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A Simple Data Reduction Scheme For Additive Synthesis

Introduction

Additive synthesis is a powerful sound generation technique because the frequency relationships and spectral characteristics of musical tones so constructed can be precisely controlled. A major problem with additive synthesis, however, is the large amount of data required for each tone. This large data requirement is one reason why additive synthesis has never been very popular, and why synthesis techniques such as frequency modulation (FM) (Chowning 1973) are extensively used.

In FM, one wave frequency modulates another. The frequencies of both the modulating wave and the carrier wave are chosen so that the generated side-bands can be used to create a spectrum. The spectrum of the tone can be controlled by a single parameter—the index of modulation. Varying the index of modulation varies the timbre of the tone.

Sinusoidal Synthesis

In general, sinusoidal synthesis requires two shapes for each partial, an amplitude envelope and a frequency contour. In 1967, Freedman introduced an additive synthesis technique that reduces the shape information to about 100 coefficients. With this technique a description of amplitude envelopes and pitch contours is provided, and a table of coefficients is used to generate the envelopes. A problem with this representation is that one set of coefficients produces a tone of one timbre, one loudness, and one pitch. A completely new set of coefficients is required when any one attribute is changed. A refinement of this technique employs envelopes defined by "straight-line" approximations (Moorer, Grey, and Strawn 1978; Wessel 1979). Even with this representation, however, the data requirements remain high.

The additive synthesis technique proposed in this paper also employs a table of data, but the data here are treated differently. The table contains a collection of data organised as a file of spectra. An index into this file selects a spectrum for each tone. Varying the index varies the spectrum. Thus the spectrum of the tone can be made to vary over the duration of the tone by attaching an envelope to the index. This index is equivalent to the index of modulation in FM; indeed, the idea of using an index was derived from FM. There remains one last problem: shapes are continuous and files are discrete. Linear interpolation is used to resolve this difficulty.

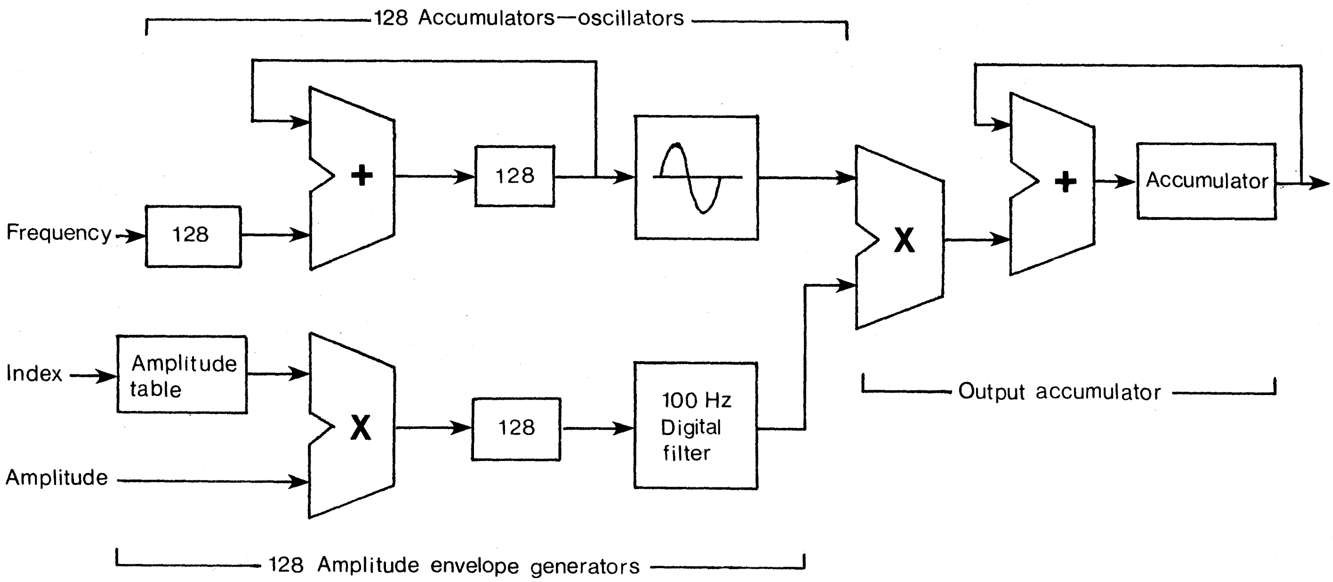
The sinusoidal synthesis technique employs two files: one of spectra v, and one of partial structures p (a partial structure is a set of frequency ratios: the ratio of each partial to the lowest frequency), and four shapes; amplitude $a(t)$, frequency $f(t)$, spectral index $k(t)$, and partial structure index $l(t)$. The output wave of the oscillator $s(t)$ can be represented as:

$$s(t) = a(t)\cdot\sum_{i=1}^{n} v_i(k(t)) \sin (2\pi P_i(l(t))f(t) \cdot t)$$

where: $v_i(k)$ is the ith amplitude of the kth spectrum,
$p_i(l)$ is the ith frequency ratio of the lth partial structure.

Each new sound requires only a new set of shapes. Thus this representation of sinusoidal synthesis is highly economical in its data requirements.

Fig. 1. The oscillator configuration. An accumulator-oscillator generates a ramp, which is converted to a sine wave. Amplitude envelopes are generated from filtered, amplitude-scaled spectra originating from the spectral tables. Frequency tables reside in main memory and are interpolated by software. The output sample is collected in an output accumulator.



A Digital Oscillator Design Based on Additive Synthesis

The sinusoidal synthesis technique can be implemented using digital techniques. An oscillator could comprise 128 sinusoidal oscillators internally grouped to form four oscillators of 32 partials each. Chamberlain (1976) and Snell (1977) have designed digital Fourier synthesis oscillators. Chamberlain's design employs no data reduction; the host system is therefore required to provide all amplitude and phase information. Chamberlain's oscillator offers 32 partials and controls the pitch by varying the sampling rate. Snell's oscillator offers up to 256 partials, employing the uniform sampling theorem. The aforementioned data reduction scheme of using many straight-line segments to approximate each envelope is used. At the end of his article, Snell mentions the technique of switching between different envelope tables; however, he provides no details.

A configuration for our oscillator is shown in Fig. 1. The configuration employs an accumulator-oscillator, which takes frequency-control information and performs a modulo-addition, producing a ramp function. The ramp is converted to a sine wave using a look-up table. An index $k(t)$ to the spectral table provides a set of 32 spectral amplitudes that are scaled by the input-amplitude level. The resulting amplitude functions are smoothed using a 100 Hz first-order digital filter (low-pass). The sine wave is scaled by the amplitude and is passed to an output accumulator that collects samples of the 32 partials, generating a sample of the output wave.

The total complement of 128 oscillators can be accommodated by time-division-multiplexing, serviced by 128-stage shift registers. The speed requirements for the oscillator module are awesome, but not impossible to attain. A sample rate of 50 kHz results in a pipeline time slice of 156 nsec. However, by employing sufficient levels of pipelining, the oscillator can be constructed using available technology.

Conclusions

Additive synthesis offers many advantages for sound synthesis; however, its high data requirements have limited its application. A sinusoidal synthesis technique has been introduced that combines the power of additive synthesis with the convenience of FM. This level of convenience is achieved with only a small sacrifice in flexibility. Thus far the synthesis technique has been realized only in software, but it has been demonstrated to be realizable in digital hardware.

References

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