WALSH EXPANSION OF A SINUSOID

The Walsh expansion of cos $(f2\pi\theta)$ is written as the following:

$$\sum_s A_{sf} \text{ cal } (s,\theta) \tag{11}$$

where A_{sf} is the Walsh coefficient of sequency s, i.e.,

$$A_{s_f} = \int_0^1 \cos\,(f2\pi\theta)\,\text{cal}\,(s,\theta)\,d\theta = \frac{1}{2}a_{fs} \tag{12}$$

from (4). Substitution of $f = 1$, $s = 1,2,3, \cdots$, in (8) yields the required coefficients. To illustrate, for $s = 7$ (cal) in a set of 16 Walsh functions, $k = 14 = 1001$ in Gray code, so $\alpha = 2$. A term within the product in (8) is a cosine or a sine, depending on whether g_x is 0 or 1. Hence

$$A_{71} = \frac{1}{2}a_{17} = (\text{sinc } 1/16)\,(\sin \pi/2) \cdot (\cos \pi/4)(\cos \pi/8)\,(\sin \pi/16) \approx 0.1266.$$

The sal coefficients may be evaluated in the same way.

ACKNOWLEDGMENT

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A Calibrated Ultrasonic Field Measuring System for General Use

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Abstract—**An instrumentation scheme is presented which provides a calibrated field intensity equivalent plot using common laboratory equipment. The parameter that is measured gives a direct indication of the merit of an ultrasonic transducer for a given application in terms of its beam shape. The composite output of the system yields a three-dimensional representation of the intensity equivalent parameter as a function of a two-dimensional spatial field. Several applications of the resulting instrumentation are given including the effects of crystal window flatness and acoustic lensing.**

I. INTRODUCTION

IN THE APPLICATION of ultrasound, there are several identifiable areas where improvements could be made provided a straightforward means for characterizing the electroacoustic efficiency of the transducer were available. Two industrial environments where ultrasound is commonly employed are flow metering and visualization of internal structures (e.g., flaw detection, medical visualization, depth finding, etc.). It is apparent here that a controlled narrow beamshape is required for high spatial resolution. For example, in the flowmeter case, a narrow beam is required to resolve the ambiguities that could result from turbulence. In broad beamwidth applications, it is also desirable to obtain a picture of the acoustic pattern being generated by the transducer(s).

A combination of a visualization capability with a knowledge of the properties of the environment in which the ultrasound will be employed permits the choice of a specific beam shape to optimize some criteria. Subsequently, by mechanically building and/or adjusting the parameters of the crystals appropriately, the desired beamshape can be approximated. This approach increases the signal-to-noise ratio (SNR) at the input of the processing circuitry by having realized essentially a spatial filter that reduces irradiation of undesired structures. The instrumentation scheme presented here is generally useful for ultrasonic applications that employ both an ultrasonic transmitter and receiver.[1]

II. BASIS OF INTENSITY EQUIVALENT MEASUREMENT

The basic aim of this instrument is to allow a rapid assessment of ultrasonic transducers by displaying equivalent field intensity. The calibrated parameter that is used as a measure of goodness G is the ratio of

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[1] That is, the scheme presented is not intended for use with ultrasonic crystals used for heating and cleaning purposes.

receiving transducer voltage output and transmitting transducer electrical power input.[2]

Although acoustic effects are of obvious concern, most common applications of ultrasonic transducers utilize electrical signal generation and detection. Thus it is sensible to obtain a characterizing parameter that represents the twice transduced signal in electrical terms rather than a singly transduced signal in acoustical terms. In addition, for applications where the transmitting and receiving crystals are physically different, but exist in the same package, more information is obtained from the doubly transduced G parameter. To obtain an equivalent characterization using a singly transduced method, one would require the acoustic field distribution of each ultrasonic crystal which must then be combined with appropriate shifts to reflect the spatial orientation of the transmitting and receiving crystals.

Furthermore, equivalent acoustic transmitted power can be measured from electrical power input to the transducer, while acoustic received power can be determined from electrical voltage output of a transducer into a high input impedance amplifier. Therefore, the G parameter is appropriate for measuring the electroacoustoelectric transfer function of an ultrasonic system.

III. REALIZATION OF A PRACTICAL VISUALIZATION TOOL

To make the instrument a useful research tool for other ultrasonic investigations and other investigators, the following instrumentation criteria were established.

1) The crystals should be tested in the same manner in which they will ultimately be used. This allows the combined field characteristics of the transmitter and receiver to be obtained.

2) The instrument must provide equal electrical power drive to all transducers at various frequencies. This permits comparisons in intensity and conversion efficiencies between transducers and transducer configurations.

3) All parameters must be calibrated or calibratable and, therefore, repeatable. This is a somewhat difficult problem as a result of the cumbersome nature of the multivariable spatial and amplitude requirements.

4) The display must be readily interpretable and provide a permanent record in a graphic hardcopy format.

5) The entire instrument should satisfy the two necessary requirements of a properly designed instrument intended for wide general use: it must be easy to use and inexpensive to build.

What has been realized is an instrument composed of common laboratory components which satisfies all of the above criteria.

To begin with, a photograph of the complete instrumentation system is shown in Fig. 1. On the left is the ultrasonic field portion which consists of a water tank, the transducer fixture and the device for controlling the reflecting target's position. The latter device uses an inverted X–Y plotter (in this case a Hewlett–Packard model 7005B) to obtain precise mechanical movements of the reflecting target with this target fastened in place of the usual felt pen.

The transducer is mounted in a watertight fixture with its radiating face directed along the horizontal and under the water surface. The fixture is physically attached to the X–Y plotter to obviate the inconvenience of mechanically referencing the transducer to the target for each experiment. Without a reference of this type, alignment becomes a problem in the three dimensional case since it requires that the plane in which the reflecting target travels be aligned with a perpendicular line emanating from the transducer face and that this alignment be repeatable. The entire assembly has the advantage of not requiring alignment to insure that the transducer and reflector are in the same transverse plane such that the locus of all lines perpendicular to the face of the transducer will be parallel to the plane in which the target moves. In addition, the transducer fixture itself can be raised and lowered to facilitate off-center measurements and the transducer can be rotated in the fixture to observe field asymmetry. Fig. 2 shows a closeup of this fixture.

A stainless-steel ball of about one-half inch in diameter is used to provide a self-aligning target. By choosing the diameter to be much greater than a wavelength, the range of relative maxima and minima in the near field is averaged [1] with the averaging "space constant" proportional to the ball diameter to a first order. The choice of the spherical ball as a reflecting target is not ideal, however, and although it possesses a self-aligning property it does bring three disadvantages.

Firstly, the reflected energy at any fixed position is a function of the reflector size. In particular, the energy received back at the transducer is proportional to the reflector's cross-sectional area. Secondly, the reflector material has a certain acoustic impedance and this implies a particular reflection and transmission coefficient. These two disadvantages are not serious drawbacks as the amount of error produced can be calculated *a priori* [2], [3].

The third problem is the most noteworthy of the three. Presume for the moment that all of the sound rays leave the transducer face in parallel. Then as the reflector is moved away from the transducer, the net surface that will reflect an acoustic ray at such an angle so as to impinge back upon the transducer varies as the square of the target distance. A similar effect exists as the ball moves transversely. However, the entire situation is complicated by the fact that the acoustic rays do not really emanate in a parallel fashion. If the direction in which all the rays were pointing was known, then a calculation could be made to correct for this r^2 type discrepancy. Although this effect might appear to consti-

[2] $G \triangleq \dfrac{\text{receiving transducer voltage output}}{\text{transmitting transducer power input}}$.

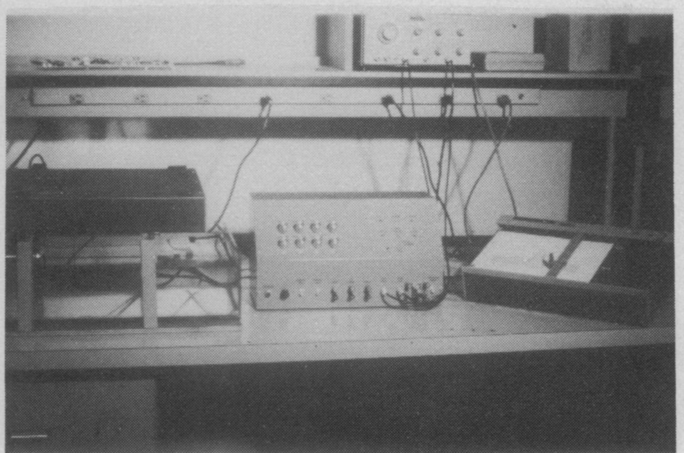


Fig. 1. Complete instrumentation system.

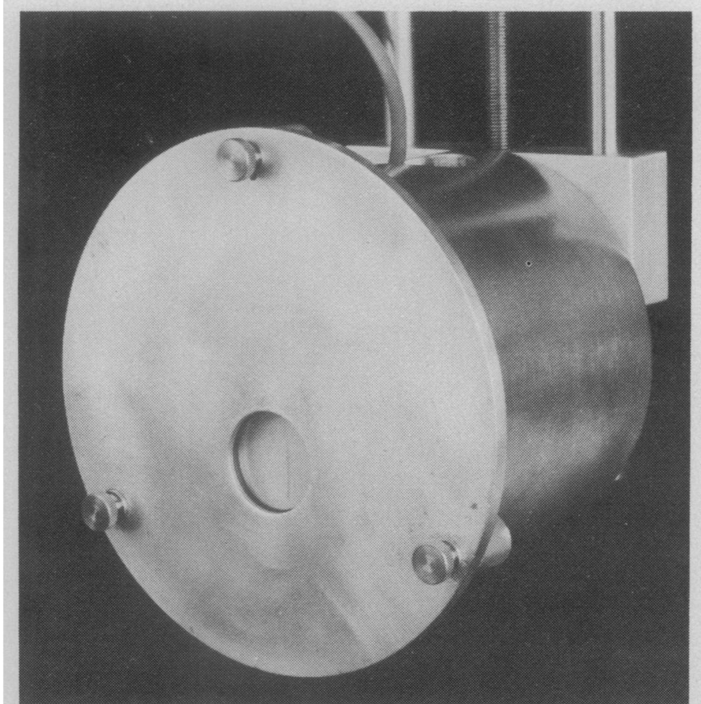


Fig. 2. Transducer fixture.

tute a strong disadvantage, it should be remembered that the intention is not to measure field intensity directly, but rather the transfer function of the total acoustic system. In this sense then, the steel ball is serving as a standardized reflector of a particular size and will, in this sense, give an indication of the response to be expected in the actual ultrasonic reflection system.

If it is desired to eliminate this distance effect, one must procure a target that presents a surface perpendicular to the acoustic beam at all points. For a parallel beam, this perfectly aligned flat surface. In practice, however, it appears difficult to obtain a parallel beam acoustic source and, in any case, cumbersome to align a flat surface perpendicularly to the beam at all points.

IV. FUNCTIONAL DESCRIPTION

A block diagram of the instrument is shown in Fig. 3. This diagram can perhaps most easily be digested by dividing it into three parts. The first part, the transmitting section, consists of an oscillator that is gated so as to drive the transmitting crystal with a sinusoidal voltage for a preset time denoted by "transmit burst width."

The second part, the receiving section, samples the reflected signal after a controlled delay (range delay) following the transmitted burst. This delay is set to correspond to the transit time of the ultrasound over the forward and return path from the transducer to reflector.

The signal that exists at the receiving amplifier's input is the result of a twice transduced signal that has been reflected from the steel ball. This signal is then peak detected during the period of the receive window (sample-and-hold gate width) and stored.

The third part, the recording section, utilizes two X–Y plotters. The first moves the reflector and the second plots the field characteristics. The X-sweep generator controls the arm of both X–Y plotters in parallel.

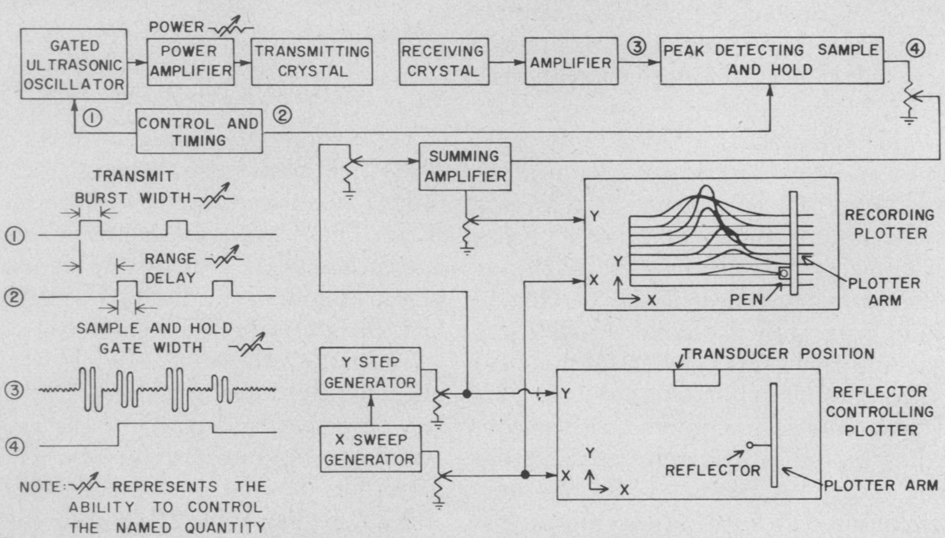
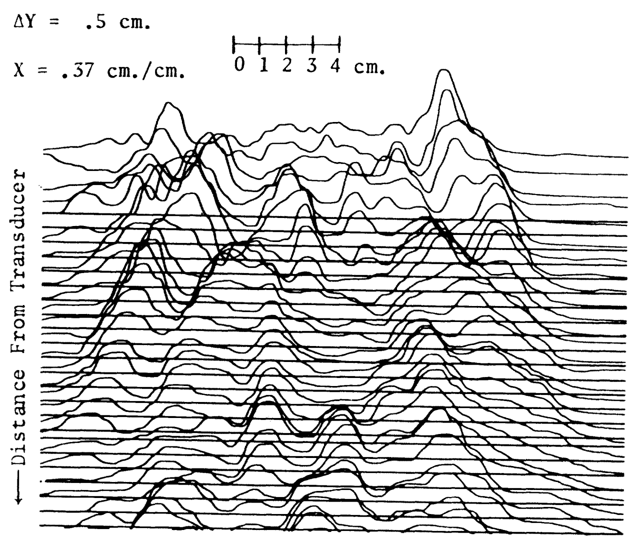
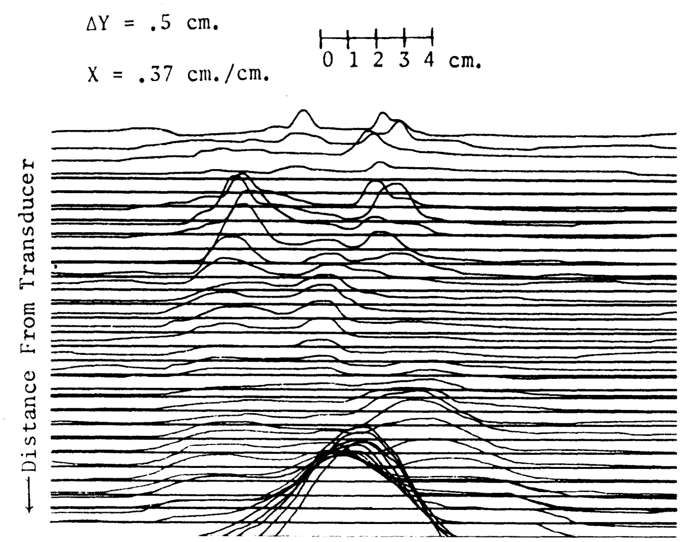


Fig. 3. Field pattern vizualization instrumentation.



A Sample Field Intensity Distribution for a Commercial Ultrasound Transducer

Fig. 4.



Field Intensity Distribution for a Ground Version of an Obstetrical Transducer

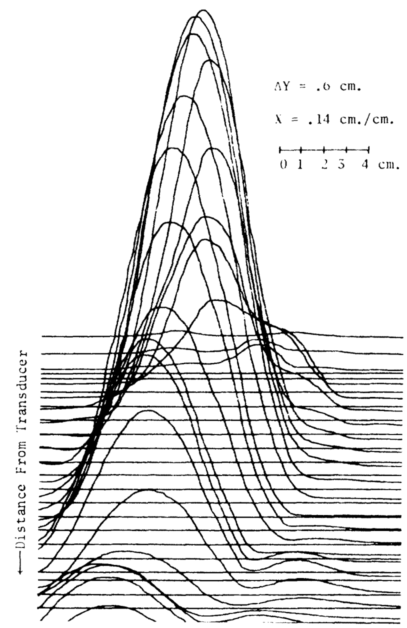
Fig. 5.

With the completion of each sweep the Y-step generator is incremented. This motion moves the ball along a line parallel to the face of the transducer and successively increases the perpendicular distance from the transducer to this line.

In the present design, a portion of the Y-step generator is added to the peak detected and held value from the receiving section and the sum is supplied to the Y axis of the recording plotter. In addition, calibration lines are drawn on alternate sweeps representing constant values of the incrementing Y position. These appear as horizontal grid lines on the "caricaturized" example shown on the X–Y recording plotter in this figure.

Fig. 4 shows a sample plot of G versus the two spatial coordinates in which the reflecting target moves. The X direction (horizontal) is easily calibrated since the length of a calibration line corresponds directly to the length of travel of the reflecting target. The Y direction (vertical) is also easily calibrated in a similar fashion and is represented as the spacing between the calibration lines. The third dimension G, representing intensity, should appear to come out of the paper perpendicularly (as though a real Z axis existed).

One of the several advantages of these calibration lines is that G can be measured on the basis of the spacing of the calibration lines which is known. That is, G is produced as a level that adds in the summing amplifier (see Fig. 3) and by knowing the gain constants of the receiving section, the correspondence of the spacing to this third dimension is trivially available. In addition, notice that the signals themselves going to the field plotter have an inherent self-calibrating feature. This makes it a relatively simple matter to readjust the controls of the recording plotter if it is required for other purposes. This has, in fact, occurred several times during the course of experiments with no significant discrepancy in the results.



Field Intensity Distribution for the Lens/Transducer Combination

Fig. 6.

V. APPLICATION OF THE INSTRUMENT

Fig. 4 shows results taken from a commercial Doppler obstetrical transducer. It consists of two D-shaped disks of lead zirconate titanate with a resonant frequency of 2 MHz. These two crystals are mounted in a plastic case and protected by an epoxy window. The important points of note in this figure are the multiplicity of peaks and the relatively wide field pattern.

Since the protective window was visibly rough and also slightly domed, the effect of making this surface flat was investigated. This was accomplished using a grinding procedure with increasingly finer grades of carborundum powder placed between the transducer face and a flat glass slab. The resultant field characteristics are shown in Fig. 5. Of note here is the spatial narrowing

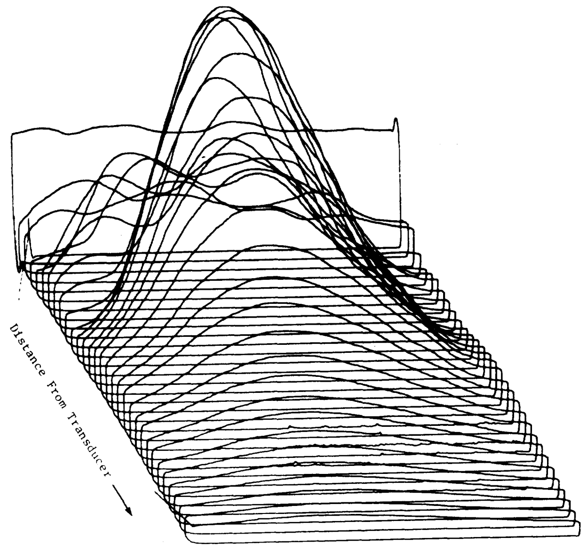


Fig. 7.

of the beam compared to Fig. 4 and also the two near field lobes merging into one in the far field.

Ideally, the beamshape could be made more ideal by increasing the acoustic power received without increasing electrical power drive. This requires narrowing of the beam so that most of the energy exists in the desired area. Equivalently, this can be considered as taking energy from the sidelobes and moving it to the central part of the beam. In addition, it would be useful to have the resultant high sensitivity of the beam occur at a specified depth. Conceptually, it can be seen that an acoustic lens which focuses the energy is the device that is required.

Plexiglas is chosen for the lens material as a good compromise among the three important acoustic properties: speed of sound different from that in water, most fluids and body tissue, low attenuation to ultrasound, and minimum acoustic impedance mismatch between transducer, lens, and water, most fluids, or body tissue. In addition, Plexiglas also possesses three very practical properties: favorable cost, ready availability, and ease of fabrication.

A 10-cm focal-length lens was fabricated using a hardened radius arm on a lathe. The resultant G pattern is shown in Fig. 6. A factor of approximately ten in G gain was obtained over the original transducer. More importantly, however, by focusing most of the acoustic energy into a smaller volume, a narrowing of the beam is obtained. The practical consequence of this is reduced spatial interference and subsequently a higher signal to noise ratio at the input to the receiver.

As it evolved, a high degree of beam narrowing was obtained with acoustic lenses. In fact, so much so that it became difficult to obtain quantitative information from the section of the plot where there are overlapping lines. This difficulty was easily overcome with a slight modification of the pseudo-three-dimensional plot.

A straightforward false perspective method has been implemented. It maintains the calibratability and simply adds some of the Y-step generator's output to the X axis by means of an additional summer. This provides a plot similar to the previous ones shown but with the distinction that with each Y increment, the X axis is shifted by a small amount to one side. Fig. 7 illustrates the false perspective solution. Notice that the calibratable feature is maintained in this format and the G lines can be arranged to appear distinctly.

VI. CONCLUSIONS

An instrumentation scheme which allows the visualization of ultrasound beam patterns has been developed. It is both easy to use and relatively inexpensive. This instrument has played a valuable part in two distinctly different but somewhat related fields. In one case, it was used to optimize the active field of an industrial ultrasonic flow meter. In the other, it is currently used to optimize medical instrumentation intended for reliable, dynamic, fetal cardiac monitoring.

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