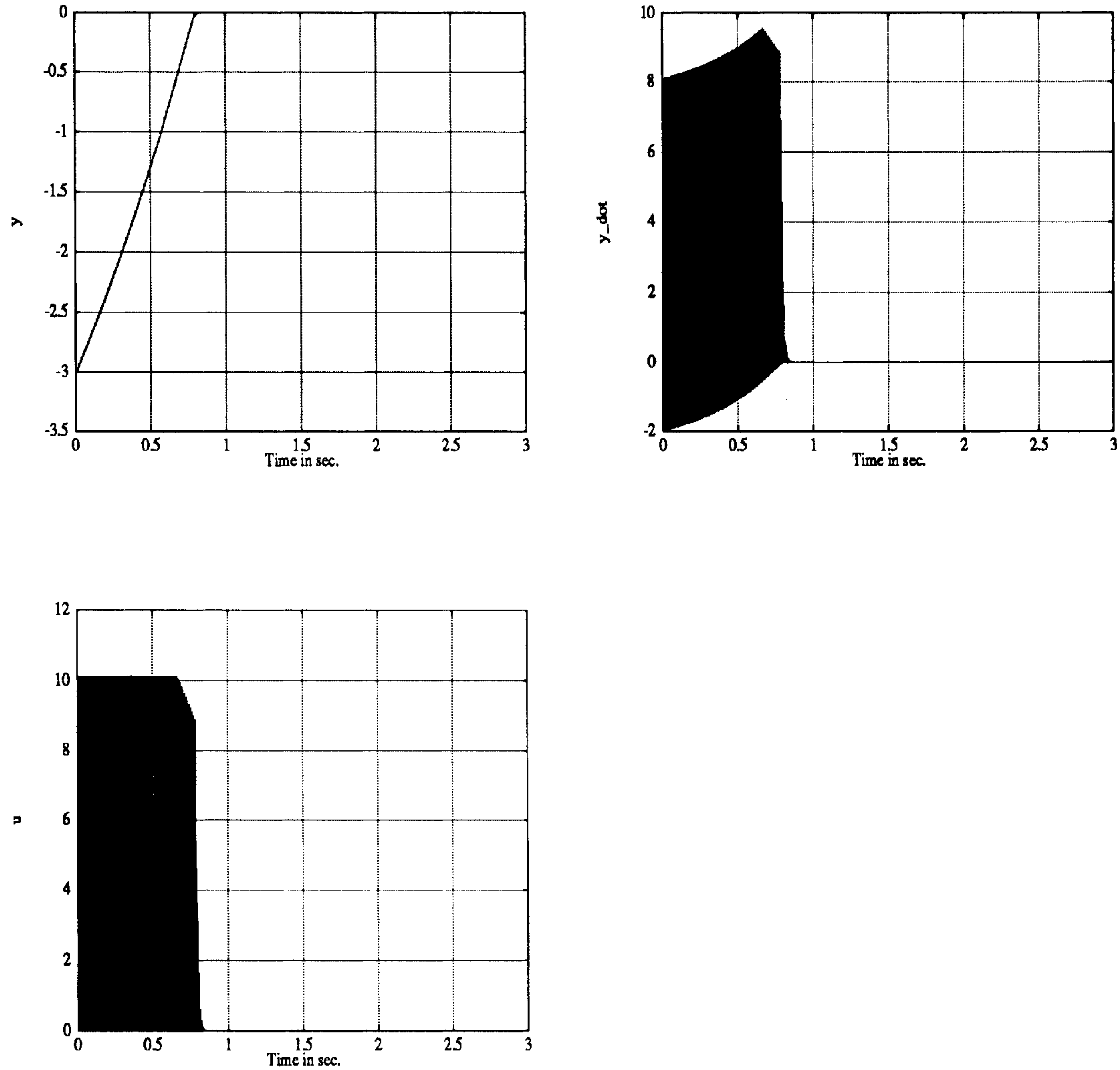

Fig. 8. Simulation results for Example I. The apparently dark areas in the waveforms are due to fast switching.

TABLE II
SA-Optimized Locations of the Apex Points of the Membership Functions for Example 1. (The Numbers in Brackets Are Initial Values)

	PL = -NL	PS = -NS	PVS = -NVS	ZE
$\frac{dy}{dt}$	0.565 (1.5)	0.066 (1.0)	–	0
y	1.968 (2.0)	0.534 (1.0)	–	0
u	10.133 (3.0)	8.866 (2.0)	4.931 (1.0)	0

TABLE III
SA-Optimized Locations of the Apex Points of the Membership Functions for Example 2. (The Numbers in Brackets Are Initial Values)

	PL = -NL	PS = -NS	ZE
$\frac{dy}{dt}$	4.613 (2.0)	0.819 (1.0)	0
u	79.99 (2.0)	47.36 (1.0)	0

The SA-optimized apex points of the membership functions are given in Table III. Simulation results are shown in Fig. 9.

VI. Conclusion

It has been shown that some unstable and unintuitive control problems cannot be stabilized by applying the usual "classical" fuzzy controller, and in view of this, a new fuzzy-controller architecture, which is inspired by considering the action of a human subject, has been derived. The basic concept of the new fuzzy controller is to separate the measurement (sensing) phase from the control phase. The proposed fuzzy controller is time-varying, and can be implemented by modulating the output of

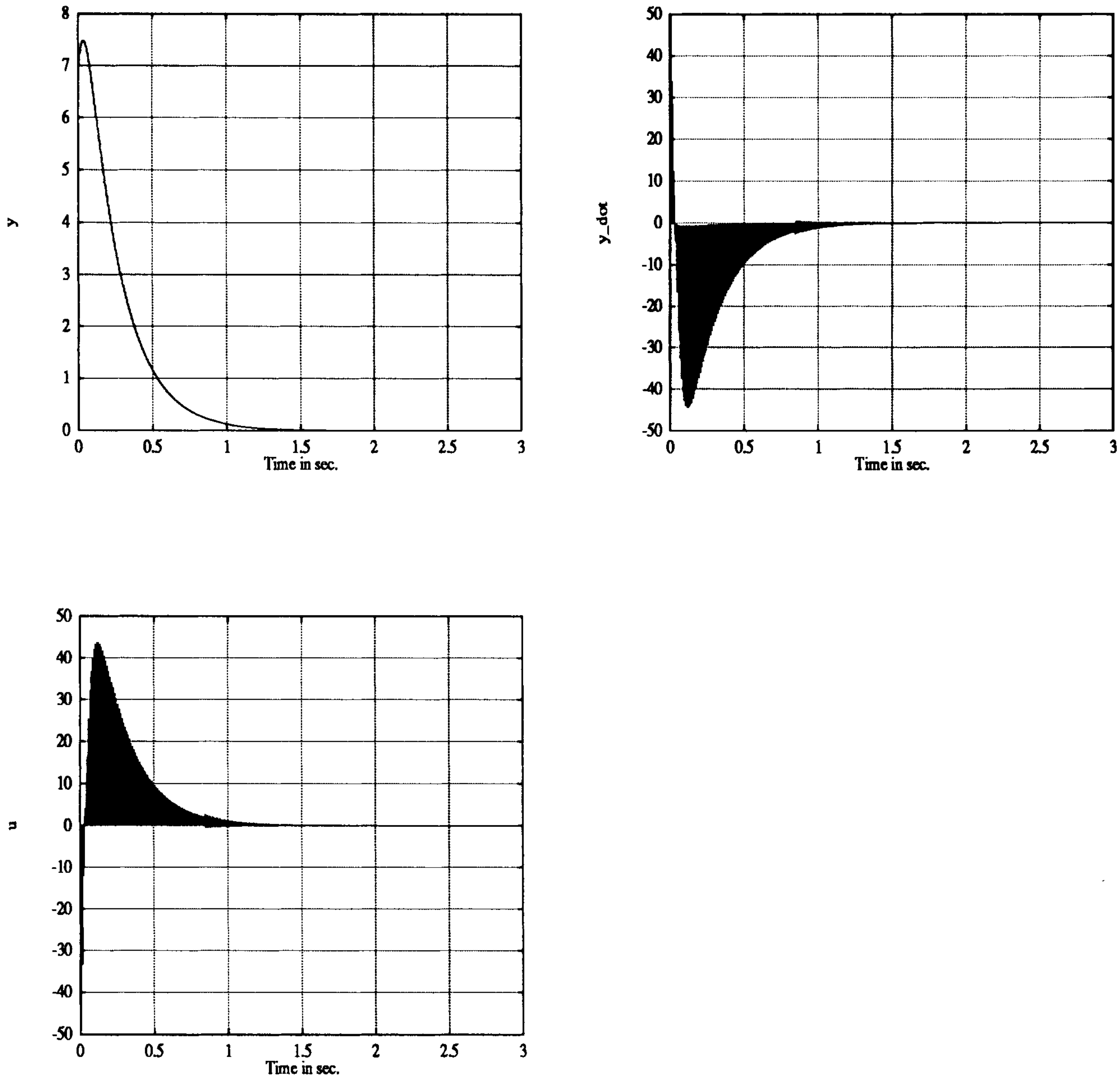

Fig. 9. Simulation results for Example 2. The apparently dark areas in the waveforms are due to fast switching.

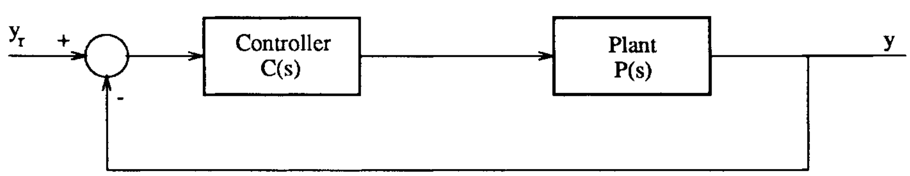

Fig. 10. A linear feedback control system.

a conventional fuzzy controller with a pulse signal. Together with a hierarchical control strategy, it is shown, through some examples, that the new fuzzy controller is capable of stabilizing a general class of unintuitive and unstable plants.

It is hoped that the approach proposed can be used to extend the capabilities of linguistic fuzzy controllers while preserving the intuitiveness of the control action.

APPENDIX A

A. A Not-Strongly-Stabilizable Plant [11]

Consider a linear feedback control system as shown in Fig. 10 where the plant $P(s)$ is open-loop unstable, and $C(s)$ is a stabilizing controller. The plant $P(s)$ and the controller $C(s)$ are both controllable and observable.

TABLE IV
RULEBASE FOR THE MODIFIED CART–POLE PROBLEM

$\frac{d^3p}{dt^3}$	$\frac{d^2p}{dt^2}$	$\frac{dp}{dt}$	p	u
–	PL	–	–	NL
–	PS	–	–	NS
PL	ZE	–	–	NS
PS	ZE	–	–	NS
NS	ZE	–	–	PS
NL	ZE	–	–	PS
–	NS	–	–	PS
–	NL	–	–	PL
ZE	ZE	POS	PL	PS
ZE	ZE	POS	PS	PS
ZE	ZE	POS	ZE	PVS
ZE	ZE	POS	NS	ZE
ZE	ZE	POS	NL	ZE
ZE	ZE	NEG	PL	ZE
ZE	ZE	NEG	PS	ZE
ZE	ZE	NEG	ZE	NVS
ZE	ZE	NEG	NS	NS
ZE	ZE	NEG	NL	NS
ZE	ZE	ZE	PL	PS
ZE	ZE	ZE	PS	PVS
ZE	ZE	ZE	ZE	ZE
ZE	ZE	ZE	NS	NVS
ZE	ZE	ZE	NL	NS

A *strongly-stabilizable* plant is a plant that can be stabilized (i.e., the overall feedback system becomes stable) by employing an asymptotically stable linear time-invariant (LTI) controller $C(s)$.

A *not-strongly-stabilizable* plant is an LTI plant for which any LTI controller $C(s)$ that is capable of stabilizing it, must itself be unstable. Such a plant has a unique pole-zero interlacing property on the positive real axis of the s-plane [11]. An example of such a plant is the linearized modified cart–pole system discussed in Section III-A, having the pole-zero pattern shown in Fig. 3. According to the discussions made in Section III-B, it is not possible to come up with a set of rules for the classical fuzzy controller such that it will stabilize such a plant.

APPENDIX B

The rulebase for the modified cart–pole problem in Section III-A is shown in Table IV. Here, $\frac{d^3p}{dt^3}$ represents the rate of change of the horizontal force, $\frac{d^2p}{dt^2}$ represents the horizontal force, $\frac{dp}{dt}$ is the horizontal velocity, p is the horizontal displacement, and u is the control force.

The rules in Table IV define a hierarchical control strategy which moves the modified cart–pole system described in Section III-A from a specified initial position to a reference position (zero). The labels in this table have the following linguistic meanings: NL—Negative Large, NS—Negative Small, NVS—Negative Very Small, NEG—Negative, ZE—Zero, POS—Positive, PVS—Positive Very Small, PS—Positive Small, PL—Positive Large.

There are two parts in this control strategy: first, an attempt is made to minimize the horizontal force (the intuitive goal) and the rate of change of force; and then, when the force and its time derivative are almost zero, to minimize the displacement (the unintuitive goal) and velocity. As with any heuristic strategy, the one described by Table IV is not unique.

The rules (the first eight in the table) for minimizing the horizontal force and its derivative are intuitive; namely, they are formed so that the control action, u, is applied to counteract them; for instance, when the horizontal force exerted by the pole (represented by $\frac{d^2p}{dt^2}$) is PL the control action, u, is NL. For this section of rules, the rate of change of the horizontal force is considered only when the horizontal force is about zero.

The rules (the remaining 15 rules in the table) for regulating the displacement and velocity are unintuitive; namely, the control action is directed away from the long-term goal; for instance, the ninth rule in the table specifies a control action which apparently moves the system away from the intended final position. This section of rules only applies when the pole is almost balanced (i.e., the horizontal force exerted by the pole and the rate of change of the force are almost zero). Here, the 15 rules are formed by considering all possible combinations of displacement and velocity.

TABLE V
RULEBASE FOR EXAMPLE 1 IN SECTION III-E

$\dot{y}$	y	u
PL	–	NL
PS	–	NS
NS	–	PS
NL	–	PL
ZE	PL	PS
ZE	PS	PVS
ZE	ZE	ZE
ZE	NS	NVS
ZE	NL	NS

TABLE VI
RULEBASE FOR EXAMPLE 2 IN SECTION III-E

$\dot{y}$	u
PL	NL
PS	NS
NS	PS
NL	PL
ZE	ZE

APPENDIX C

The rulebase for Example 1 in Section III-E is shown in Table V. The linguistic meanings of the labels follow those of Appendix B.

APPENDIX D

The rulebase for Example 2 in Section III-E is shown in Table VI. The linguistic meanings of the labels follow those of Appendix B.

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